SISE 9128: Introduction to Machine Learning

Alex Smola, RSISE ANU
Problem Sheet — Week 2

Teaching Period
October 8-19, 2001

The due date for these problems is Thursday, October 18

Problem 8 (Linear Programs and ε-insensitive Loss)
Assume we have the following loss function

$$c(\mathbf{x}, y, f(\mathbf{x})) = |y - f(\mathbf{x})|_\varepsilon \text{ where } |\xi|_\varepsilon := \begin{cases} 0 & \text{if } |\xi| \leq \varepsilon \\ \xi - \varepsilon & \text{if } \xi > \varepsilon \\ -\xi - \varepsilon & \text{otherwise} \end{cases}$$

1. *Rewrite $|\xi|_\varepsilon$ as a linear optimization problem (analogous to the rewrite of $|\xi|$ which was discussed in the lecture).* **Hint:** *all you need to do is modify the constraints.*

2. *Rewrite the regularized risk functional for a linear model $f(x) = \langle \mathbf{w}, \mathbf{x} \rangle$. as a quadratic optimization problem with constraints.* **Hint:** *you only need to take care of the empirical risk term.*

   Recall that the regularized risk is given by

   $$R_{\text{reg}}[f] = \frac{1}{m} \sum_{i=1}^{m} |y_i - f(\mathbf{x}_i)|_\varepsilon + \frac{\lambda}{2} \|\mathbf{w}\|^2$$

Problem 9 (Prior Probabilities)
Assume a prior probability $p(f) = c \exp(-\frac{1}{2}(\|f\|^2 + \|f'\|^2))$ on $[0, 2\pi]$ for suitably chosen c.

1. *For the class of functions $\mathcal{F}$ given by*

   $$\mathcal{F} := \{f | f(x) = \alpha_0 + \alpha_1 \cos x + \beta_1 \sin x\}$$

   with $\alpha_i, \beta_i \in \mathbb{R}$ compute the normalization constant such that $\int_{\mathcal{F}} p(f) df = 1$.

2. *Now assume the class*

   $$\mathcal{F} := \left\{ f | f(x) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \cos(ix) + \beta_i \sin(ix) \right\}.$$

   What is the value of the normalization constant in this case. Rewrite $p(f)$ directly in terms of the coefficients α_i and β_i.

3. *Consider the series $f_n := \sin x + \frac{1}{n} \sin nx$. Show that the series f_n converges to $\sin x$ for $n \to \infty$, yet that $p(f_n)$ does not converge to $p(\sin x)$.*

4. **Bonus question:** *Interpret the previous result.*