Graphical Models II

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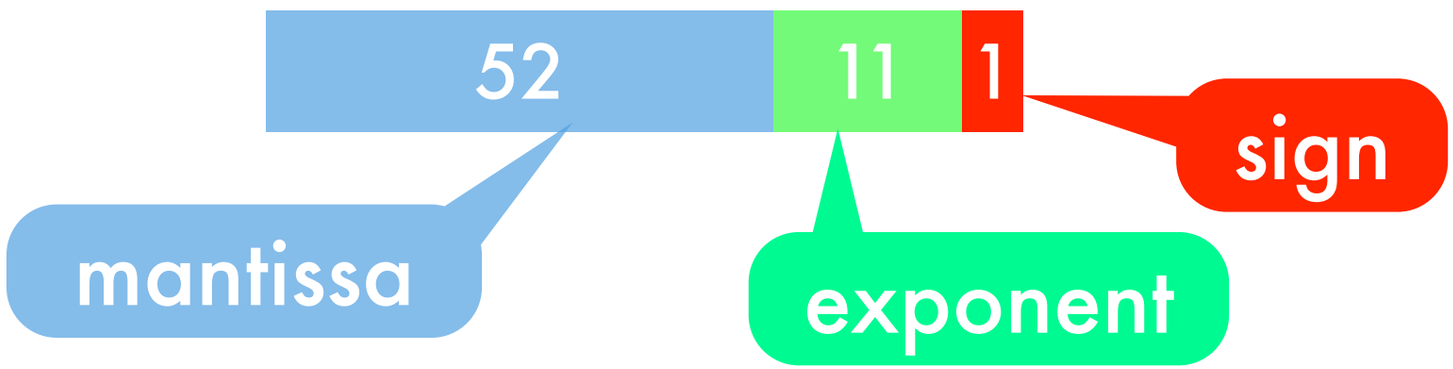
Issues with Probabilities

Logarithms are good

- Floating point numbers



$$\pi = \log p$$

- Probabilities can be very small. In particular products of many probabilities. Underflow!
- Store data in mantissa, not exponent

$$\prod_i p_i \rightarrow \sum_i \pi_i \qquad \sum_i p_i \rightarrow \max \pi + \log \sum_i \exp\left[\pi_i - \max \pi\right]$$

- Known bug e.g. in Mahout Dirichlet clustering

Zoology
What, How, Why

YAHOO!®

Models

Statistics

YAHOO!®

Inference Methods

Efficient Computation

Models

l1, l2 Priors

Conjugate Prior

Statistics

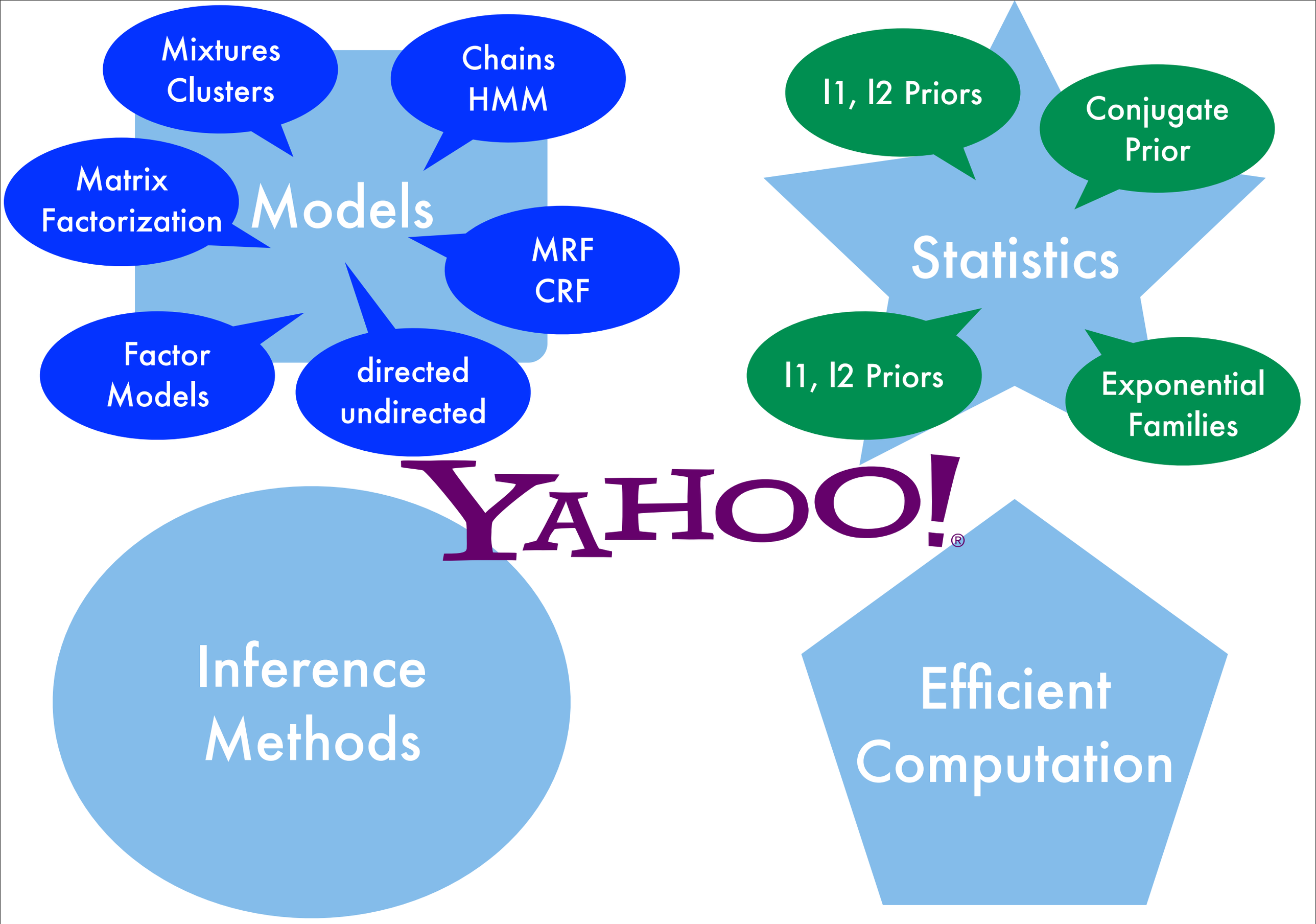
l1, l2 Priors

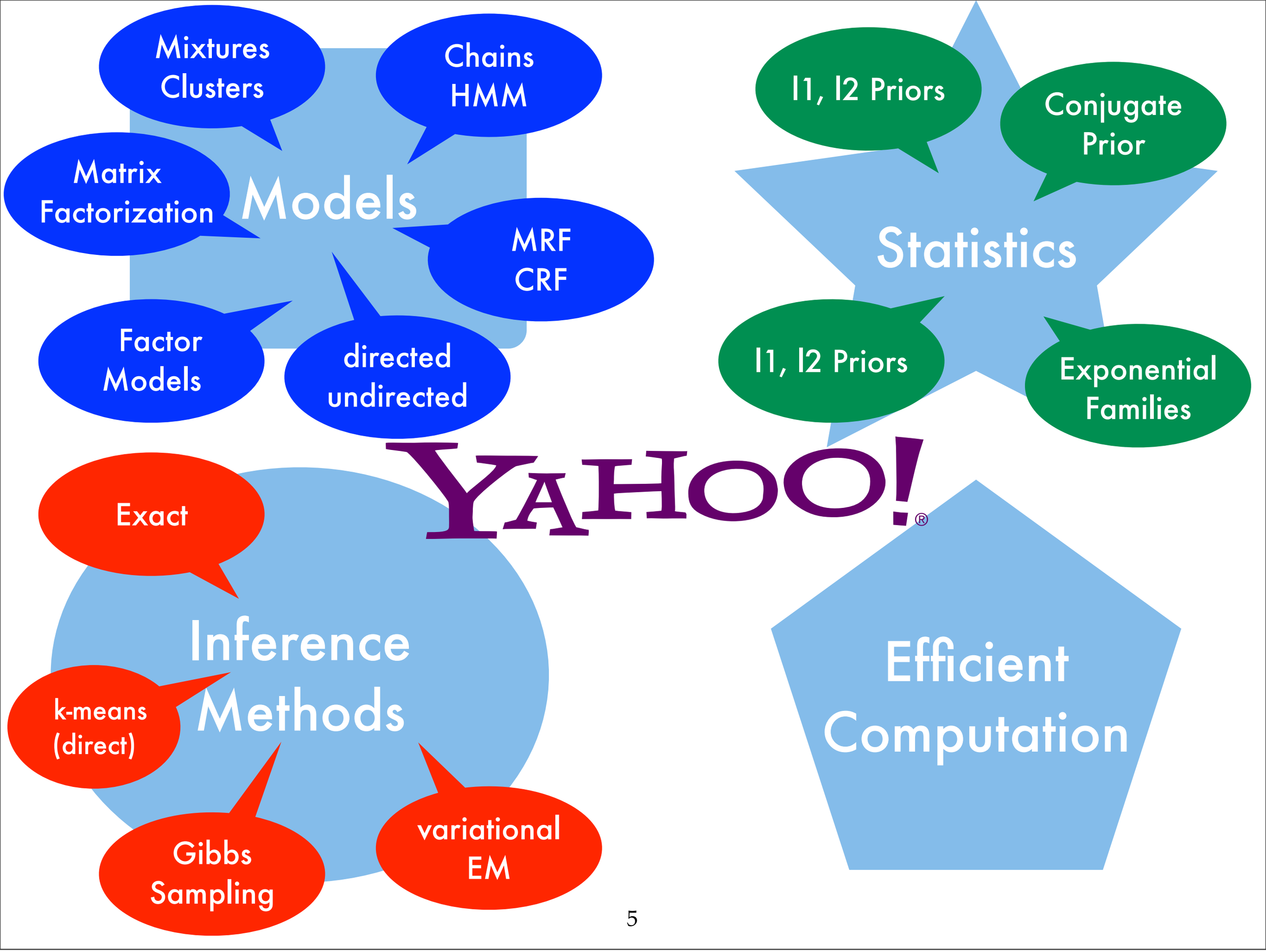
Exponential Families

YAHOO!®

Inference Methods

Efficient Computation


Mixtures
Clusters
Chains
HMM
Matrix
Factorization
Models
MRF
CRF
Factor
Models
directed
undirected
l1, l2 Priors
Conjugate
Prior
Statistics
l1, l2 Priors
Exponential
Families
YAHOO!
Inference
Methods
Efficient
Computation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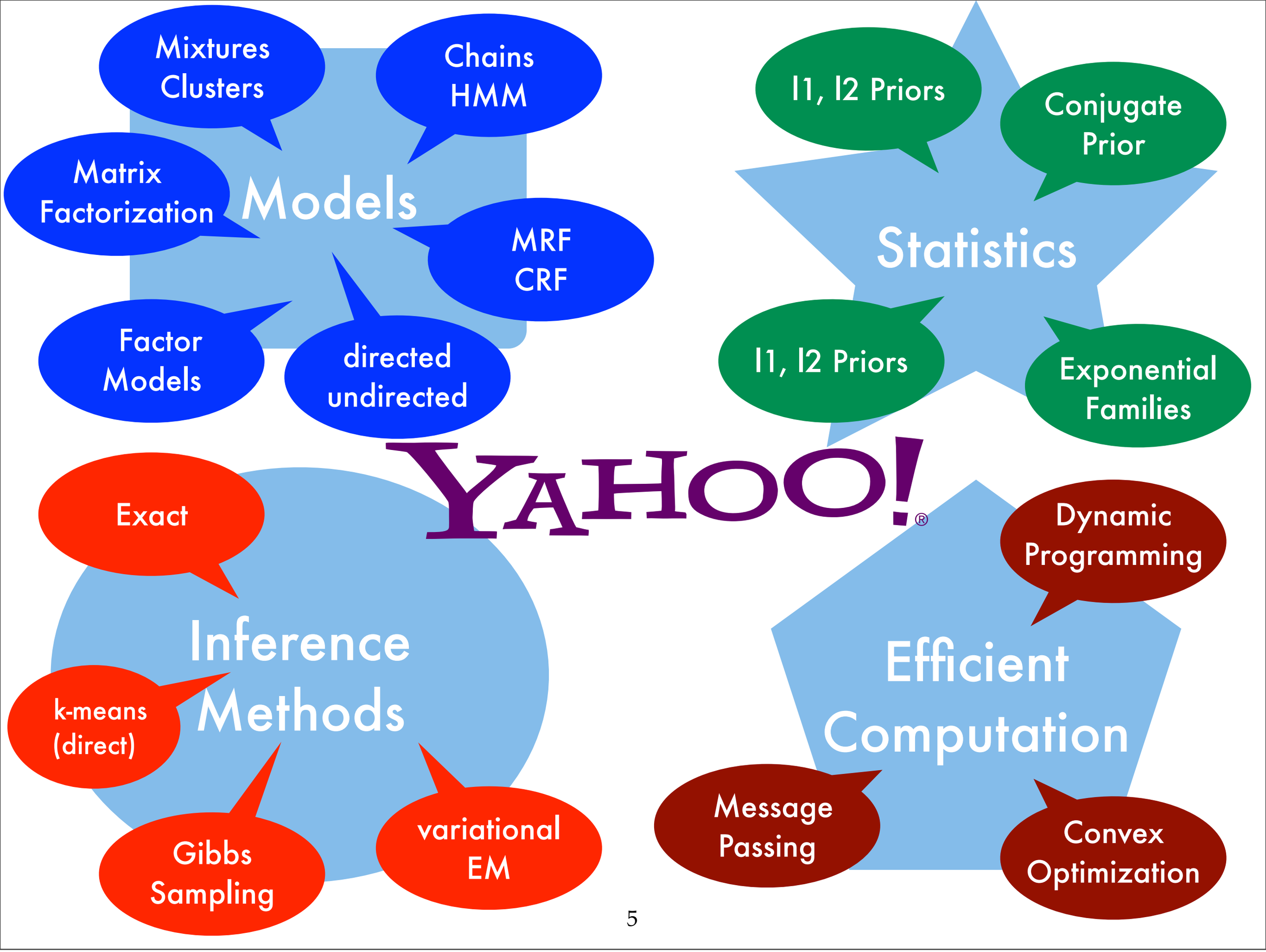
YAHOO!
Exact
Inference
Methods
k-means
(direct)
Gibbs
Sampling
variational
EM
Efficient
Computation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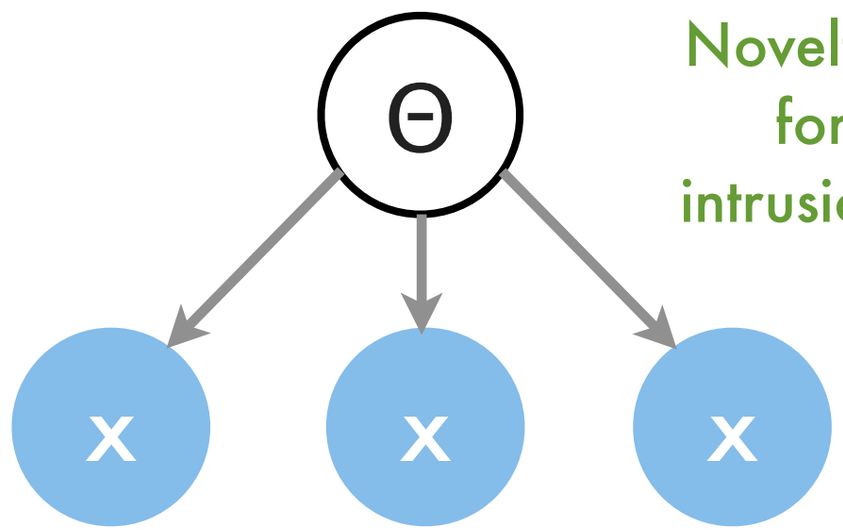
Efficient
Computation
Message
Passing
Convex
Optimization


Spam Filtering
Classification
Exploration
Segmentation
Annotation
YAHOO!®
Prediction (time series)
Clustering
Novelty Detection
Document Understanding
User Modeling
Advertising

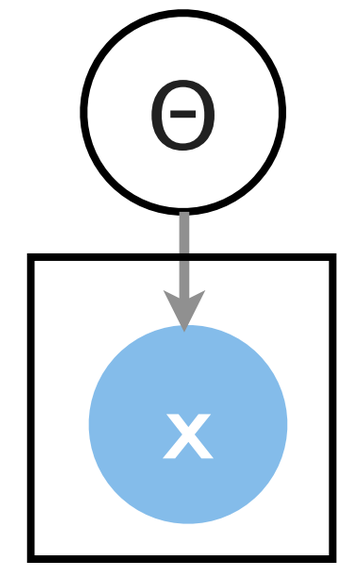

Spam Filtering
Classification
Exploration
Segmentation
Annotation
System Design
YAHOO!®
Prediction (time series)
Clustering
Novelty Detection
Document Understanding
Advertising
User Modeling
Debugging
Performance Tuning

‘Unsupervised’ Models

Density Estimation



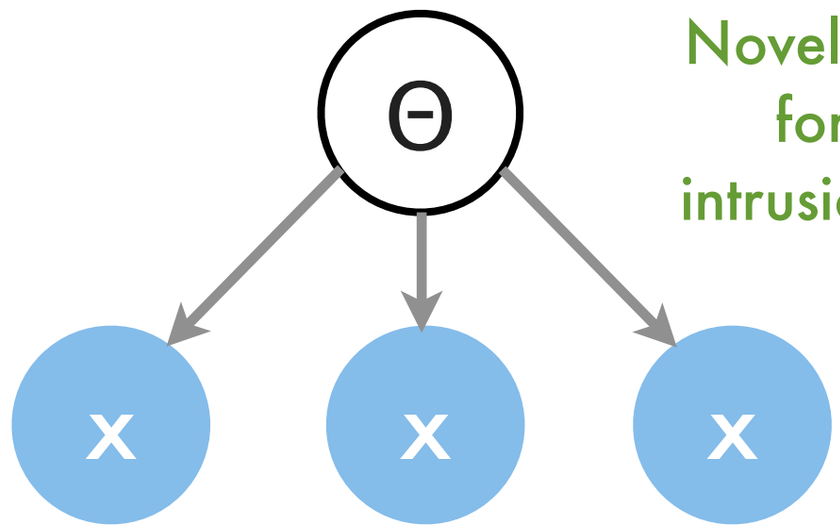
Novelty Detection
forecasting
intrusion detection



webpages
news
users
ads
queries
images

‘Unsupervised’ Models

Density Estimation



Novelty Detection
forecasting
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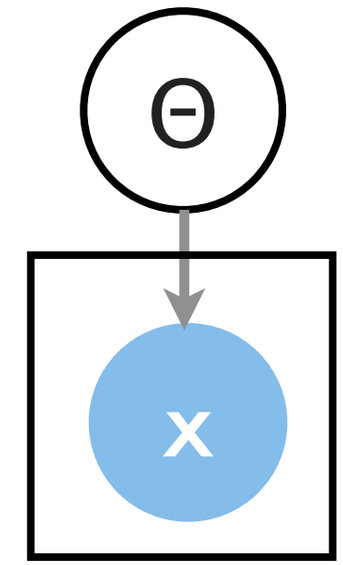
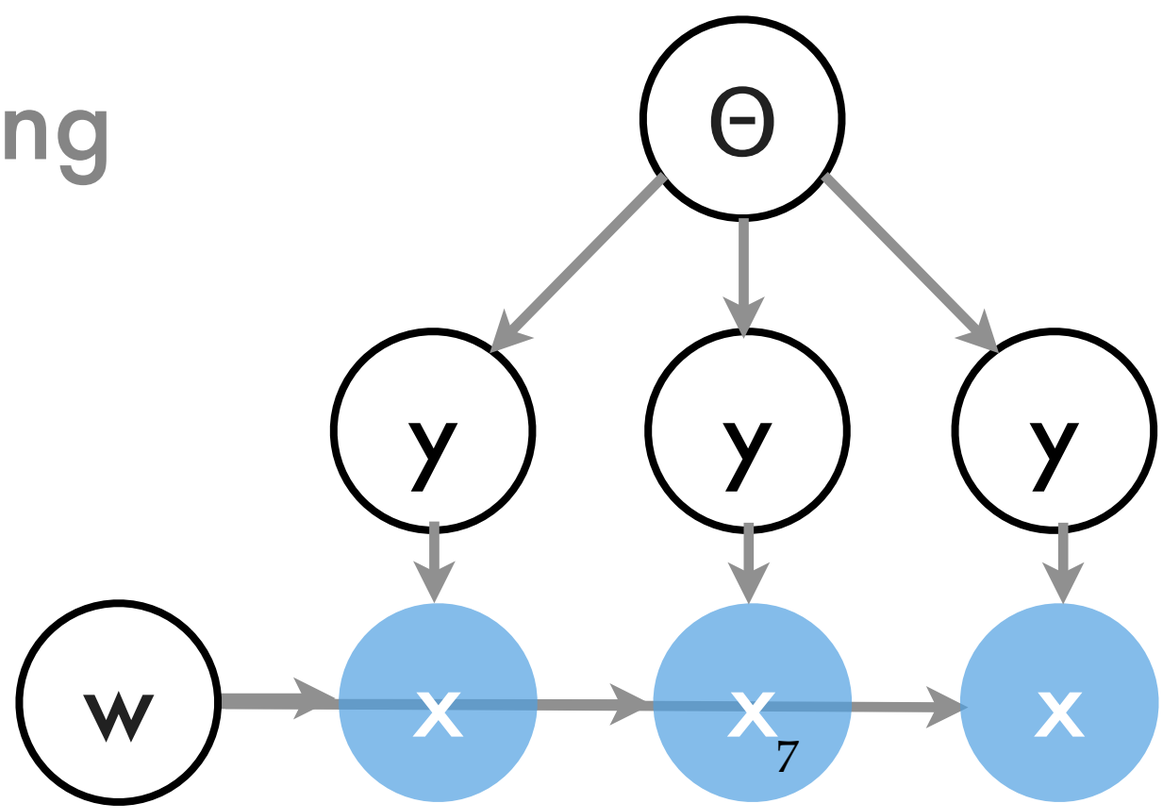
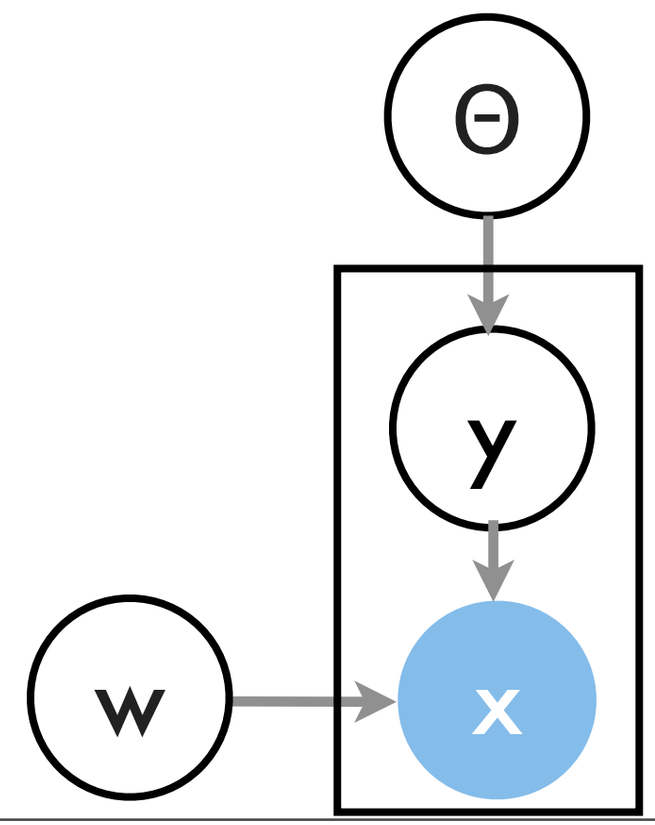


Clustering

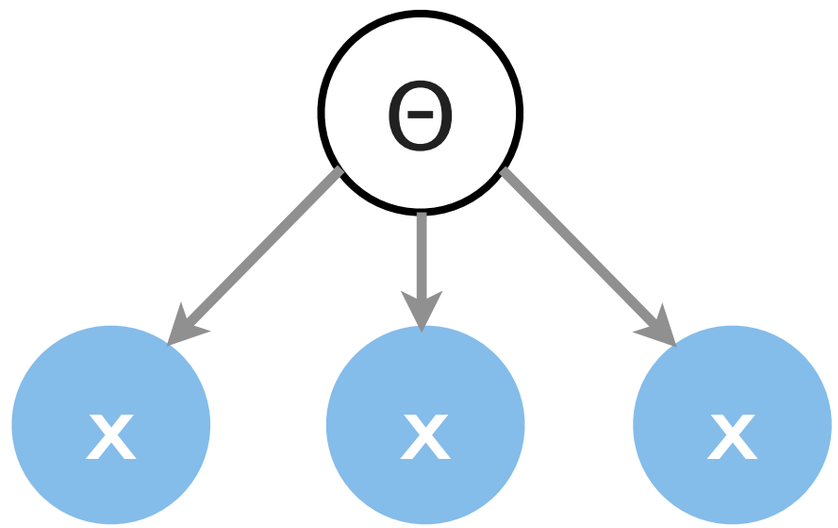
webpages
news
users
ads
queries
images

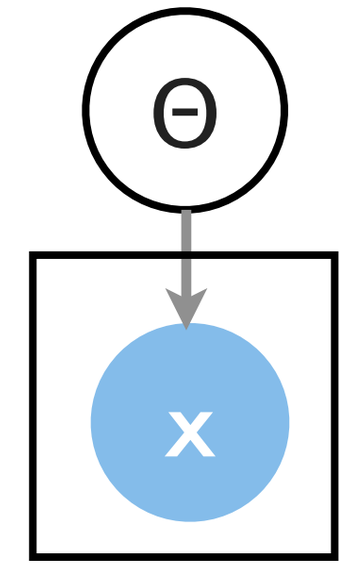




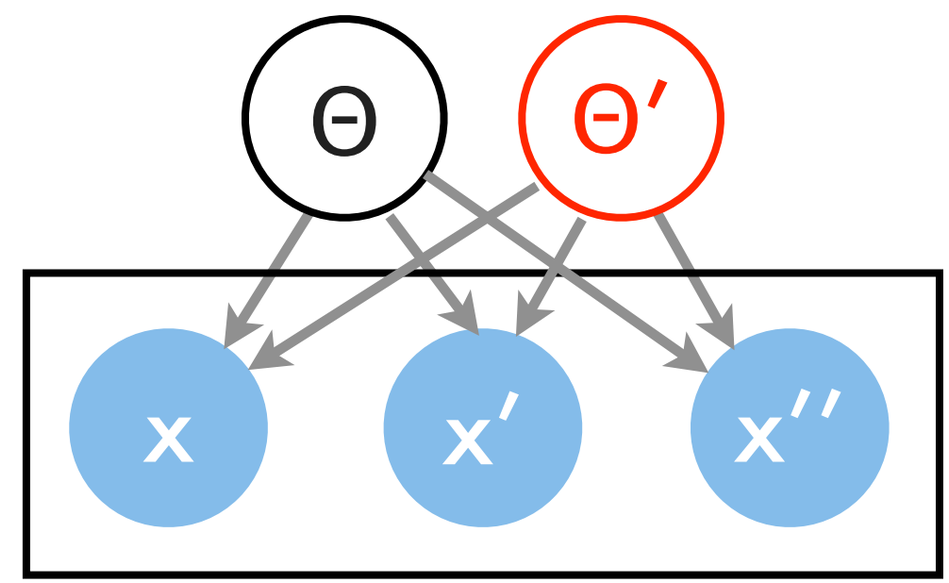
‘Unsupervised’ Models

Density Estimation





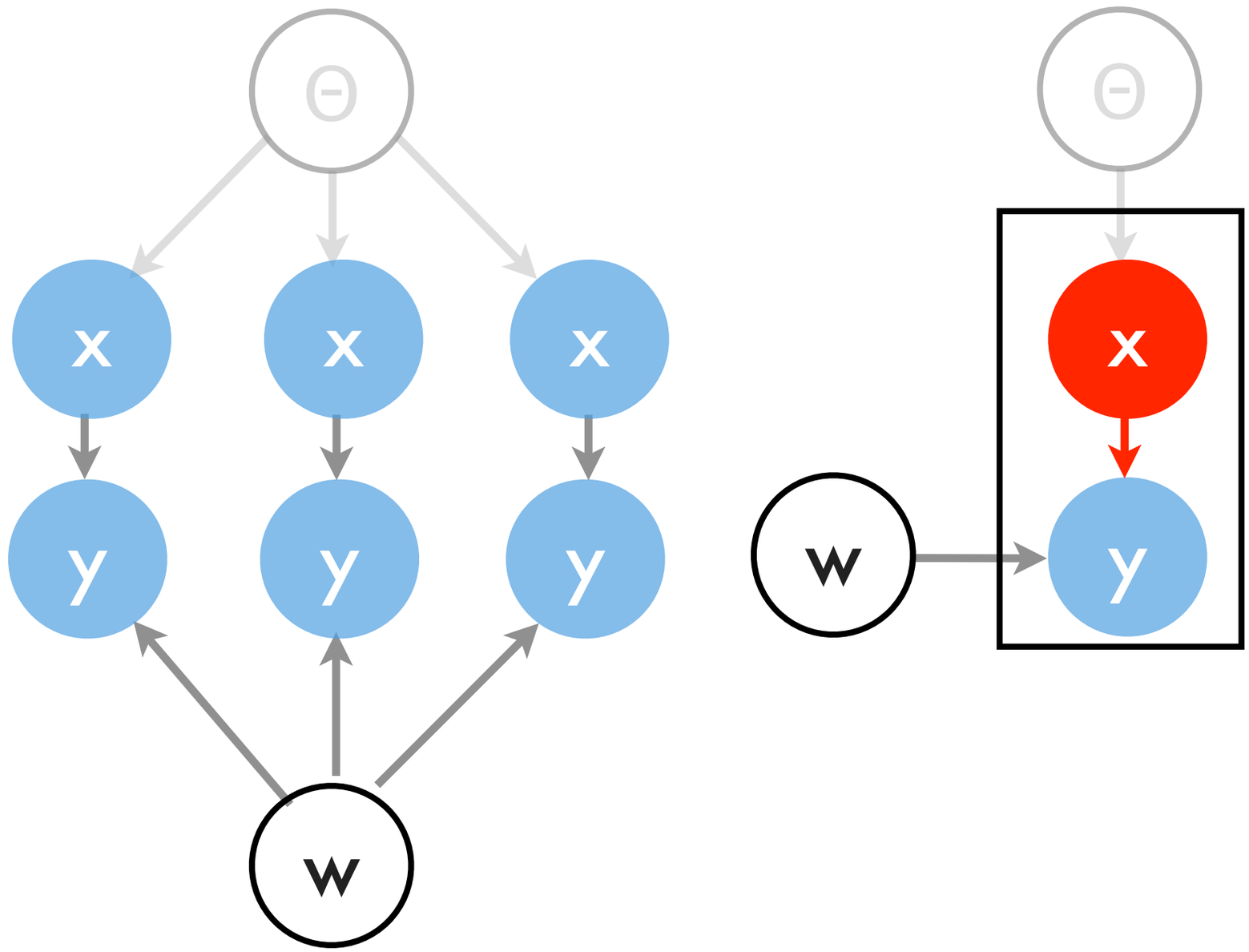
Factor Analysis



‘Supervised’ Models

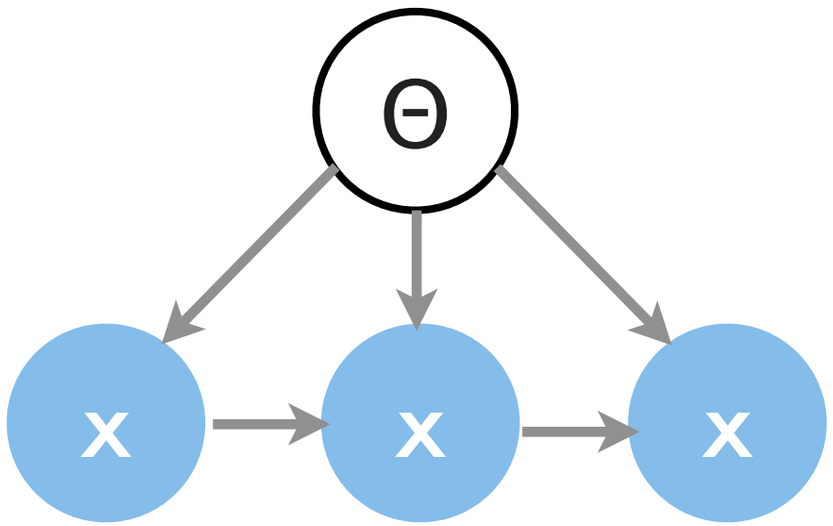
Classification
Regression

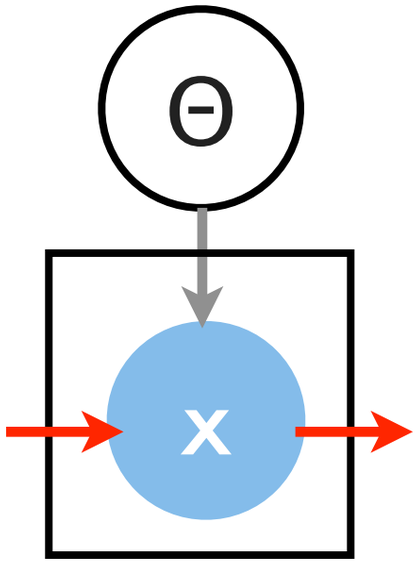
spam filtering
tiering
crawling
categorization
bid estimation
tagging



Chains

Markov Chain





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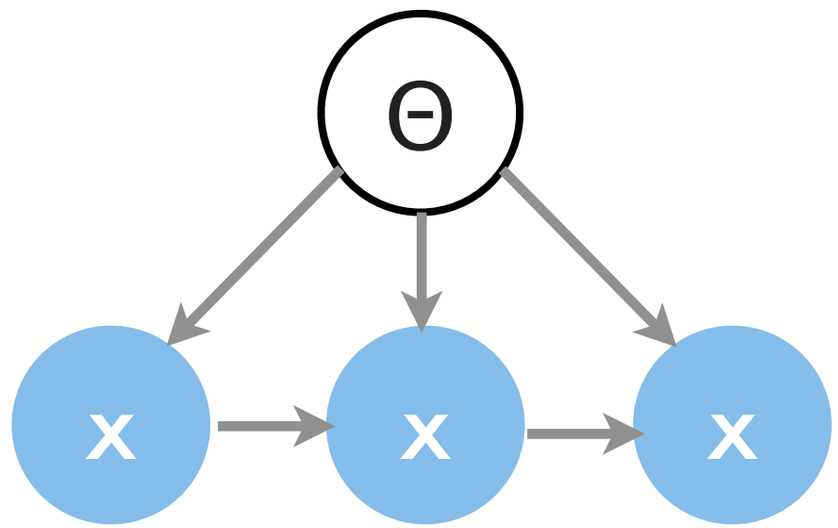
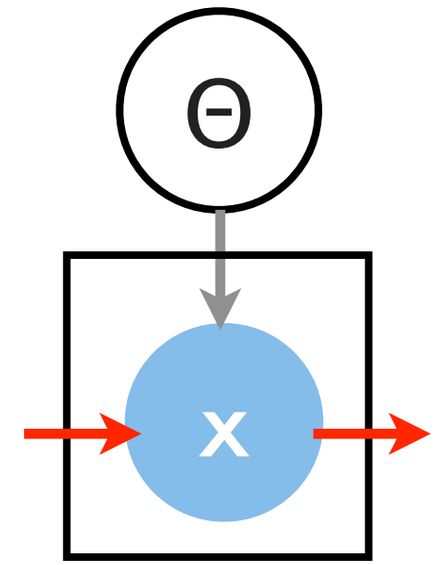
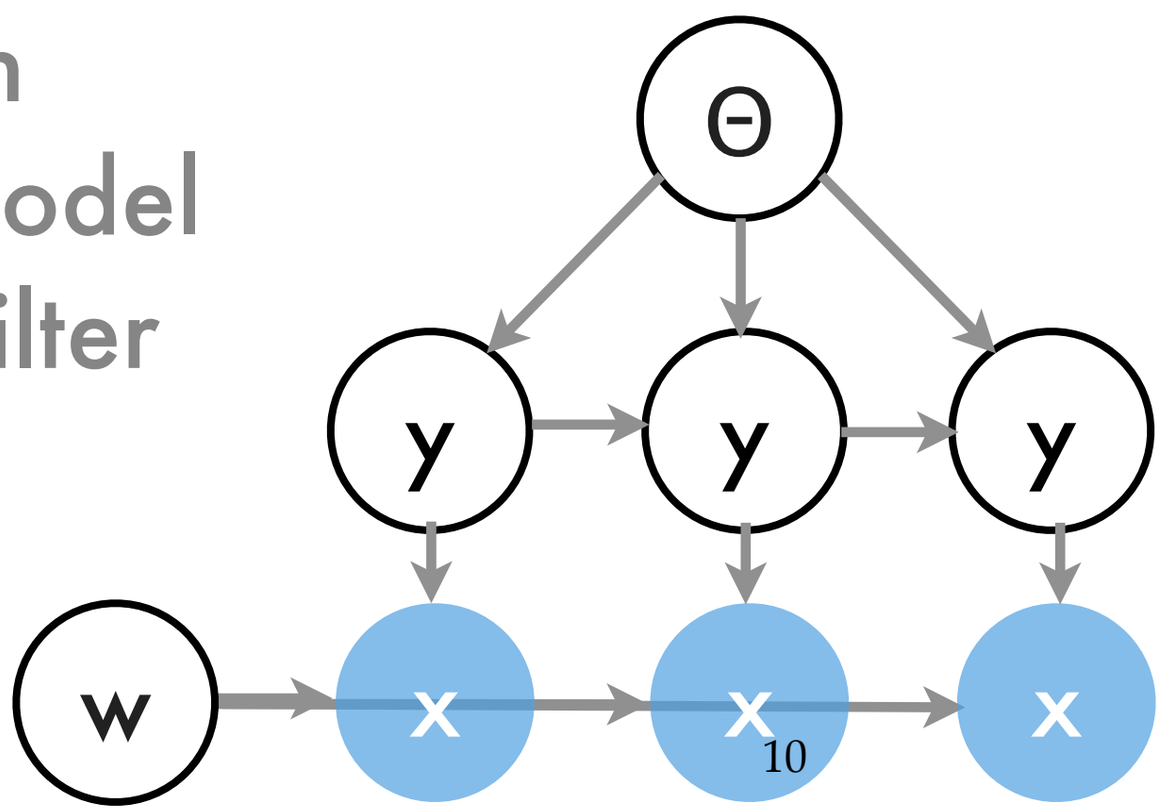
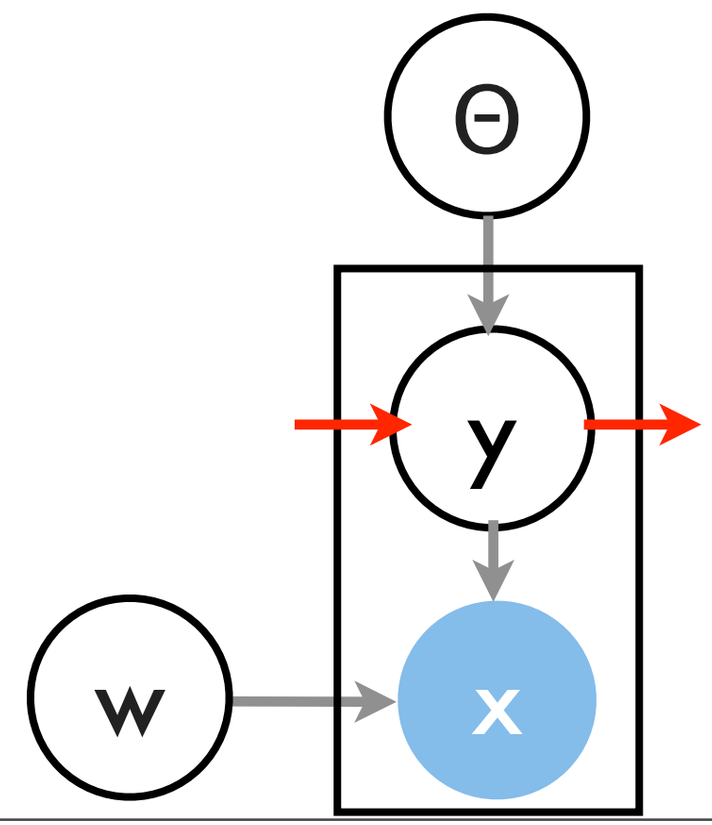




Hidden Markov Model
Kalman Filter

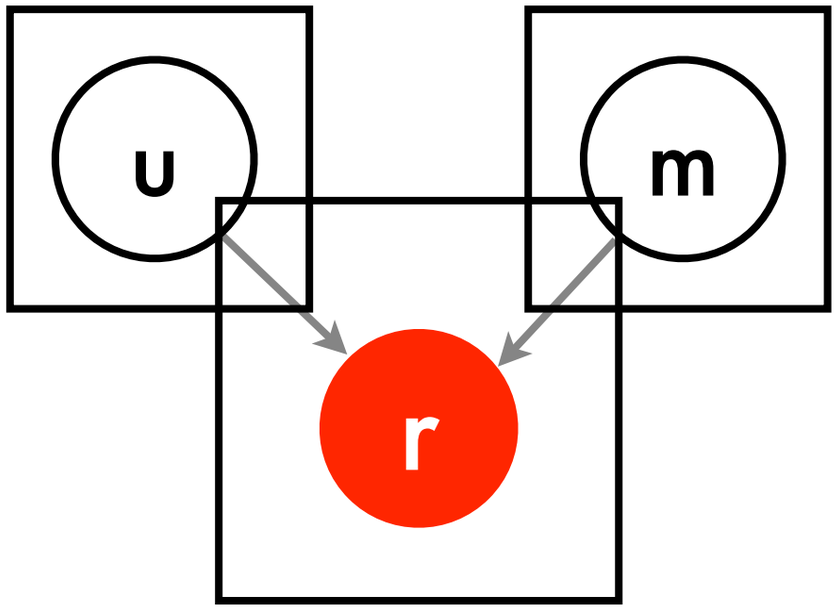




Collaborative Models

Collaborative Models

Collaborative Filtering



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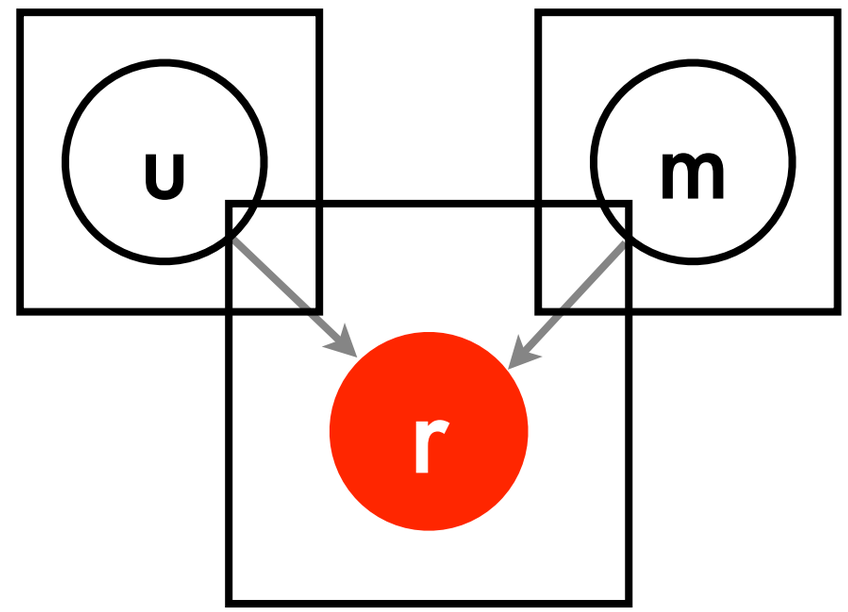
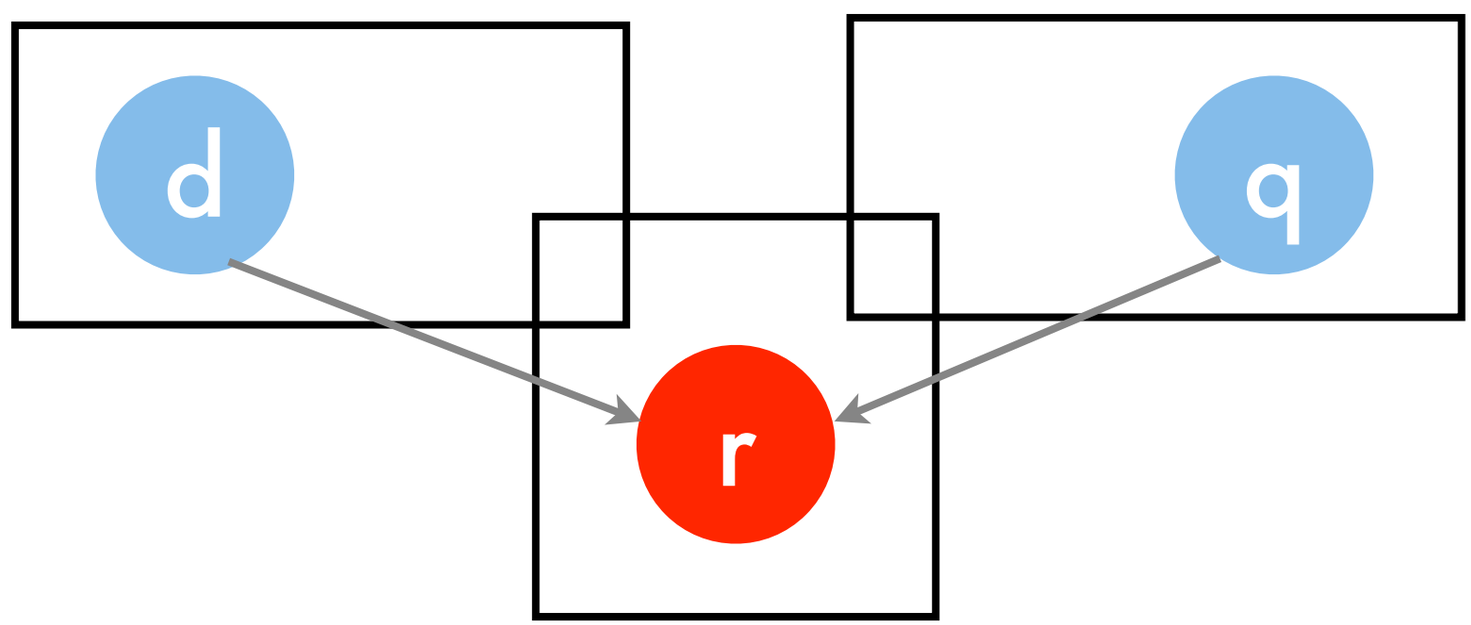


Current Webpage Ranking



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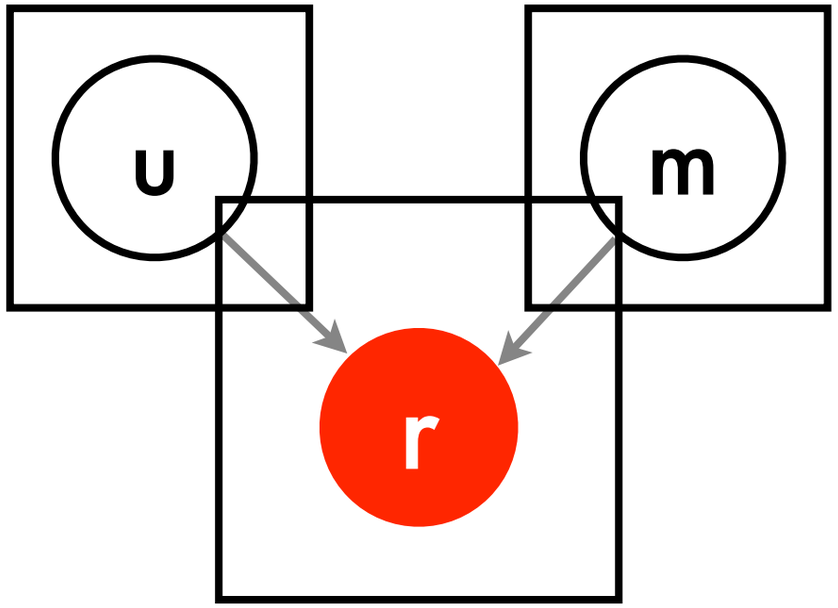

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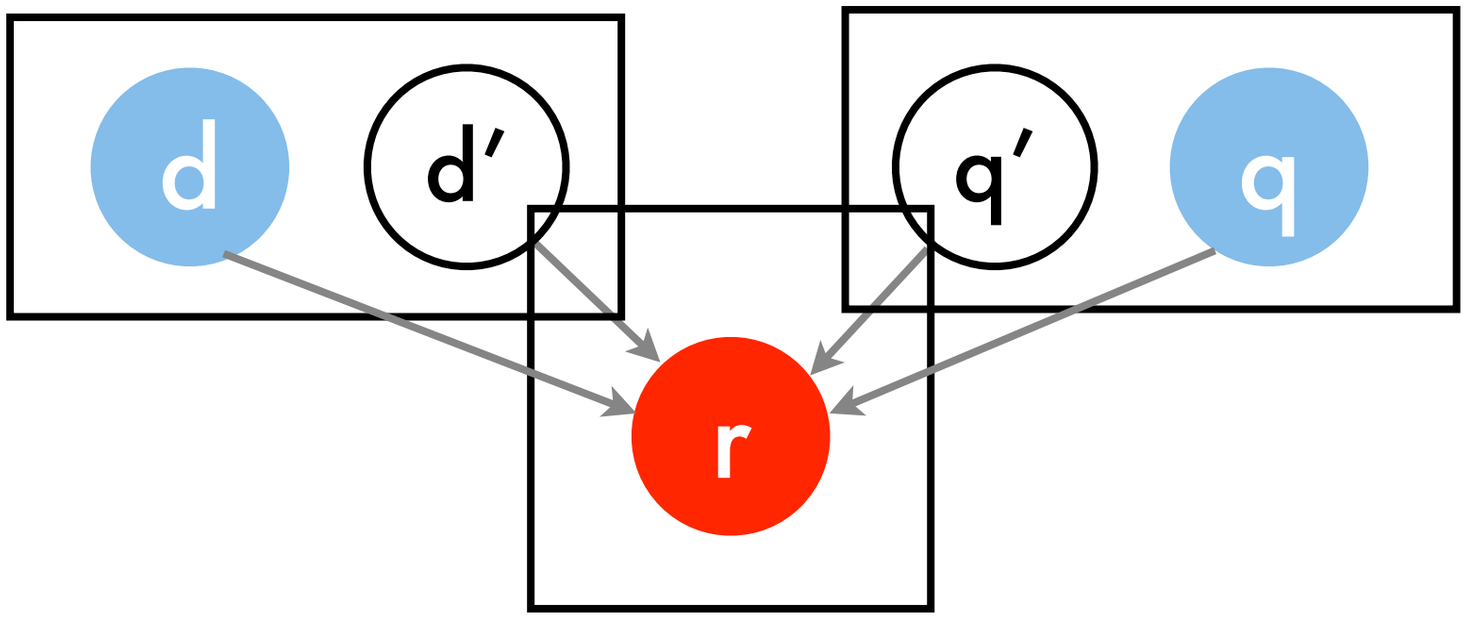

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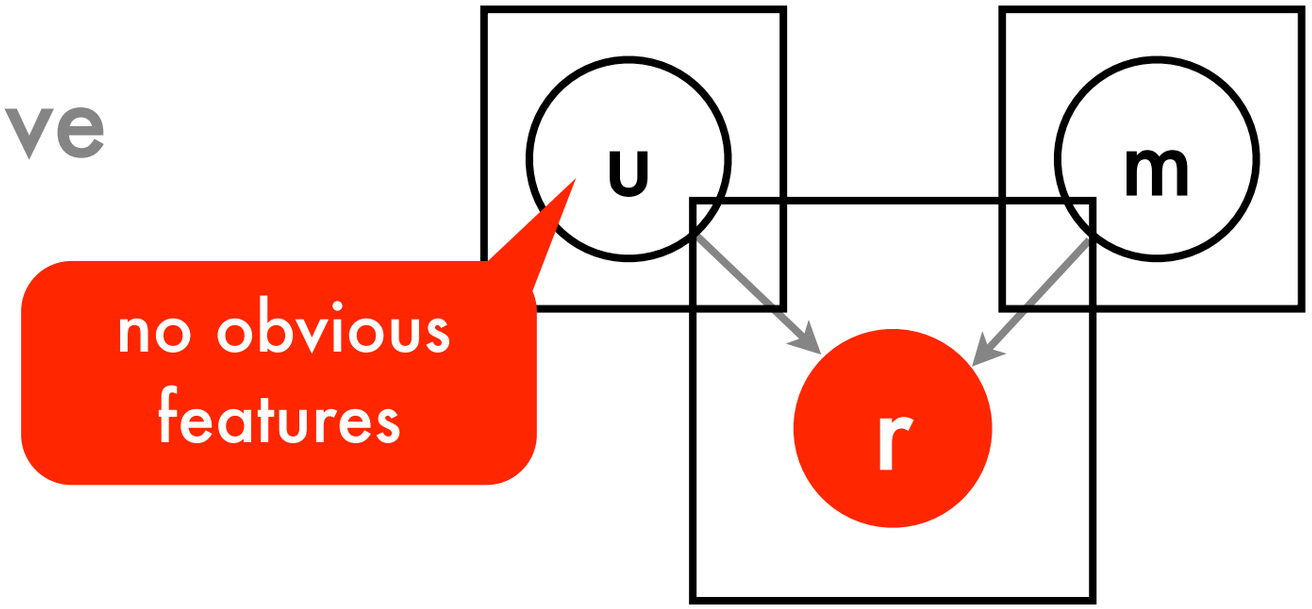

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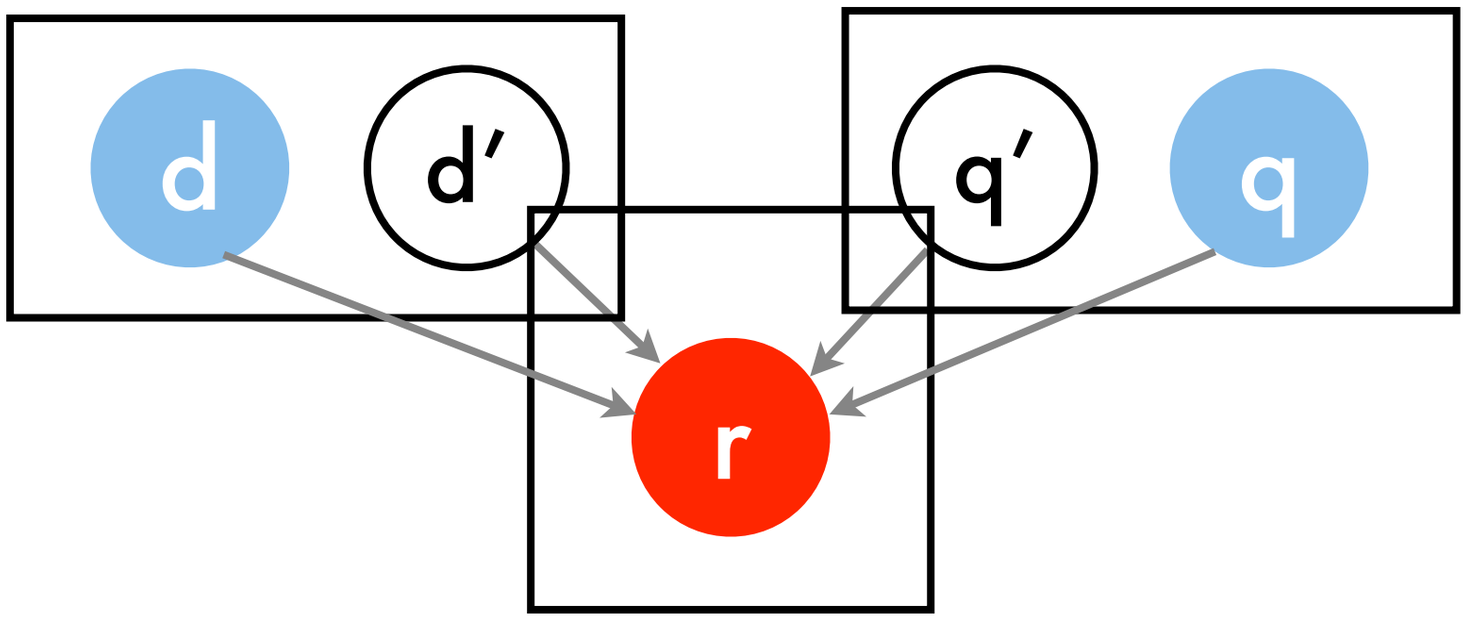

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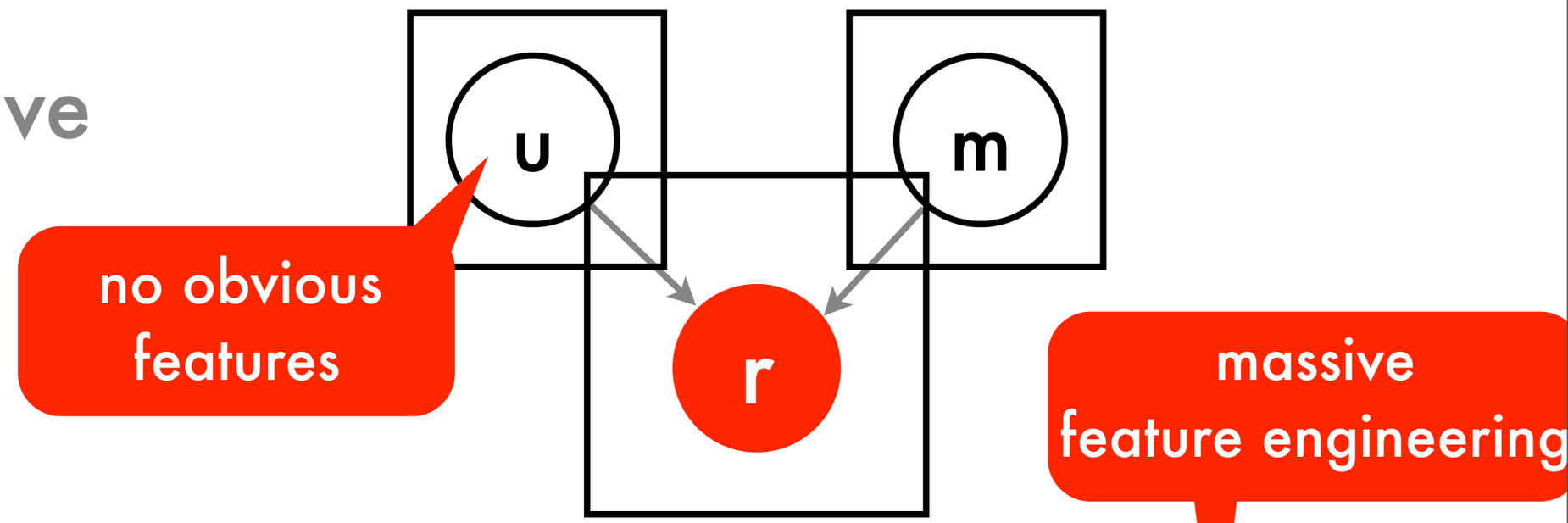

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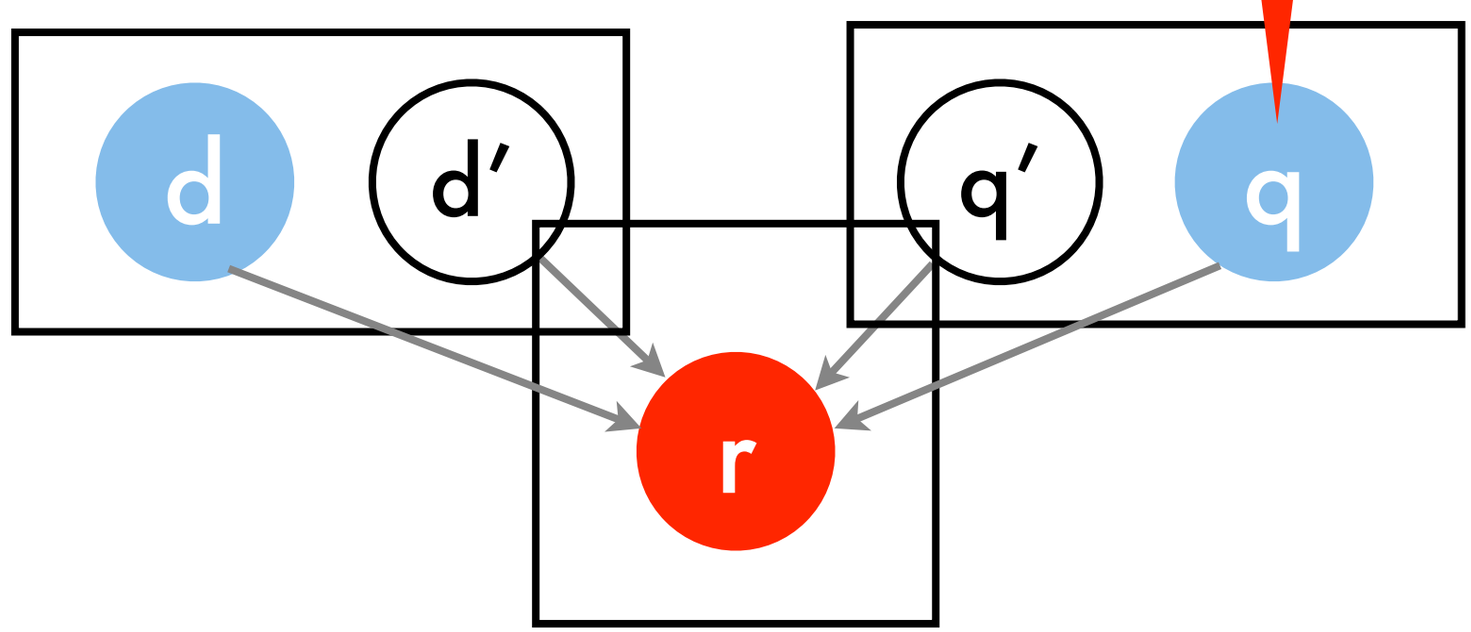

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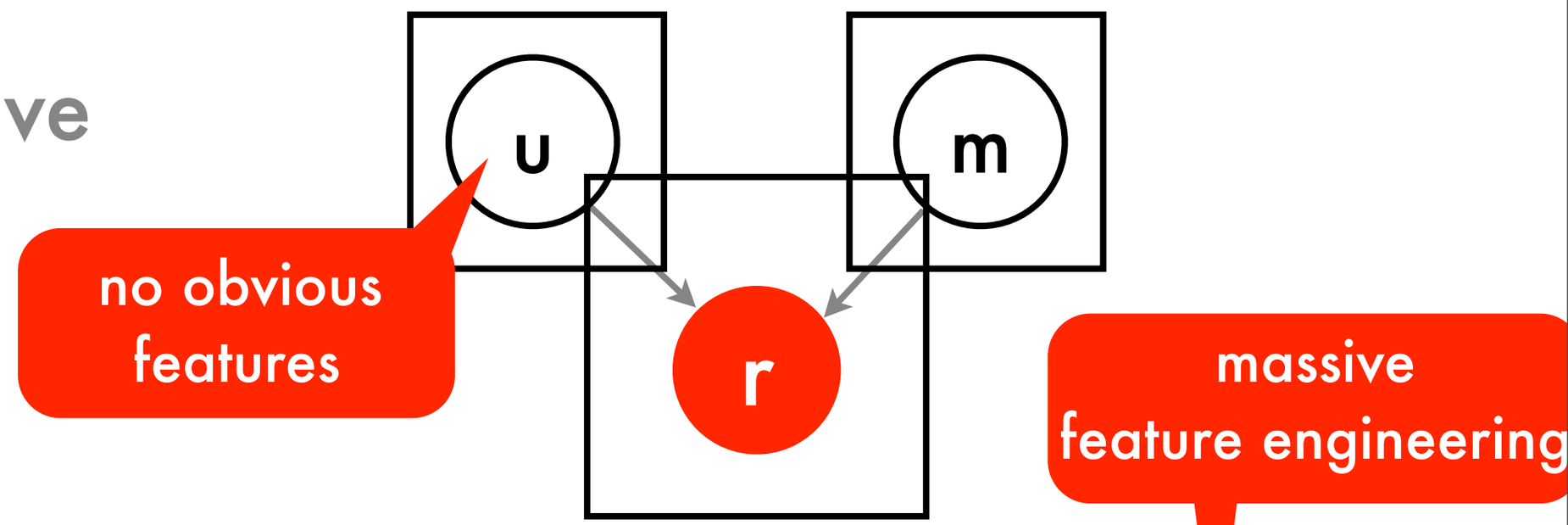

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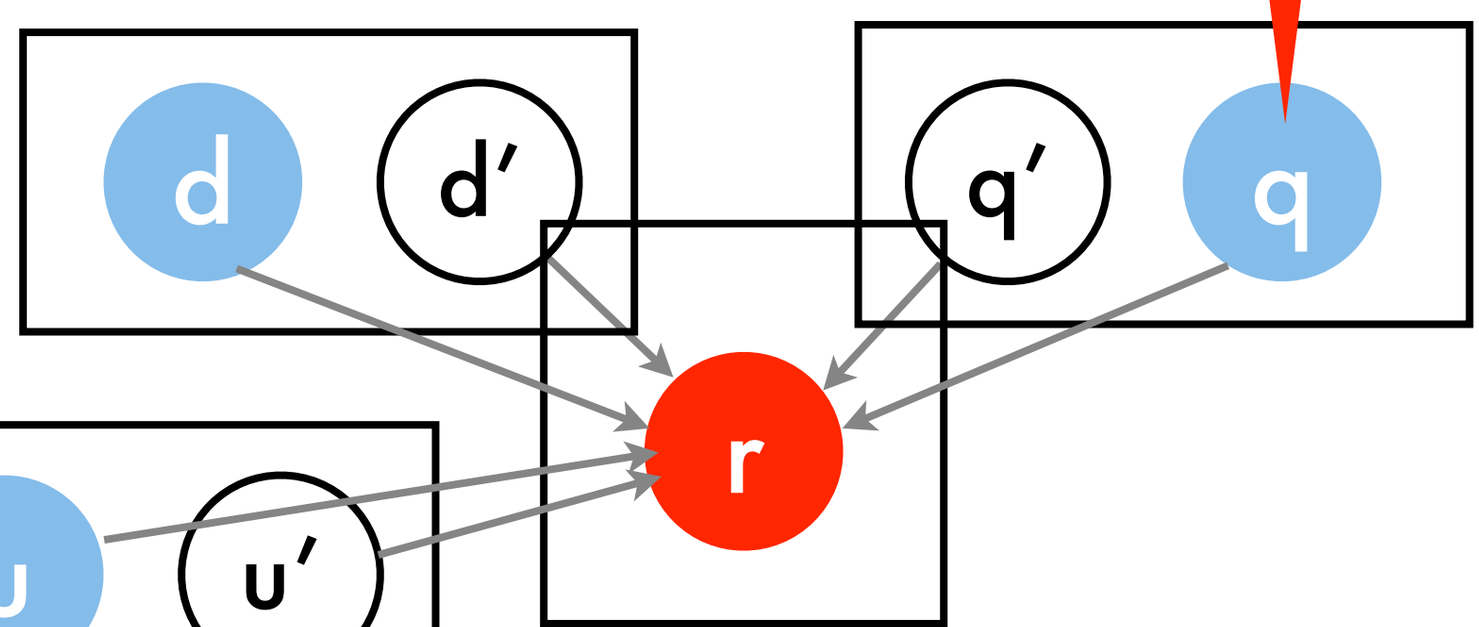
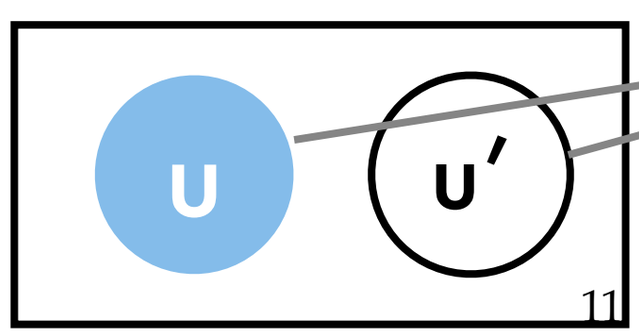
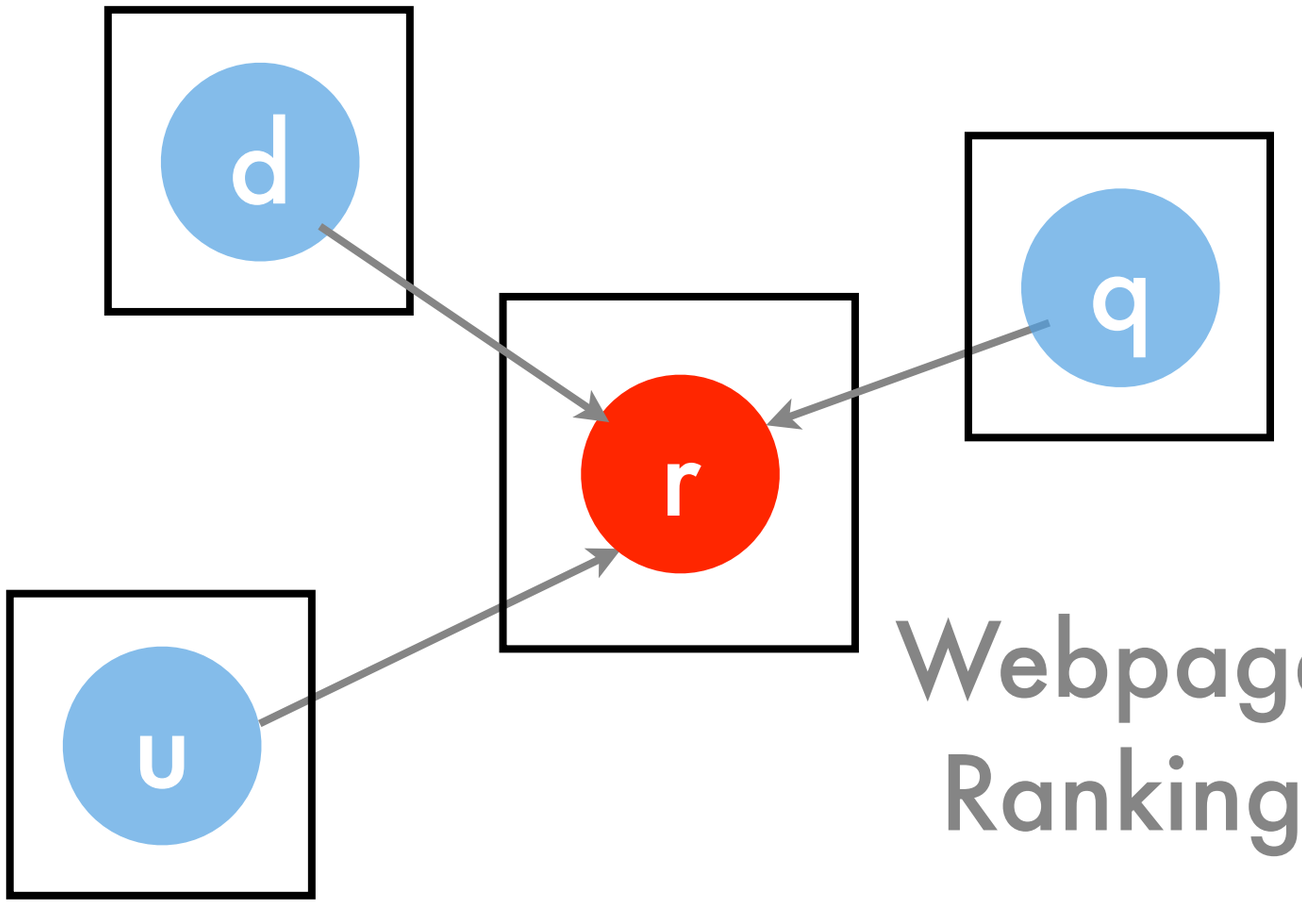


personalized



Data Integration



Webpage Ranking

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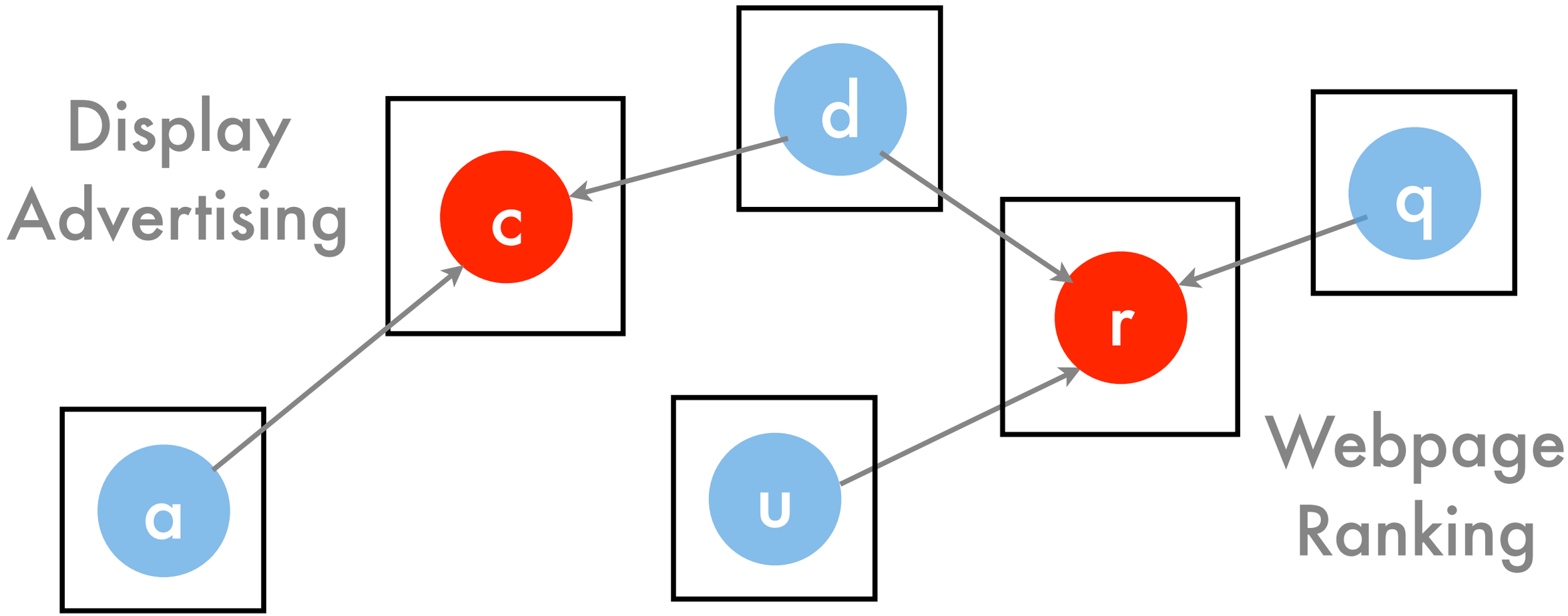

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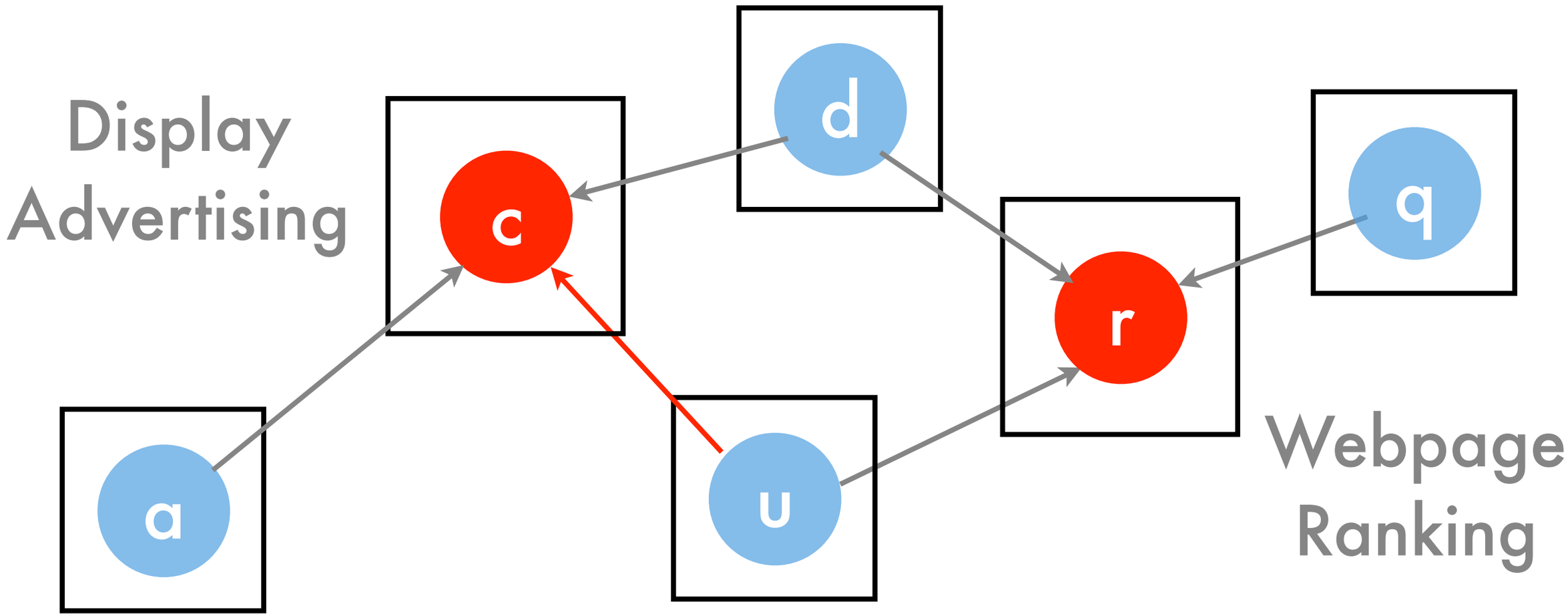

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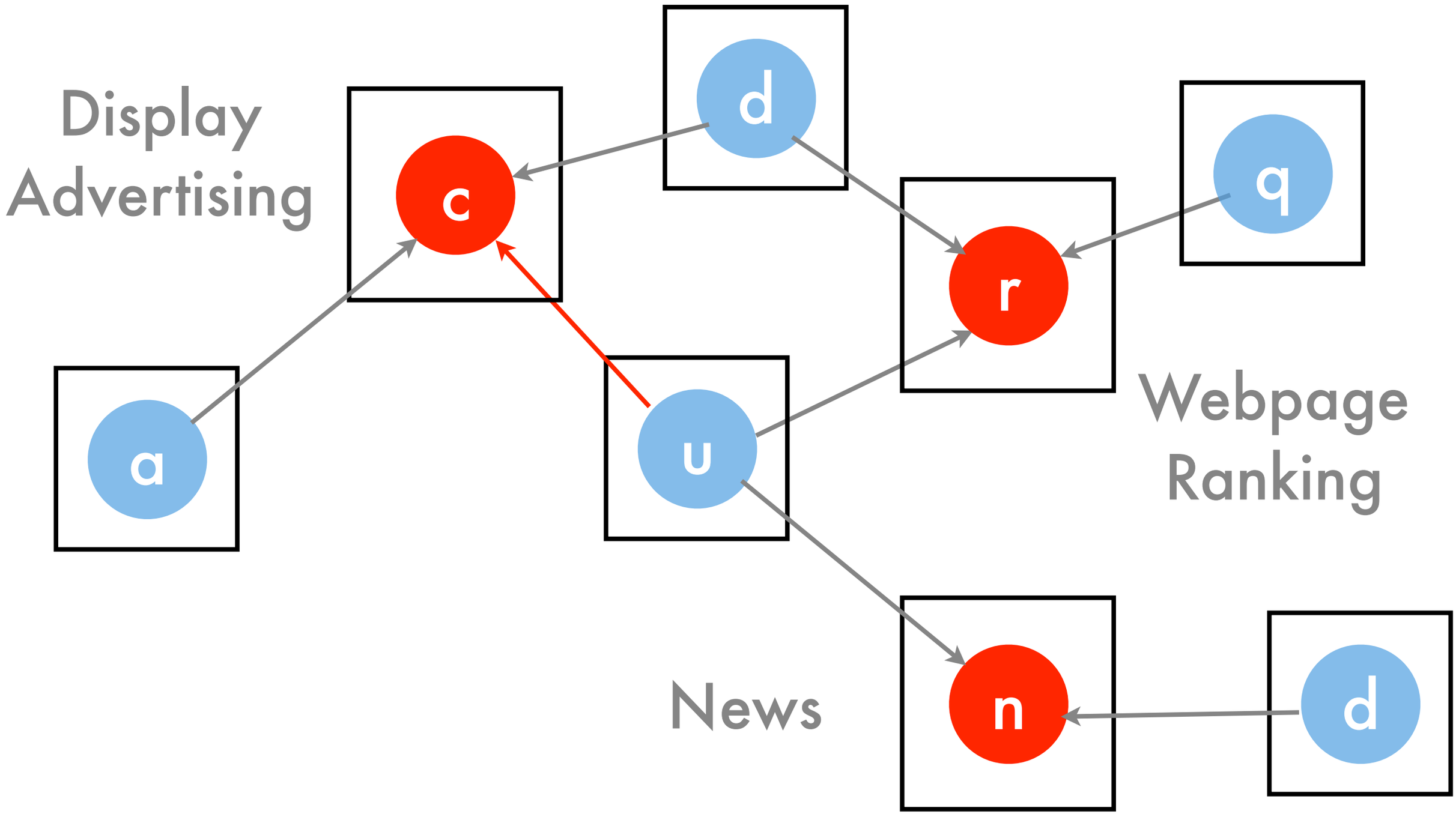

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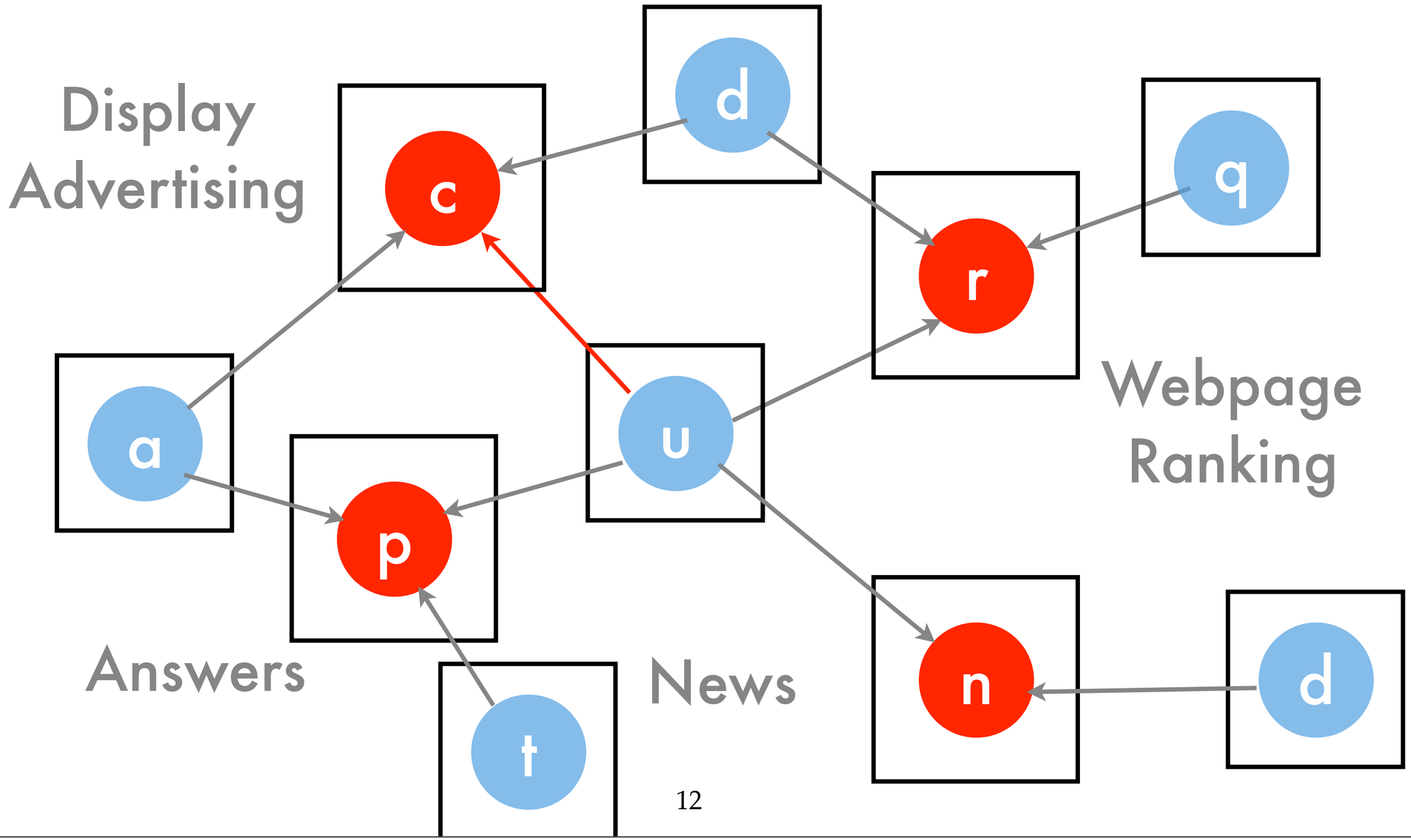

Topic Models

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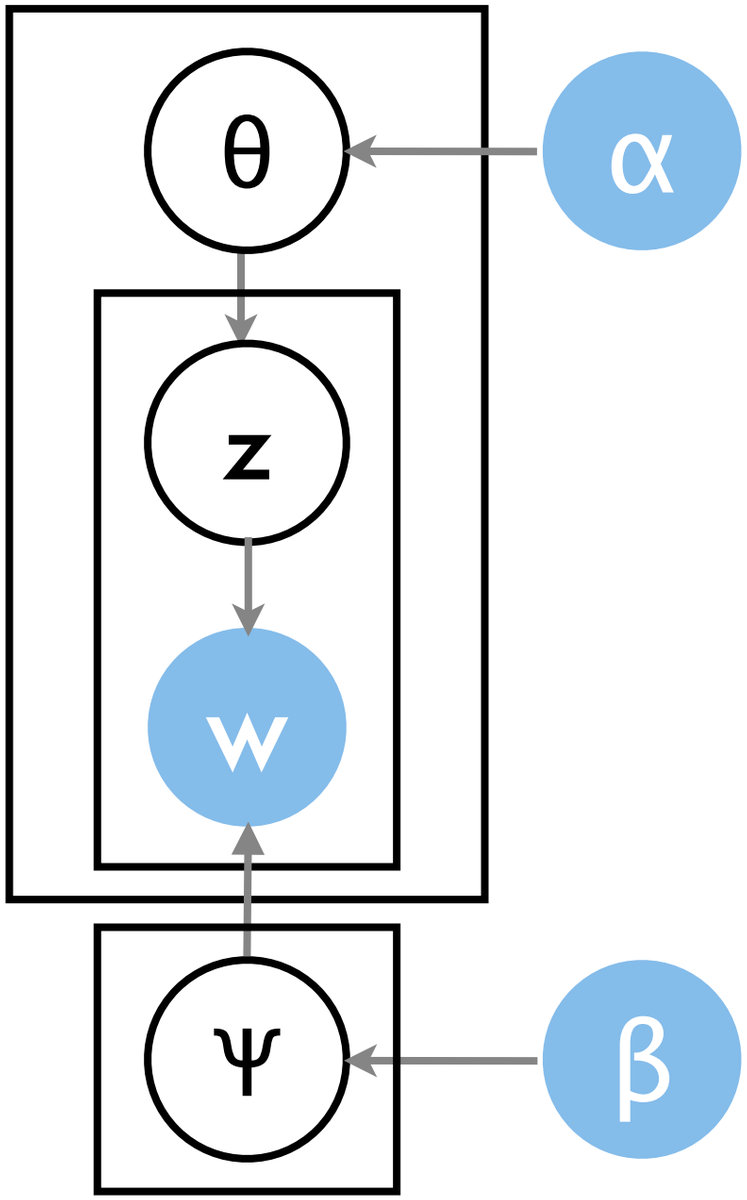
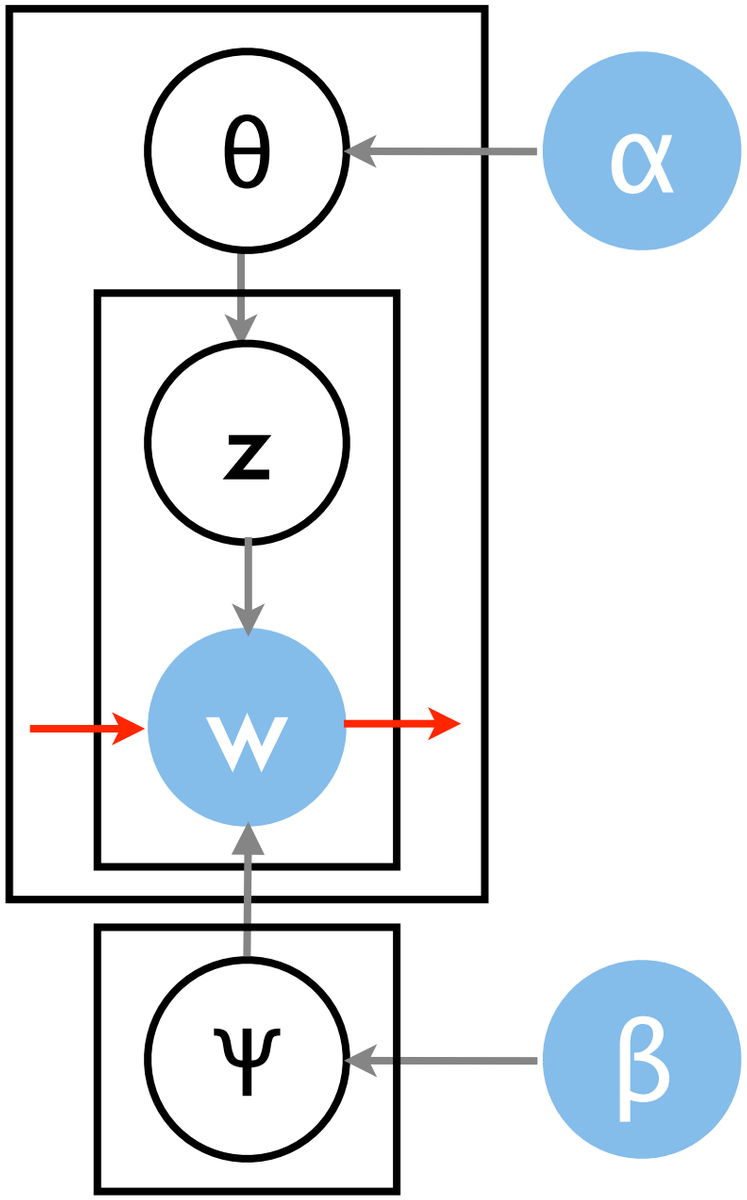


Topic Models

Topic Models

Simplical Mixtures



Topic Models

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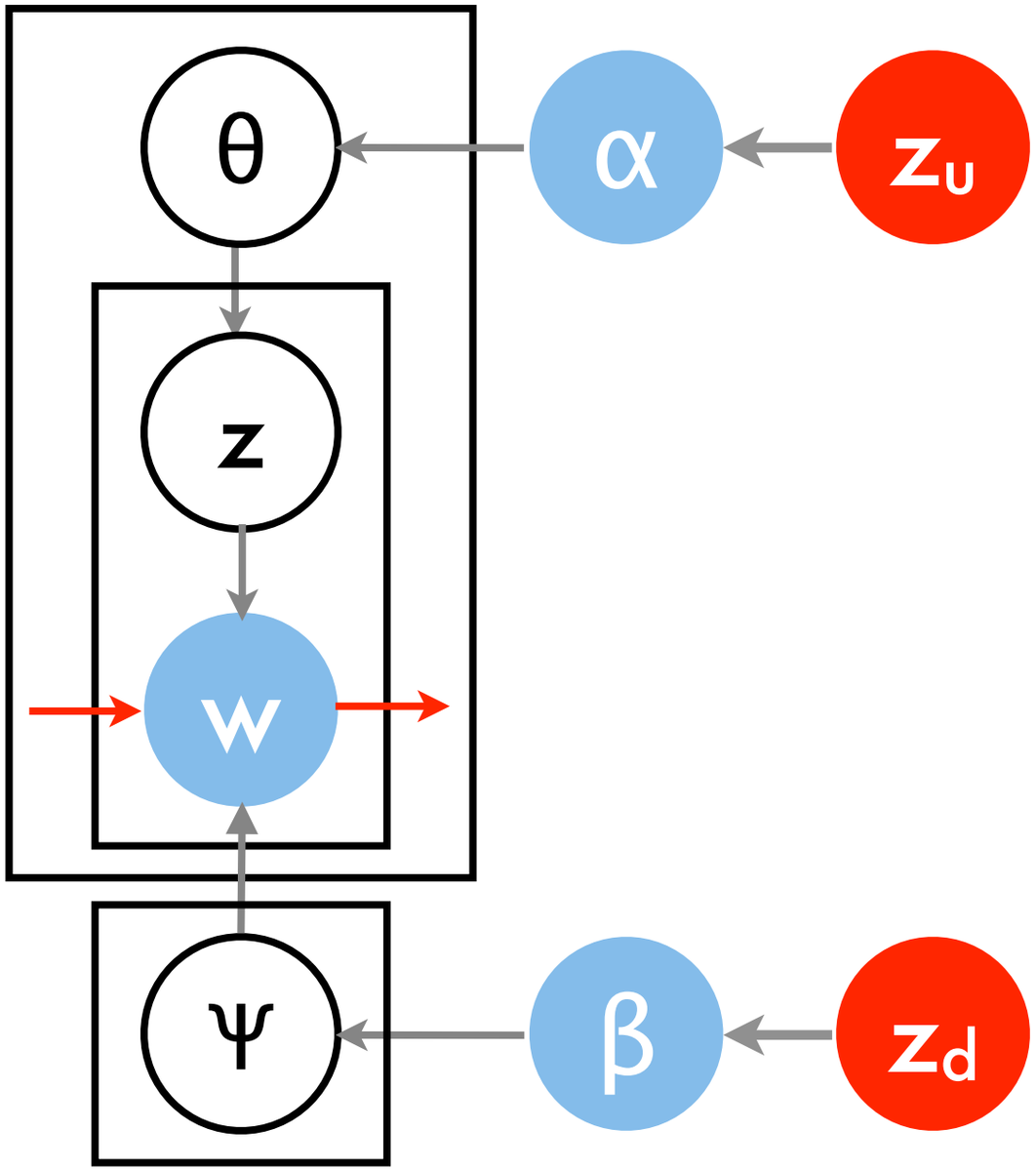


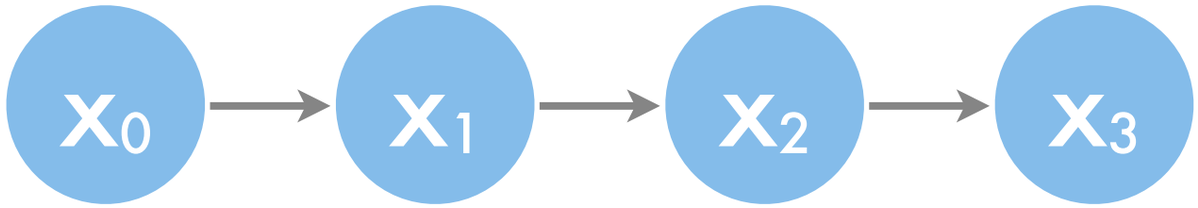
Upstream Conditioning

Downstream Conditioning

Dynamic Programming 101

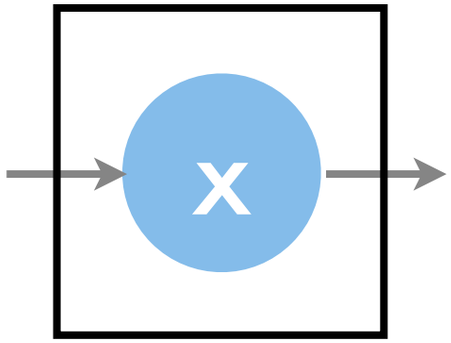
Chains

$$p(x;\theta) = p(x_0;\theta)\prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta)$$



$$p(x_i) = \sum_{x_0,\ldots x_{i-1},x_{i+1}\ldots x_n} \underbrace{p(x_0)}_{:=l_0(x_0)} \prod_{j=1}^{n} p(x_j|x_{j-1})$$

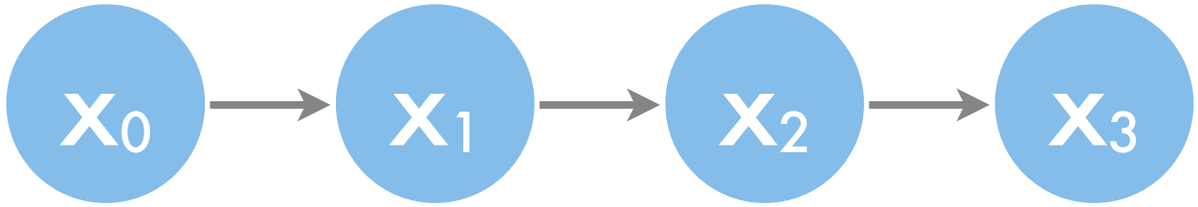
$$= \sum_{x_1,\ldots x_{i-1},x_{i+1}\ldots x_n} \underbrace{\sum_{x_0}[l_0(x_0)p(x_1|x_0)]}_{:=l_1(x_1)} \prod_{j=2}^{n} p(x_j|x_{j-1})$$



$$= \sum_{x_2,\ldots x_{i-1},x_{i+1}\ldots x_n} \underbrace{\sum_{x_1}[l_1(x_1)p(x_2|x_1)]}_{:=l_2(x_2)} \prod_{j=3}^{n} p(x_j|x_{j-1})$$

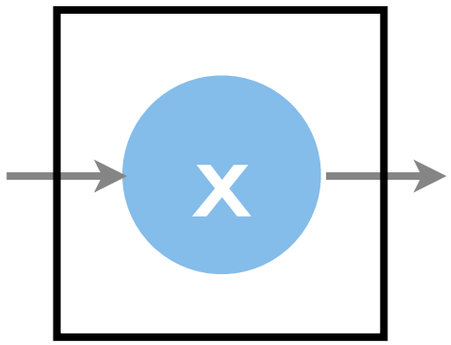
Chains

$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta)$$



$$p(x_i) = l_i(x_i) \sum_{x_{i+1}\dots x_n} \prod_{j=i}^{n-1} p(x_{j+1}|x_j)$$

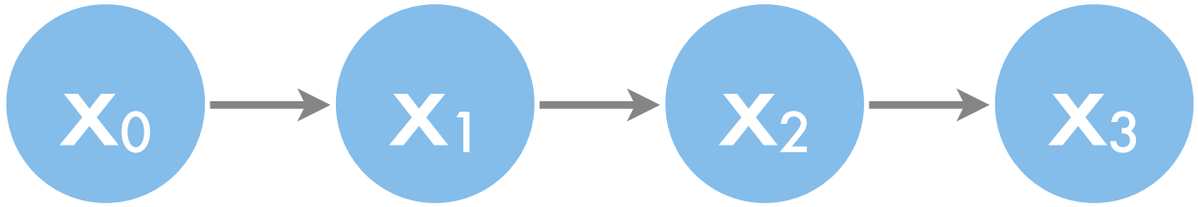
$$= l_i(x_i) \sum_{x_{i+1}\dots x_{n-1}} \prod_{j=i}^{n-2} p(x_{j+1}|x_j) \underbrace{\sum_{x_n} p(x_n|x_{n-1})}_{:=r_{n-1}(x_{n-1})}$$



$$= l_i(x_i) \sum_{x_{i+1}\dots x_{n-2}} \prod_{j=i}^{n-3} p(x_{j+1}|x_j) \underbrace{\sum_{x_{n-1}} p(x_{n-1}|x_{n-2}) r_{n-1}(x_{n-1})}_{:=r_{n-2}(x_{n-2})}$$

Chains

$$p(x;\theta) = p(x_0;\theta)\prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta)$$



- **Forward recursion**

$$l_0(x_0) := p(x_0) \text{ and } l_i(x_i) := \sum_{x_{i-1}} l_{i-1}(x_{i-1})p(x_i|x_{i-1})$$

- **Backward recursion**

$$r_n(x_n) := 1 \text{ and } r_i(x_i) := \sum_{x_{i+1}} r_{i+1}(x_{i+1})p(x_{i+1}|x_i)$$

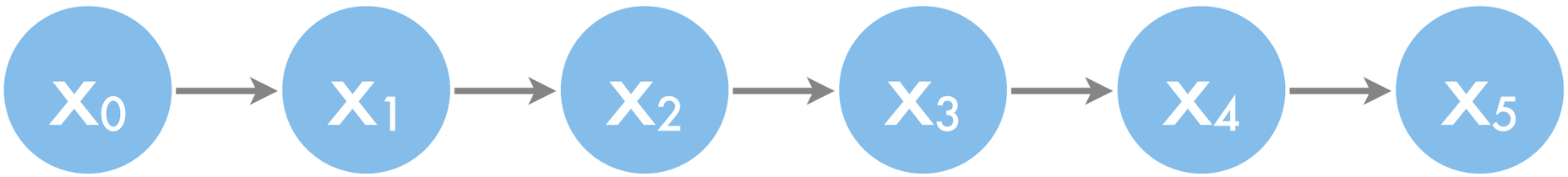
- **Marginalization & conditioning**

$$p(x_i) = l_i(x_i)r_i(x_i)$$

$$p(x_{-i}|x_i) = \frac{p(x)}{p(x_i)}$$

$$p(x_i, x_{i+1}) = l_i(x_i)p(x_{i+1}|x_i)r_i(x_{i+1})$$

Chains



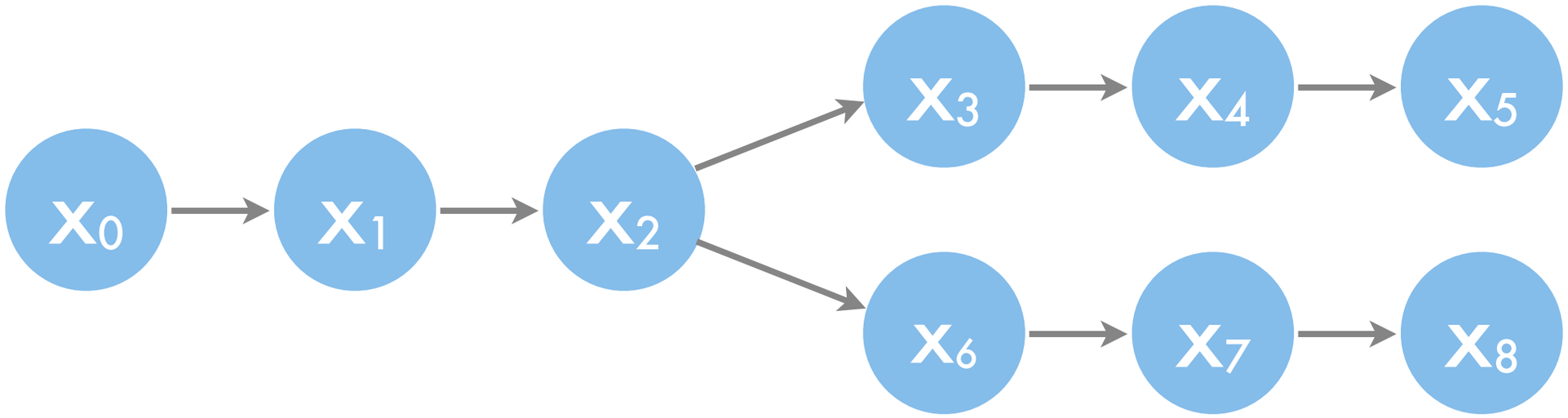
- Send forward messages starting from left node

$$m_{i-1\to i}(x_i) = \sum_{x_{i-1}} m_{i-2\to i-1}(x_{i-1}) f(x_{i-1}, x_i)$$

- Send backward messages starting from right node

$$m_{i+1\to i}(x_i) = \sum_{x_{i+1}} m_{i+2\to i+1}(x_{i+1}) f(x_i, x_{i+1})$$

Trees



- Forward/Backward messages as normal for chain
- When we have more edges for a vertex use ...

$$m_{2\to 3}(x_3) = \sum_{x_2} m_{1\to 2}(x_2) m_{6\to 2}(x_2) f(x_2, x_3)$$

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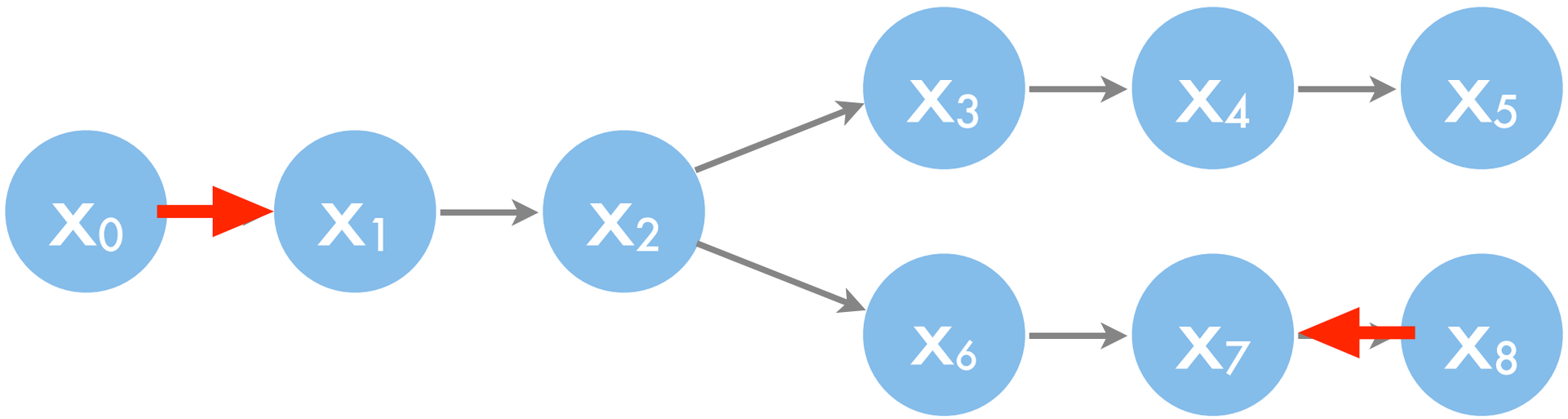
Trees



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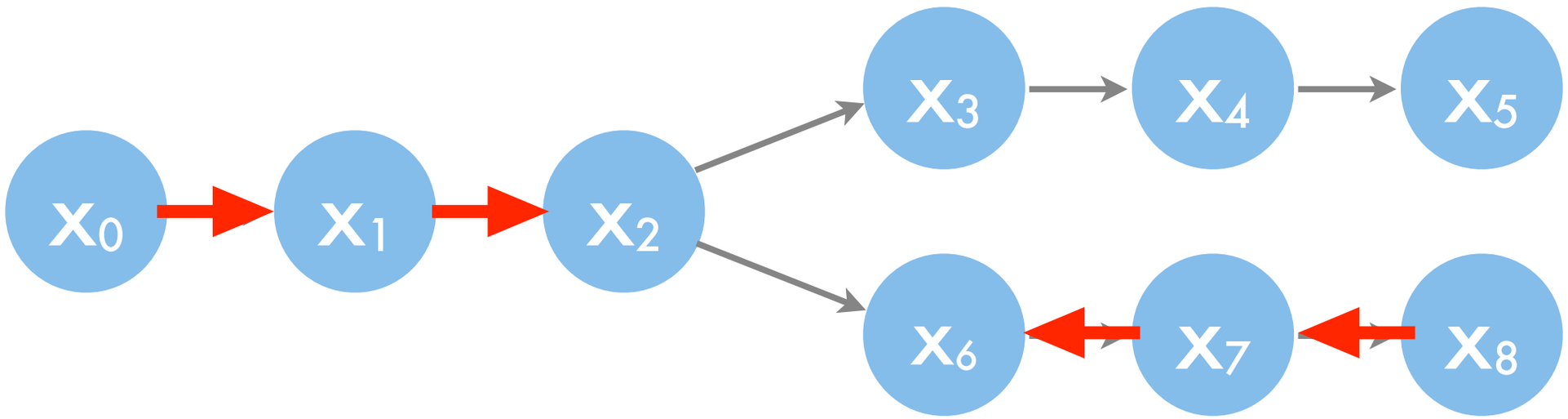

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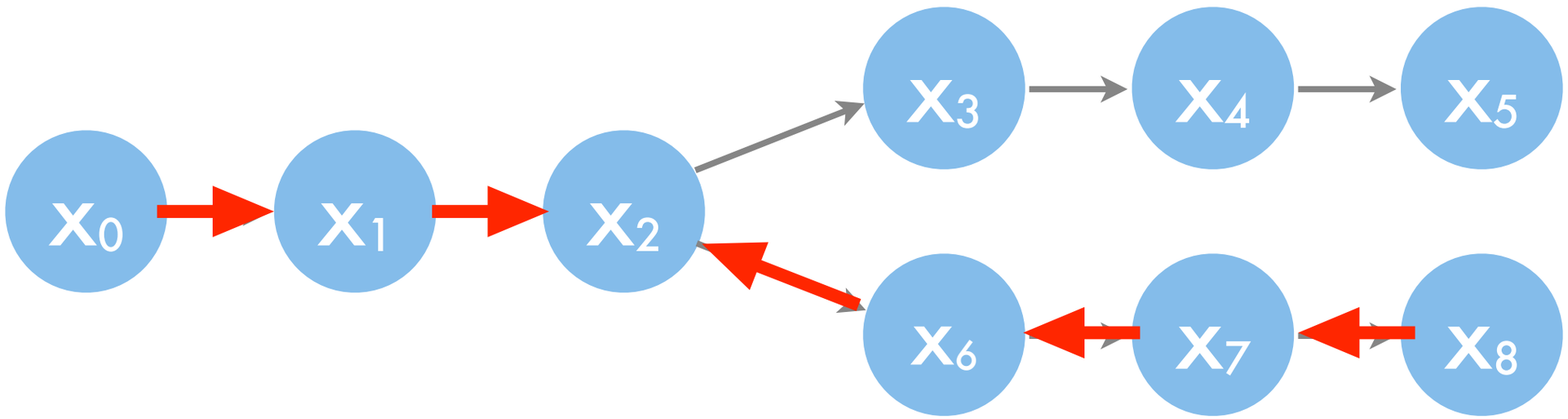

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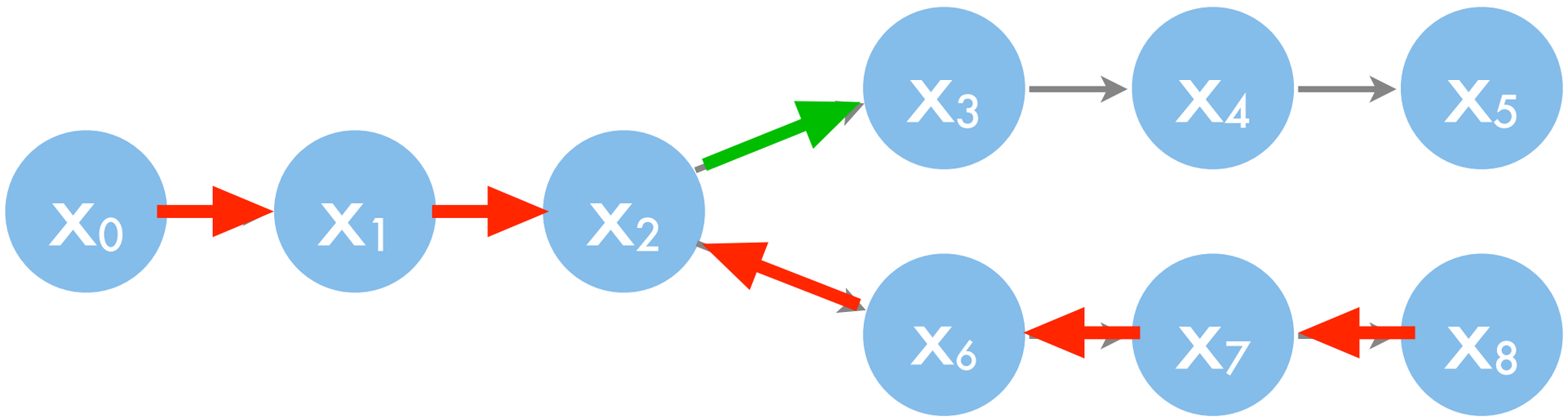

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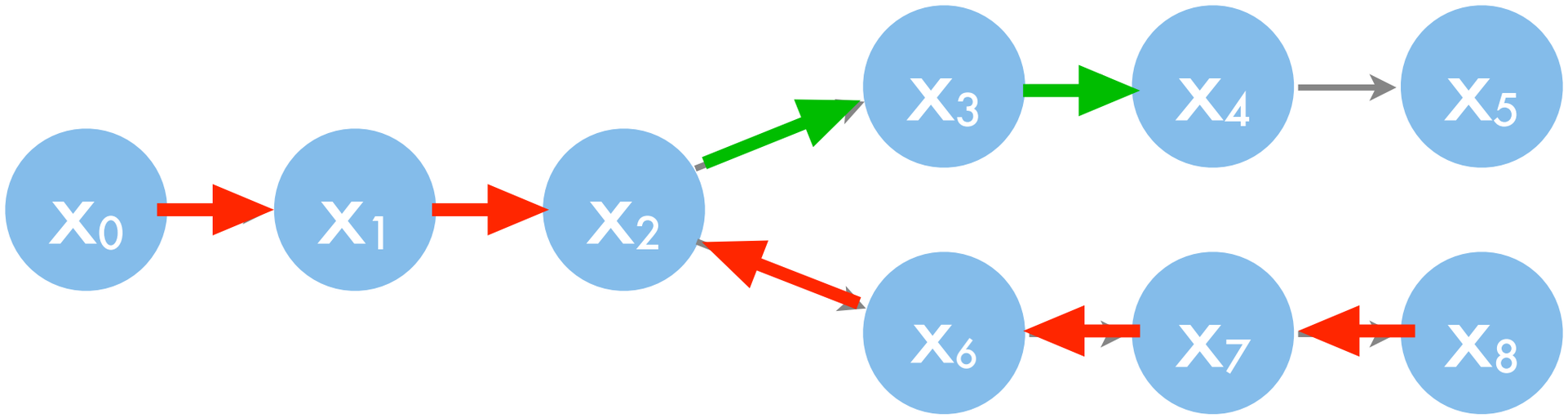
Trees



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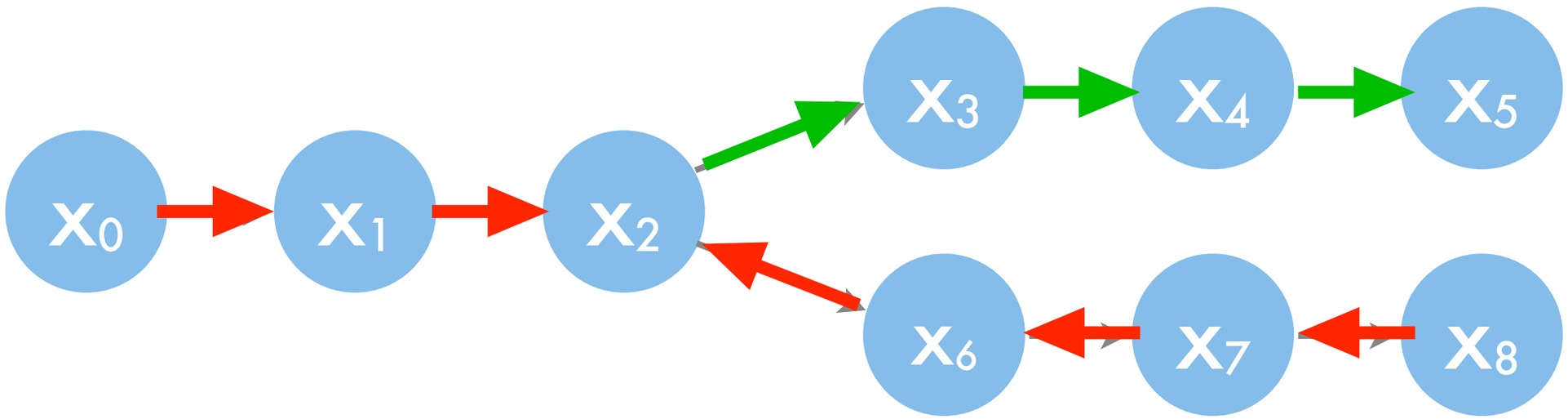

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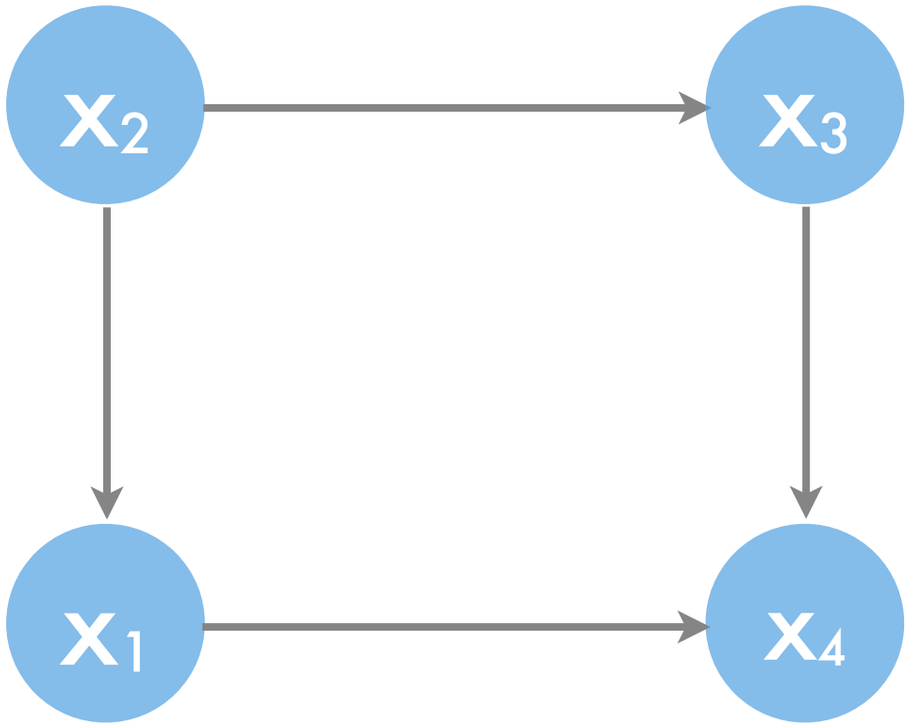
No loops allowed

$$p(x_2)p(x_3|x_2)p(x_1|x_2)p(x_4|x_1,x_3)$$



If we use it anyway – Loopy Belief Propagation (Turbo Codes, Markov Random Fields, etc.)

No loops allowed

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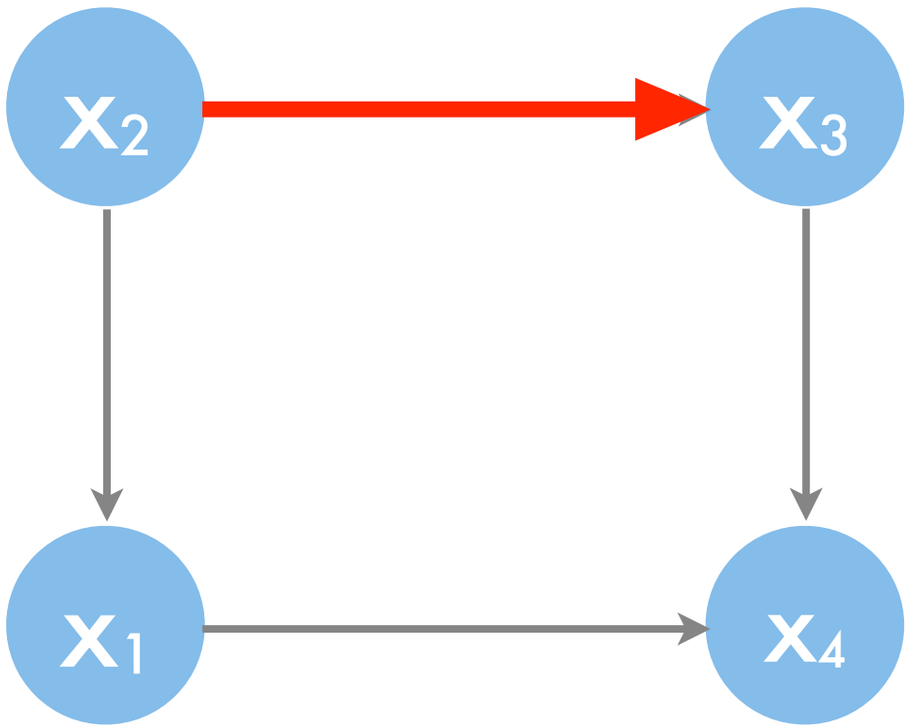

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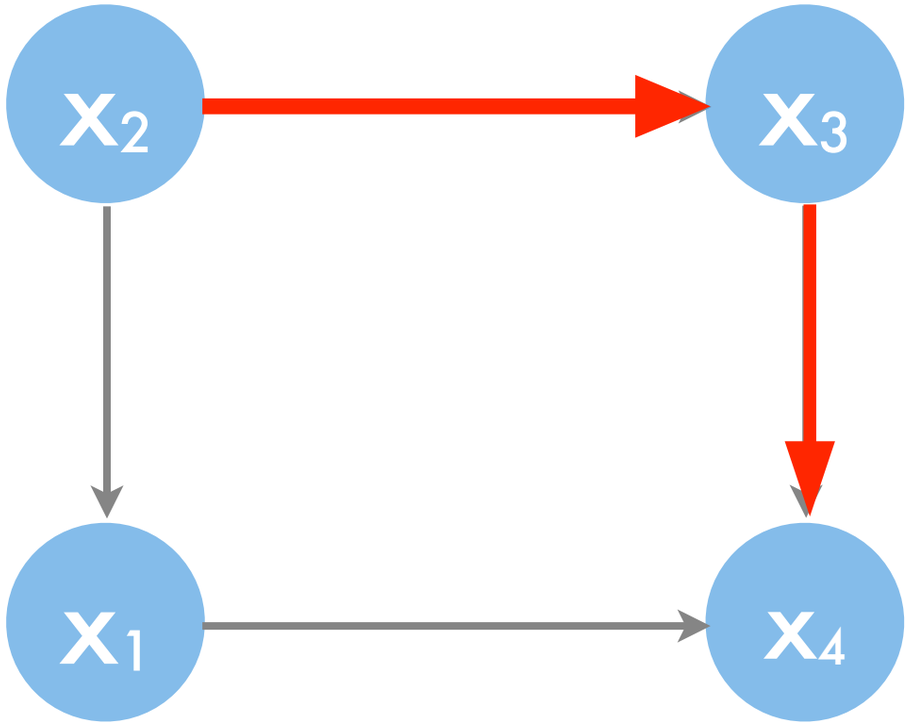

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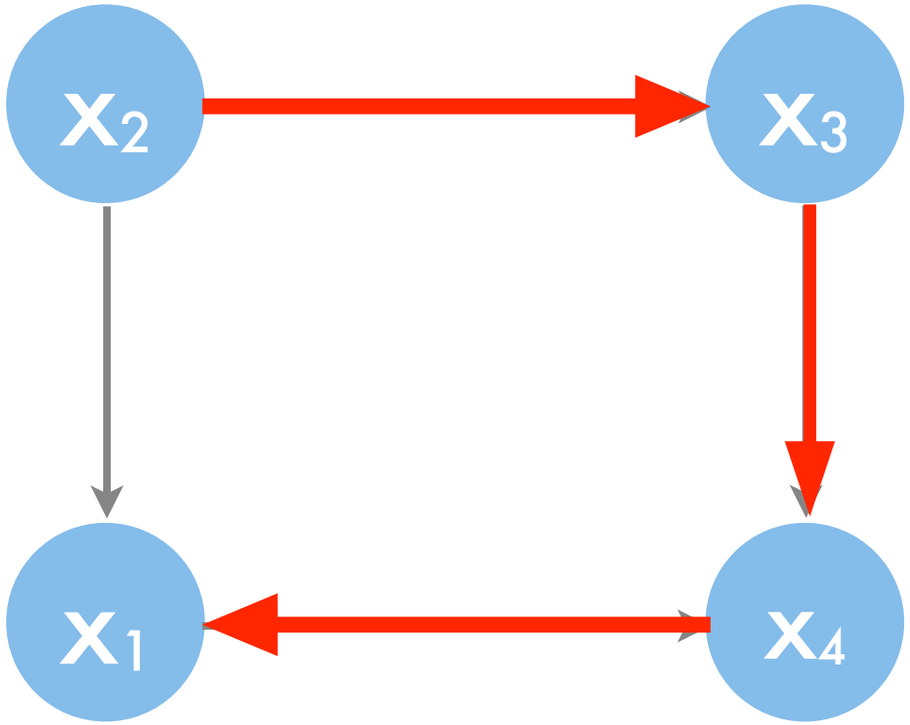

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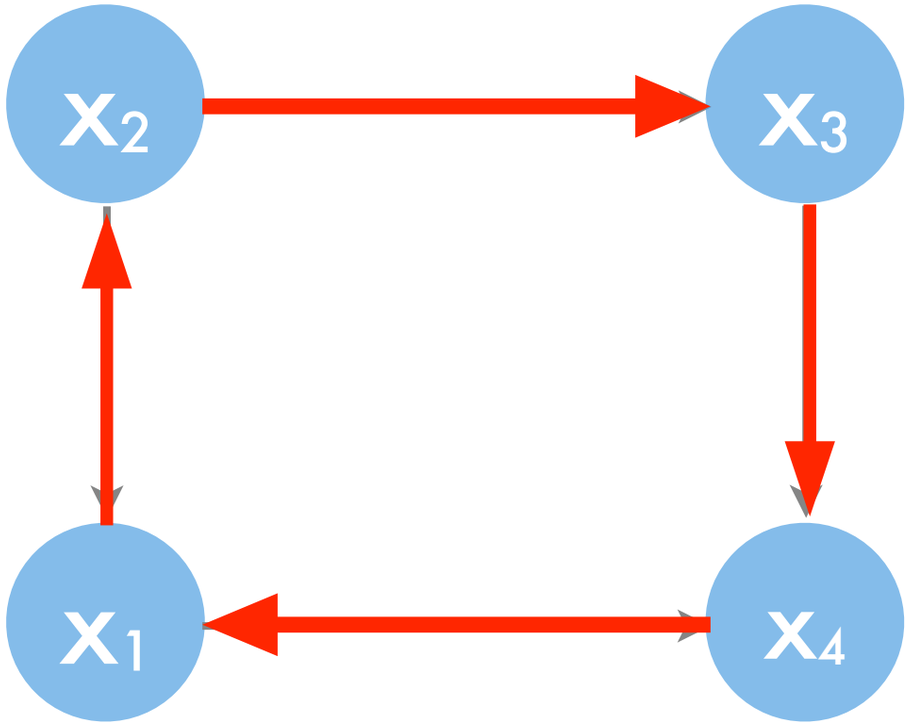

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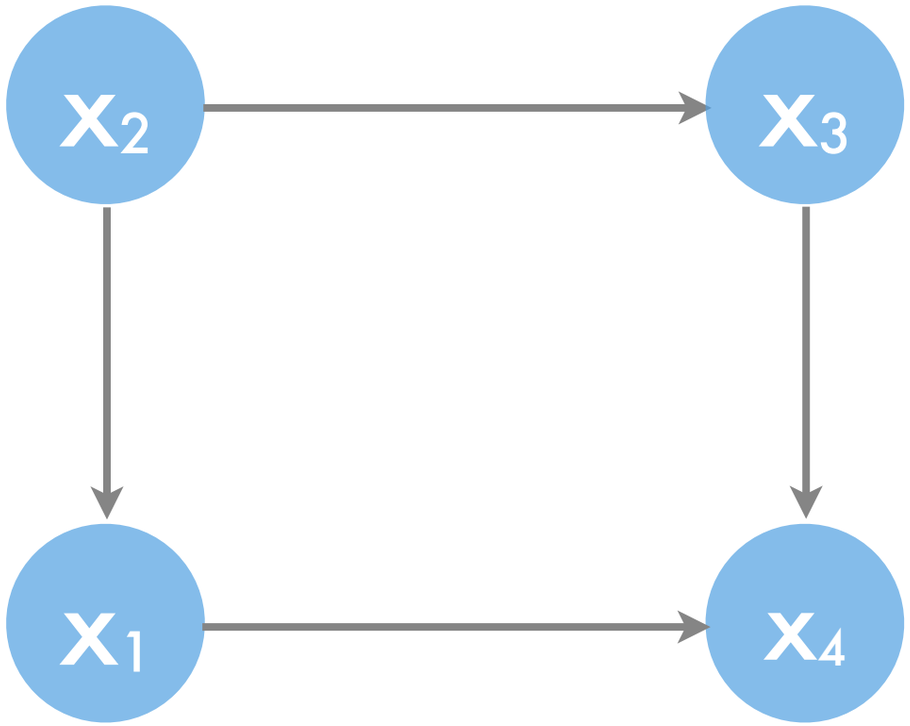

If we use it anyway – Loopy Belief Propagation (Turbo Codes, Markov Random Fields, etc.)

Hidden Markov Models

Variational Optimization

- Lower bound on likelihood

$$\log p(x;\theta) \geq \int dq(y) \log p(x,y;\theta) - \int dq(y) \log q(y)$$

This inequality is tight for p(y|x) = q(y)

- Expectation step

$$q(y) = p(y|x;\theta)$$

- Maximization step

$$\theta^* = \operatorname*{argmax}_{\theta} \int dq(y) \log p(x,y;\theta)$$

Variational Optimization

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find bound

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- Expectation step

$$q(y) = p(y|x;\theta)$$

find bound

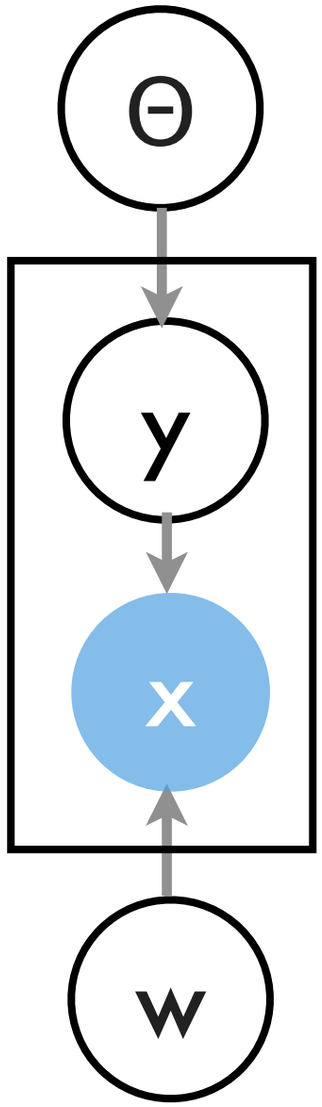
- Maximization step

$$\theta^* = \operatorname*{argmax}_\theta \int dq(y) \log p(x,y;\theta)$$

maximize it

Clustering

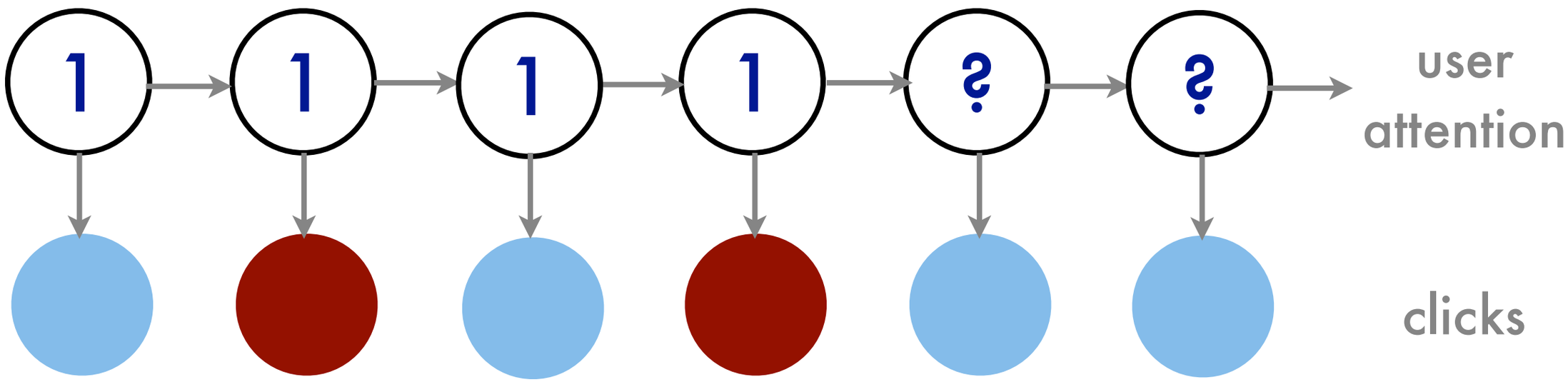
$$p(X, Y|\theta, \sigma, \mu) = \prod_{i=1}^{n} p(x_i|y_i, \sigma, \mu)p(y_i|\theta)$$



- Expectation Step
  Compute $q_i(y) = p(y|x_i, \sigma, \mu, \theta)$
- Maximization Step
  Maximize expected loglikelihood

$$\sum_{i=1}^{n} \sum_{y} q_i(y) \left[\log p(x_i|y, \sigma, \mu) + \log p(y|\theta)\right]$$

Hidden Markov Model

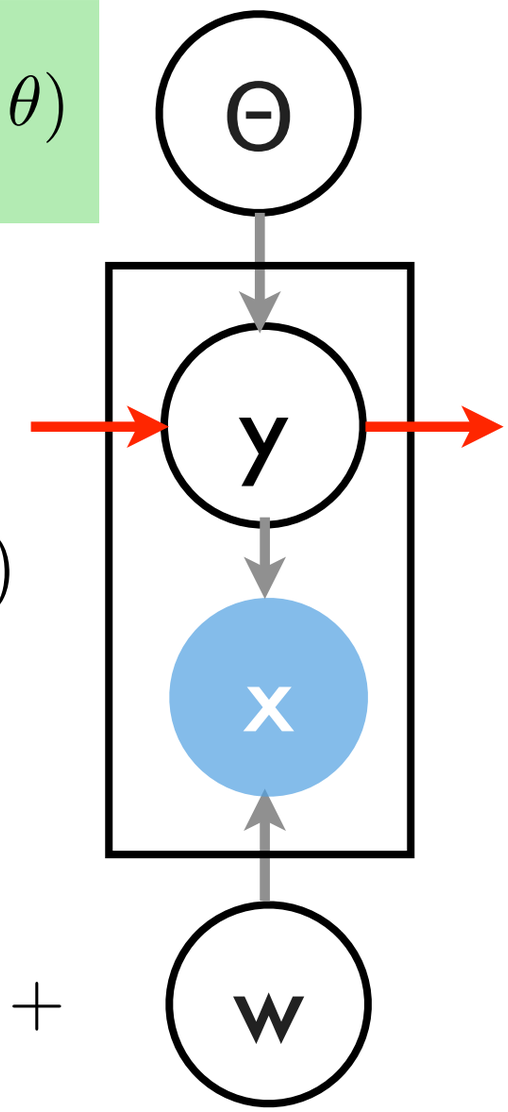


Chapelle & Zhang, 2009

- Sequence of observations (clicks, objects, video, ...)
- Hidden State

Hidden Markov Model

$$p(X,Y|\theta,\sigma,\mu) = \prod_{i=1}^{n} p(x_i|y_i,\sigma,\mu) \cdot p(y_1|\theta) \cdot \prod_{i=2}^{n} p(y_i|y_{i-1},\theta)$$



- Expectation Step
  Compute q (pick a Markov Chain)

$$q(Y) = p(Y|X,\theta,\sigma,\mu) = q(y_1) \cdot \prod_{i=2}^{n} q(y_i|y_{i-1})$$

- Maximization Step
  Maximize expected loglikelihood

$$\sum_{i=1}^{n} \sum_{y} q_i(y) \log p(x_i|y,\sigma,\mu) + \sum_{y_1} q_1(y_1) \log p(y_1|\theta) +$$

$$\sum_{i=2}^{n} \sum_{y_{i-1},y_i} q(y_{i-1},y_i) \log p(y_i|y_{i-1},\theta)$$

Expectation Step

$$p(X,Y|\theta,\sigma,\mu) = \underbrace{p(x_1|y_1,\sigma,\mu)p(y_1|\theta)}_{f_1(y_1)} \cdot \prod_{i=2}^{n} \underbrace{p(x_i|y_i,\sigma,\mu)p(y_i|y_{i-1},\theta)}_{f_i(y_i,y_{i-1})}$$

$$= f_1(y_1) \prod_{i=2}^{n} f_i(y_i, y_{i-1})$$

- Forward recursion

$$l_1(y_1) = f_1(y_1) \text{ and } l_i(y_i) = \sum_{y_{i-1}} l_{i-1}(y_{i-1}) f(y_i, y_{i-1})$$

- Backward recursion

$$r_n(y_n) = 1 \text{ and } r_i(y_i) = \sum_{y_{i+1}} r_{i+1}(y_{i+1}) f(y_{i+1}, y_i)$$

$$q(y_i) = l_i(y_i) r_i(y_i) \text{ and } q(y_i, y_{i+1}) = l_i(y_i) f(y_{i+1}, y_i) r_{i+1}(y_{i+1})$$

Maximization Step

$$\sum_{i=1}^{n} \sum_{y} q_i(y) \log p(x_i|y, \sigma, \mu) + \sum_{y_1} q_1(y_1) \log p(y_1|\theta) +$$

$$\sum_{i=2}^{n} \sum_{y_{i-1}, y_i} q(y_{i-1}, y_i) \log p(y_i|y_{i-1}, \theta)$$

Update Mixture
Identical to Clustering

$$n_y = \sum_{i} q_i(y)$$

$$\mu_y = \frac{1}{n_y} \sum_{i=1}^{n} q_i(y) x_i$$

$$\Sigma_y = \frac{1}{n_y} \sum_{i=1}^{n} q_i(y) x_i x_i^\top - \mu_y \mu_y^\top$$

Maximization Step

$$\sum_{i=1}^{n} \sum_{y} q_i(y) \log p(x_i|y, \sigma, \mu) + \sum_{y_1} q_1(y_1) \log p(y_1|\theta) +$$

$$\sum_{i=2}^{n} \sum_{y_{i-1}, y_i} q(y_{i-1}, y_i) \log p(y_i|y_{i-1}, \theta)$$

Start Probability $\quad p(y_1|\theta) = q_1(y_1)$

Transition Probability

$$n_{y,y'} := \sum_{i=2}^{n} q(y_{i-1} = y, y_i = y') \text{ and } n_y := \sum_{i=2}^{n} q(y_i = y)$$

$$p(y_{i+1} = y'|y_i = y; \theta) = \frac{n_{y,y'}}{n_y}$$

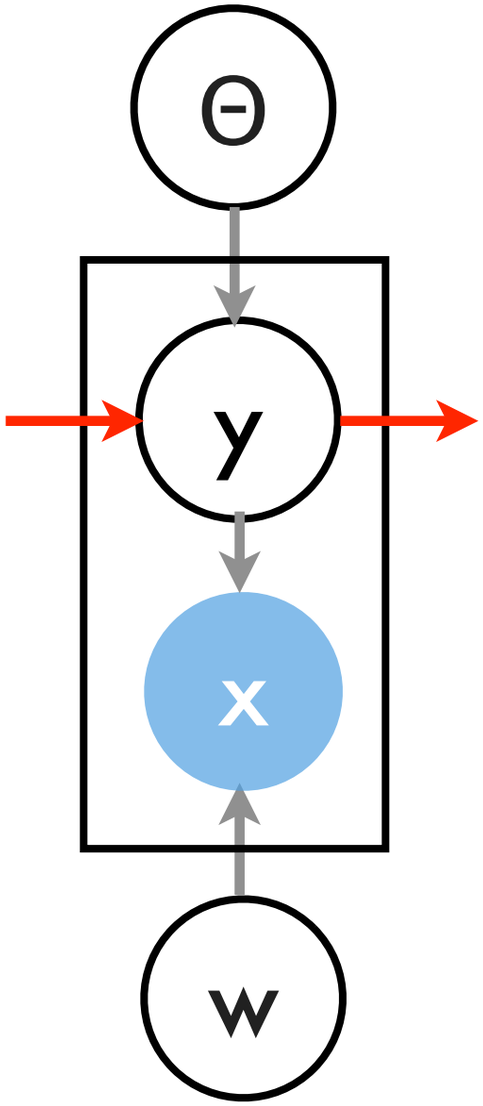
Using HMMs

State Tracking

- Assume we have some idea of current state $p(y_t)$
- Observe x. This updates our idea of the current state
- Predict future state

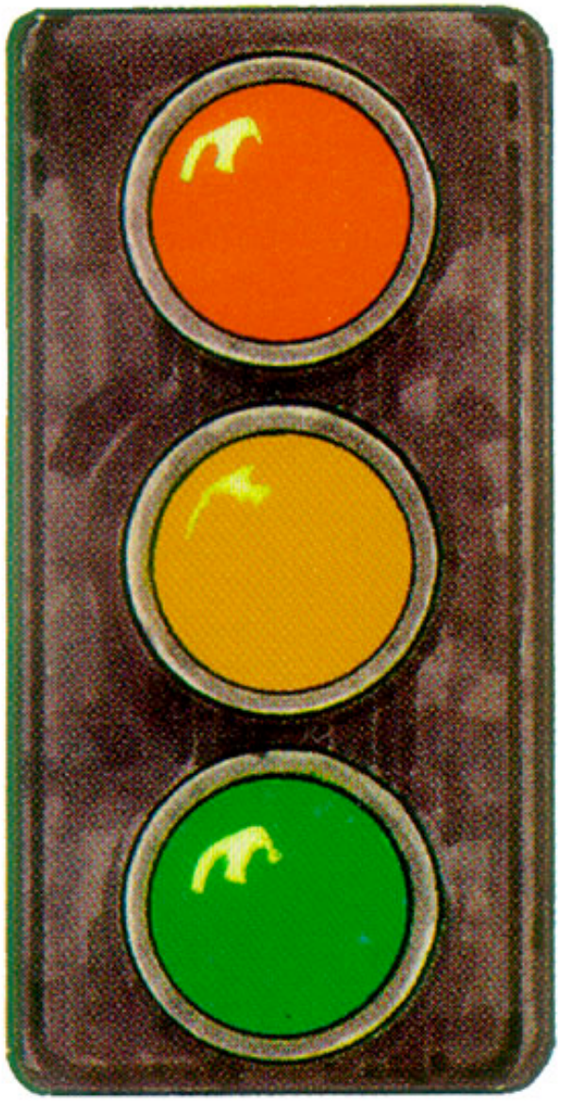
$$p(y_{t+1}|x_t) \propto \sum_{y_t} p(y_{t+1}|y_t)p(x_t|y_t)p(y_t)$$

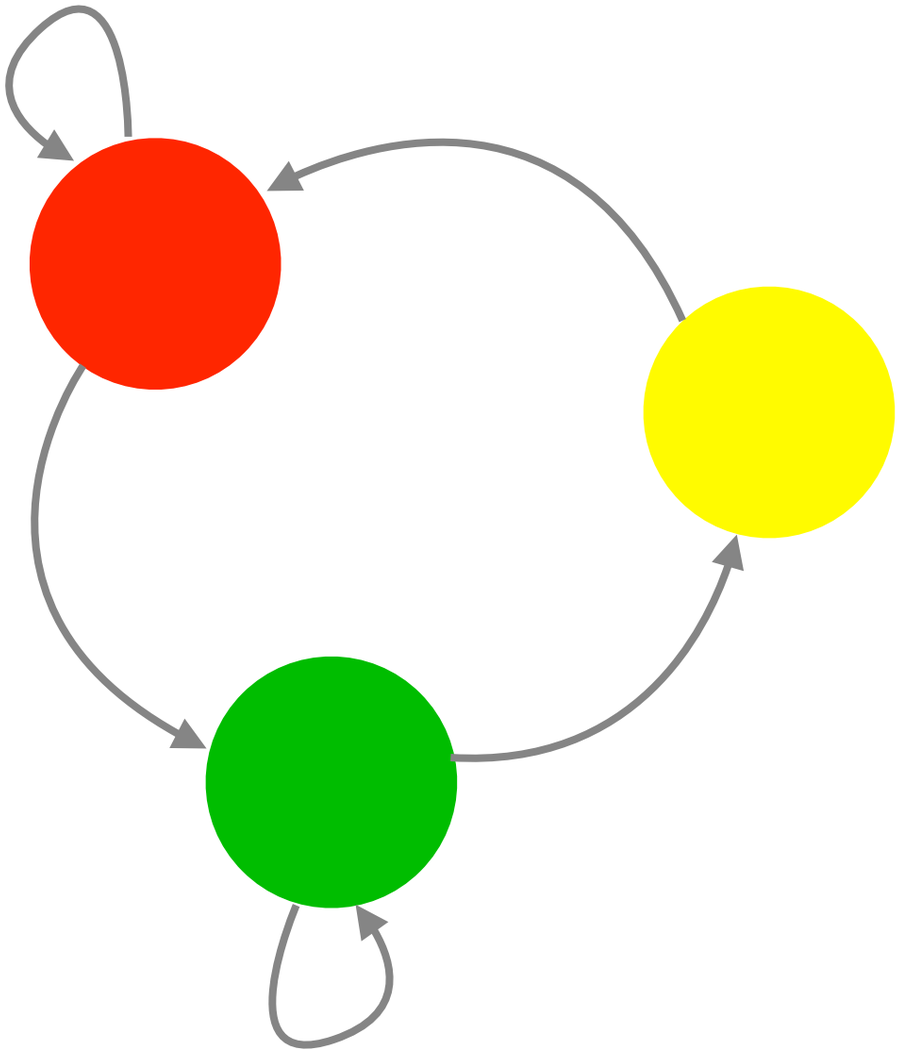
- Repeat ...



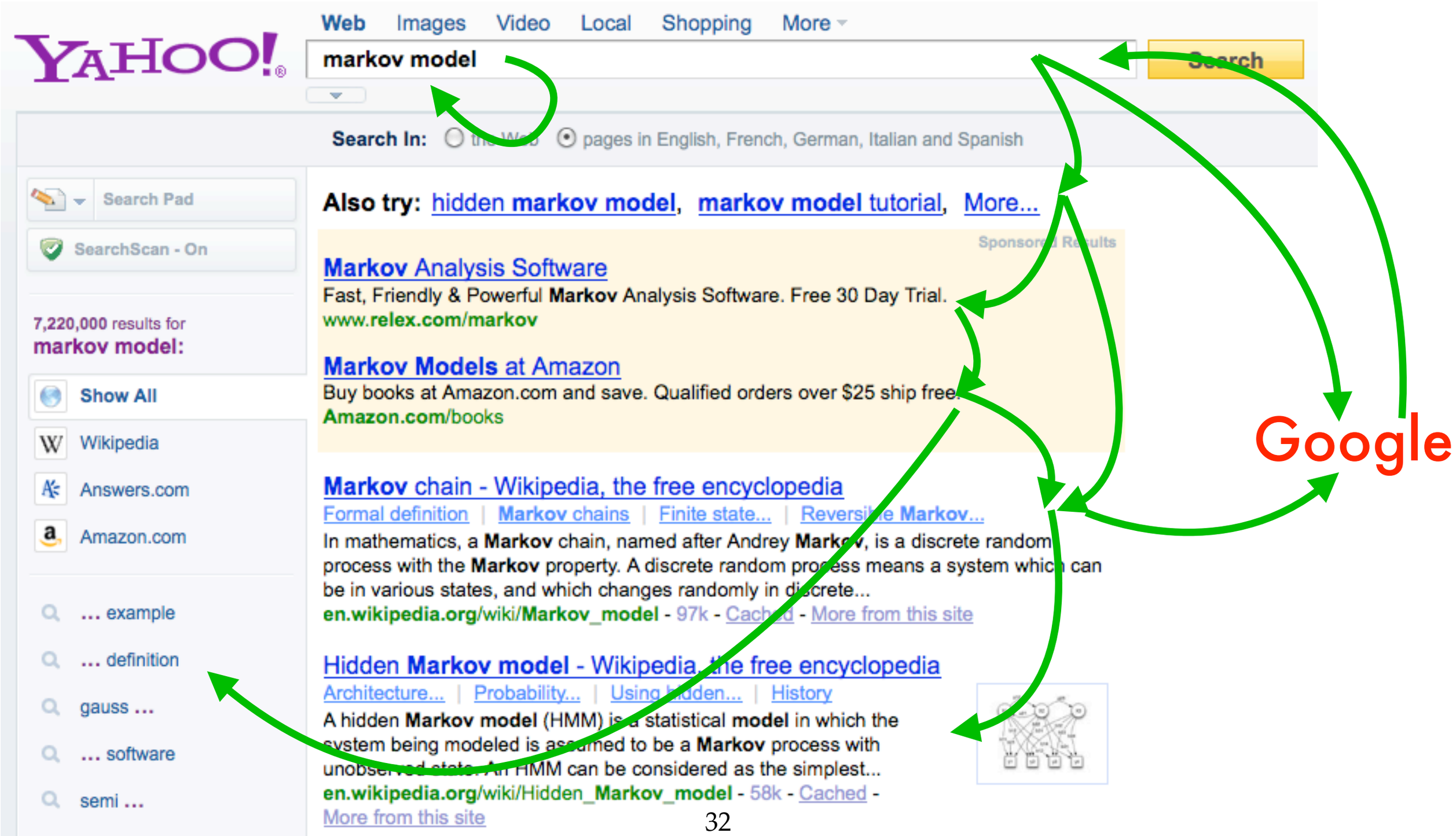
Grammars

- Sometimes we know the state transitions





Grammars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Buy books at Amazon.com and save. Qualified orders over $25 ship free.
Amazon.com/books
Markov chain - Wikipedia, the free encyclopedia
Formal definition | Markov chains | Finite state... | Reversible Markov...
In mathematics, a Markov chain, named after Andrey Markov, is a discrete random process with the Markov property. A discrete random process means a system which can be in various states, and which changes randomly in discrete...
en.wikipedia.org/wiki/Markov_model - 97k - Cached - More from this site
Hidden Markov model - Wikipedia, the free encyclopedia
Architecture... | Probability... | Using hidden... | History
A hidden Markov model (HMM) is a statistical model in which the system being modeled is assumed to be a Markov process with unobserved state. An HMM can be considered as the simplest...
en.wikipedia.org/wiki/Hidden_Markov_model - 58k - Cached - More from this site
Google

Kalman Filter

Kalman Filter

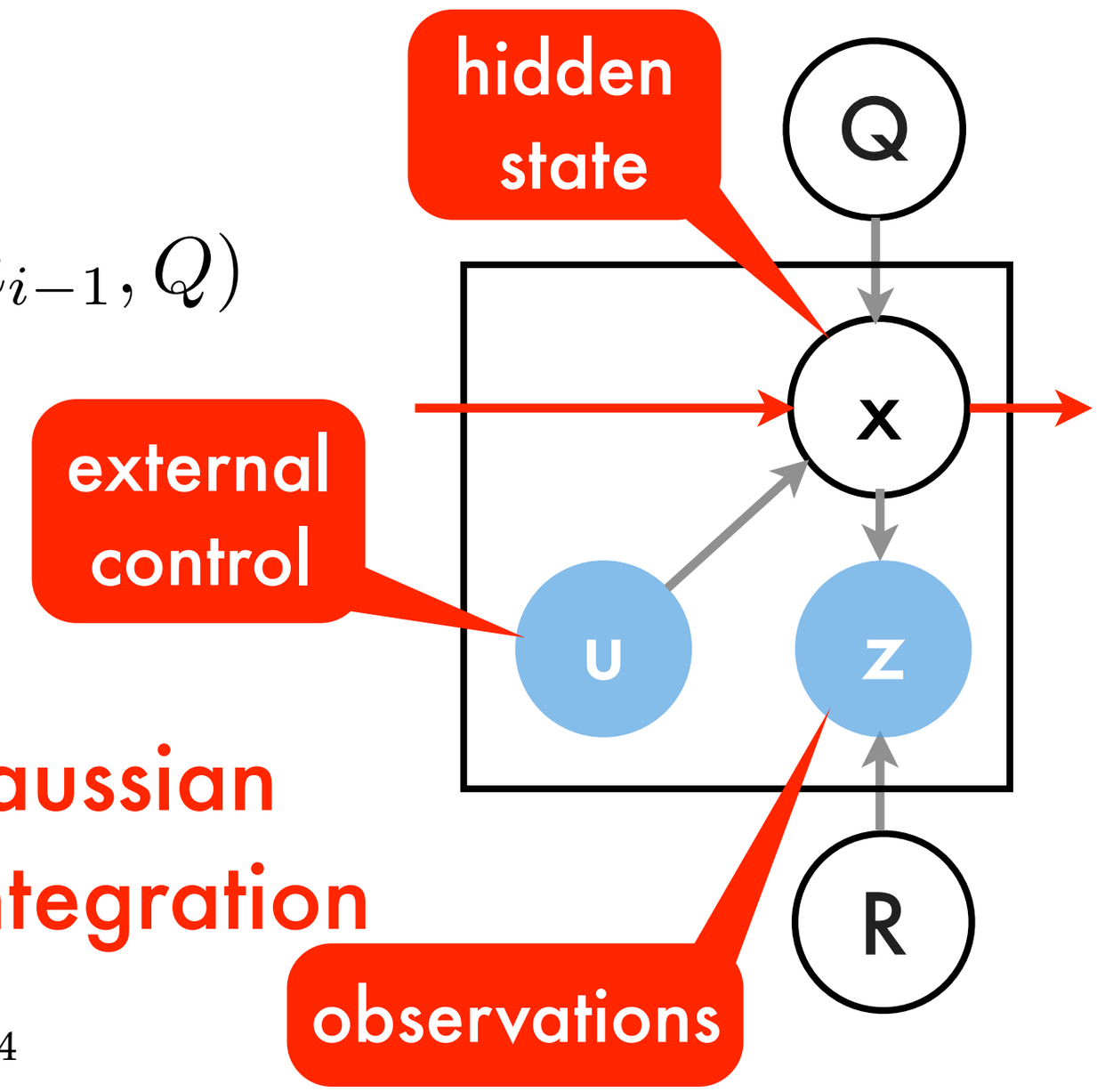
$$p(X, Z|U, Q, R, A, B, H) = \prod_{i=1}^{n} p(x_i|x_{i-1}, u_{i-1}, A, B, Q)p(z_i|x_i, H, R)$$

- **Hidden State**

$$x_i \sim \mathcal{N}(Ax_{i-1} + Bu_{i-1}, Q)$$

- **Observations**

$$z_i \sim \mathcal{N}(Hx_i, R)$$



All distributions are Gaussian
We can perform exact integration

Forward Filtering

- Assume we have some idea of the initial state

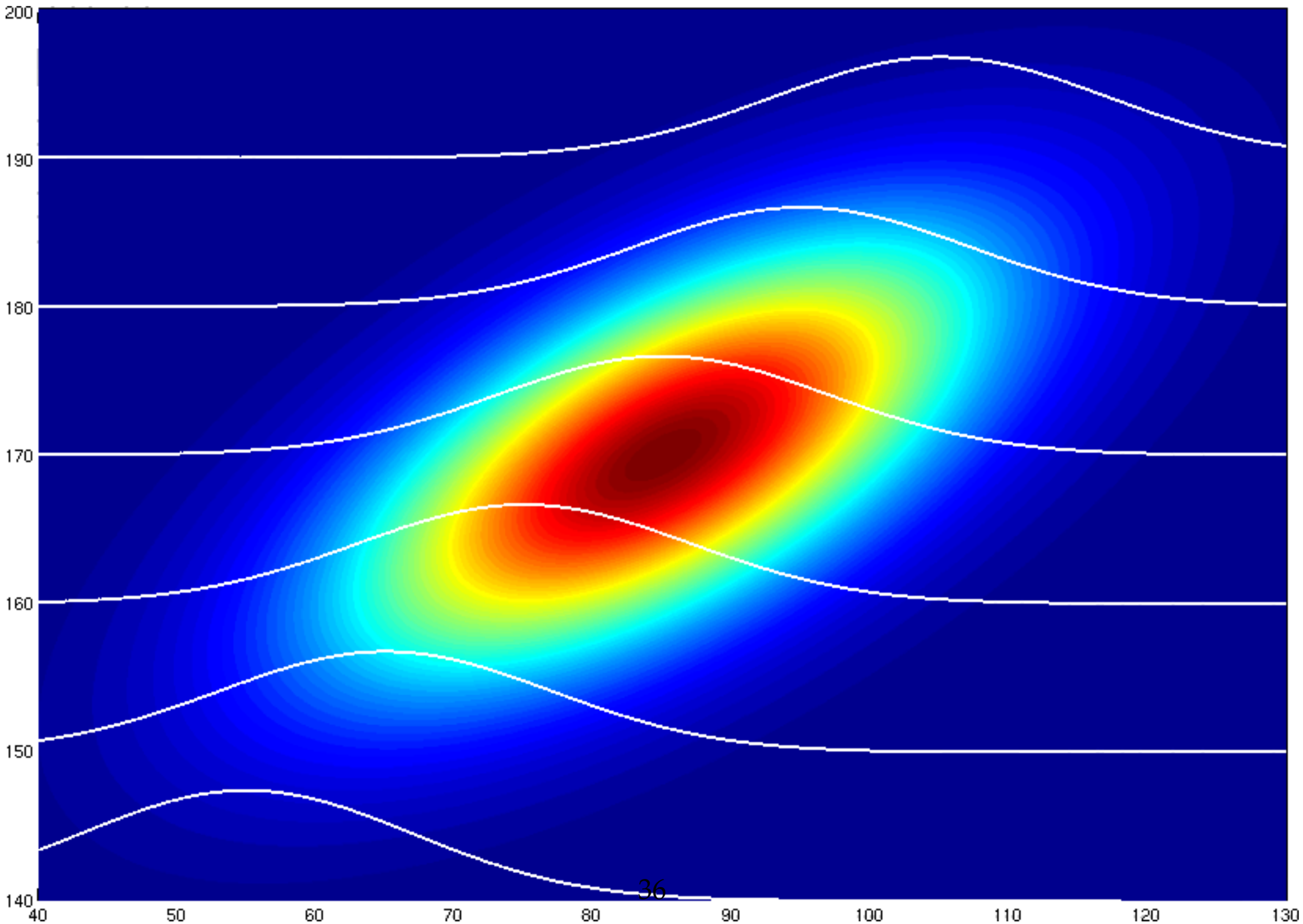
$$x_{i-1} \sim \mathcal{N}(\mu_{i-1}, \Sigma_{i-1})$$

- Hence we know how the state evolves

$$\begin{aligned}
(x_k, z_k) &= (Ax_{k-1} + Bu_{k-1} + w_{k-1}, Hx_k + v_k) \\
&= (Ax_{k-1} + Bu_{k-1} + w_{k-1}, HAx_{k-1} + HBu_{k-1} + Hw_{k-1} + v_k) \\
&\sim \mathcal{N}\left[\begin{bmatrix} Ax_{k-1} + Bu_{k-1} \\ HAx_{k-1} + HBu_{k-1} \end{bmatrix}, \begin{bmatrix} Q & QH^\top \\ HQ & HQH^\top + R \end{bmatrix}\right] \\
&\sim \mathcal{N}\left[\begin{bmatrix} A\mu_{k-1} + Bu_{k-1} \\ HA\mu_{k-1} + HBu_{k-1} \end{bmatrix}, \begin{bmatrix} Q + A\Sigma_{i-1}A^\top & QH^\top + A\Sigma_{i-1}H^\top A^\top \\ HQ + AH\Sigma_{i-1}A^\top & HQH^\top + R + HA\Sigma_{i-1}A^\top H^\top \end{bmatrix}\right]
\end{aligned}$$

- So we can estimate $z_k | x_k$
  This is normally distributed (all are Gaussian)

Conditioning



Kalman Filter Intuition

- Kalman filter pass
  - Given $(x_i, z_i) \sim \mathcal{N}(\mu, \Sigma)$ **estimate** $x_i | z_i \sim \mathcal{N}(\mu_i, \Sigma_i)$
  - Use this to predict $x_{i+1} \sim \mathcal{N}(Ax_i + Bu_i, Q)$
  - Get Normal distribution for (x_{i+1}, z_{i+1}) ...
- Rauch-Tung-Striebel (backward) filter pass
  With the benefit of hindsight, find better estimates of the hidden state x
- Parameter estimation
  Use maximum likelihood / MAP for A,B,H,Q,R

See also http://www.cs.nyu.edu/~roweis/papers/NC110201.pdf

Topic Models

Topics in a document

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

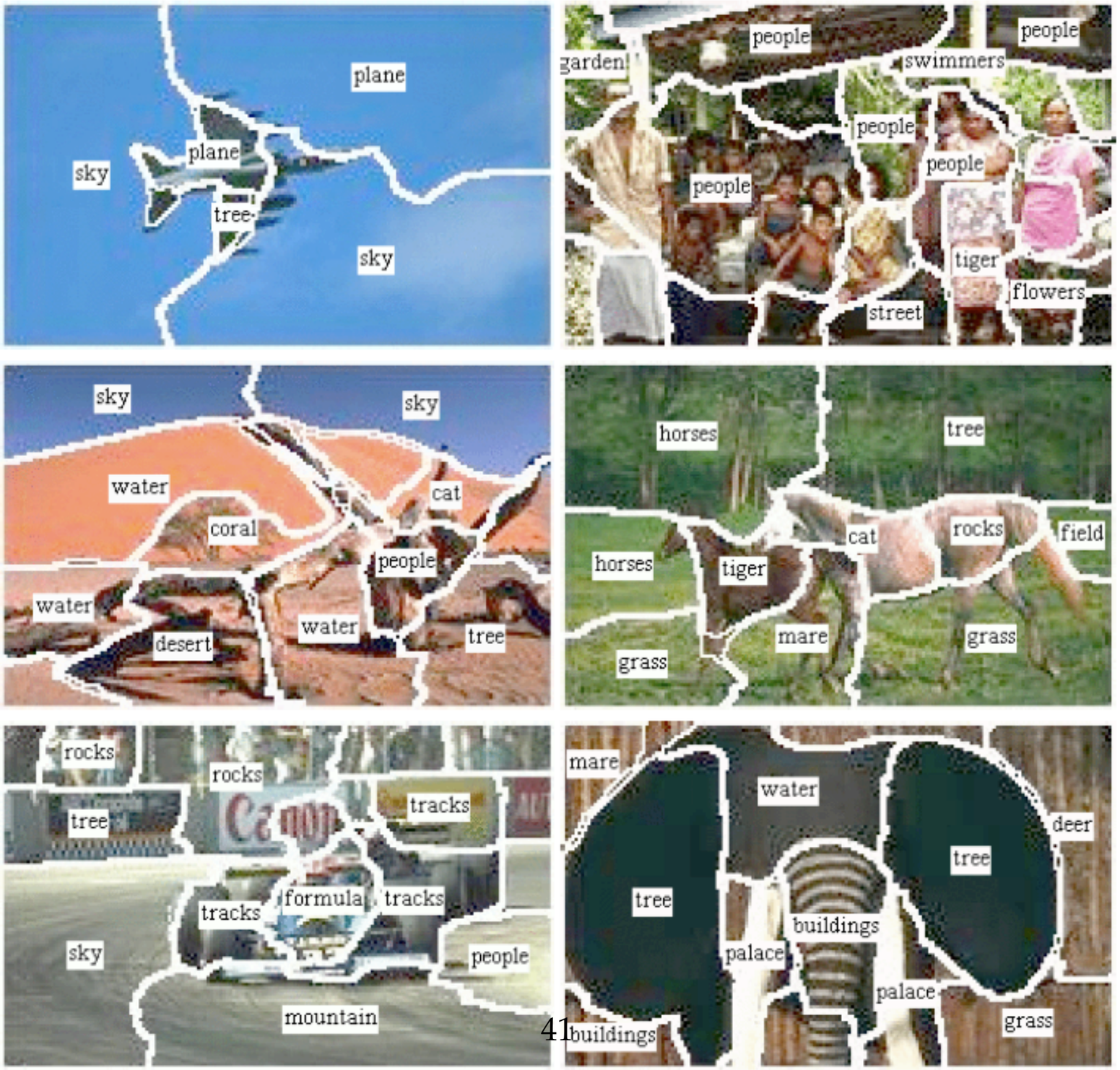
The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

Topics vs. Clustering

Topics vs. Clustering

Topics vs. Clustering

Images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grass
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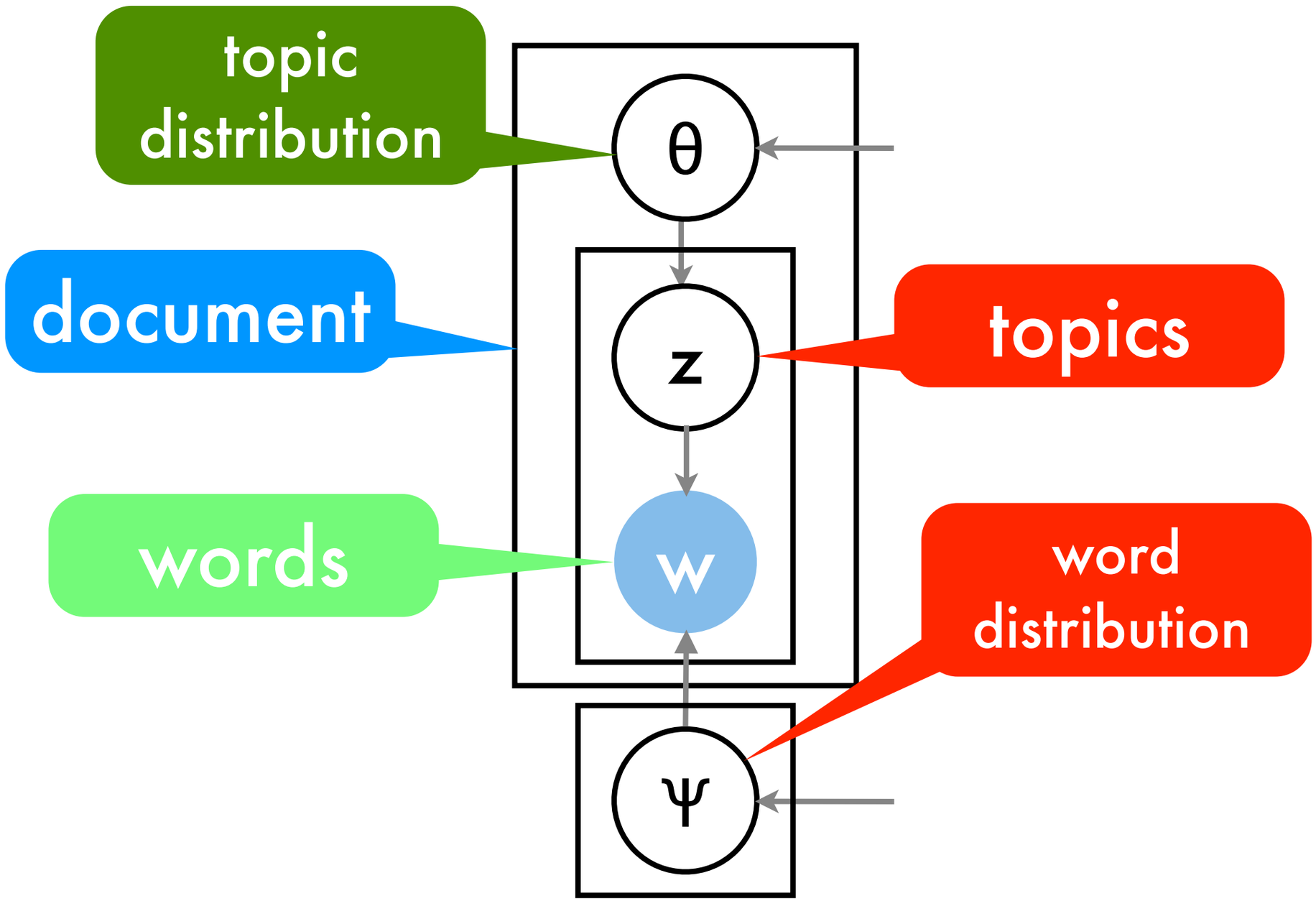
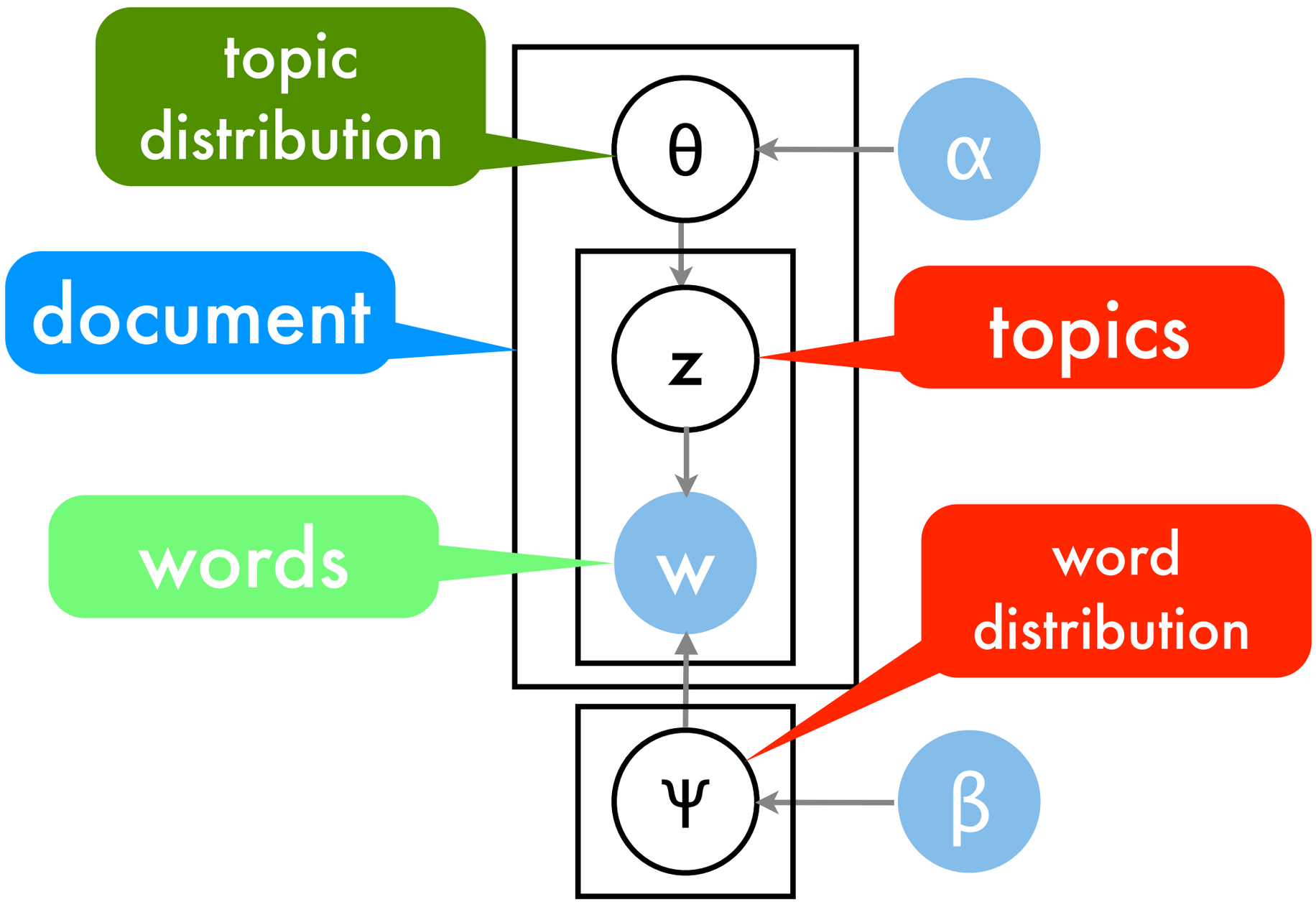
Topic Models



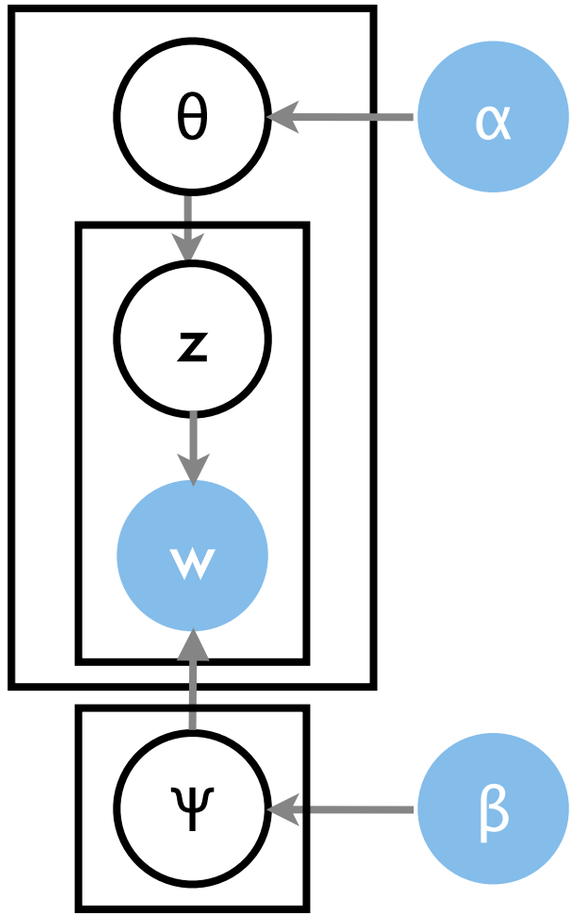
Topic Models



Joint Probability

$$p(w, z, \theta, \psi|\alpha, \beta) = \prod_{i=1}^{m} p(\theta^i|\alpha) \prod_{i,j}^{m,m_i} p(z_{ij}|\theta^i) p(w_{ij}|z_{ij}, \psi) \prod_{i=1}^{k} p(\psi^i|\beta)$$

- Estimating topics by maximizing $p(\psi|w, \alpha, \beta)$ is intractable
- EM Algorithm is still intractable
  $q(\theta, z) = p(\theta, z|\psi, w, \alpha, \beta)$
  does not decompose
- Variational approximation
  $q(\theta, z) = \prod_{i} q_i(\theta^i) \prod_{ij} q_{ij}(z_{ij})$



The gory math

- Variational E-Step

$$q^* = \operatorname*{argmin}_q D\left(\prod_i q_i(\theta^i) \prod_{ij} q_{ij}(z_{ij}) \| p(\theta, z | w, \alpha, \beta, \psi)\right)$$

Dirichlet parameters for topics $a_{it} = \alpha_t + \sum_j q_{ij}(t)$

Topic probabilities for i,j $q_{ij}(t) \propto \psi_{w_{ij},t} \exp \Psi(a_{it})$

- M-Step

$$\psi_{wt} \propto \sum_{ij} q_{ij}(t) \{w_{ij} = w\}$$

In practice use a collapsed sampler instead (smaller memory footprint, more accurate)

Topics from Y!Directory

- game baseball state runs run win inning season hits innings conference home field series scored rbi team lead doubleheader hit victory games afternoon sweep pitcher saturday week sunday single university score southern loss play bottom owls ninth double opener complete junior seventh fourth record named tuesday weekend split sixth eagles earned top senior pitched stadium allowing friday tigers ranked eighth hitter bulldogs lions final matt winning left mercer night wednesday wildcats straight back led homer allowed pair defeated action won college scoring ryan pitching walk baseman smith base vmi mike wins streak struck drove player picked wagner pitch strikeouts dropped
- movies teen cock pussy girls ass blonde tits naked hot sexy big girl babe fucked thumbnails cum pix panties babes fucking fuck pics sucking nude hard young black teens brunette butt busty horny amateur huge wet blow lesbian tight lingerie slut clips boobs cute nipple mpg breast shaved upskirt sweet asian facial bikini dick lesbians chick cunt panty blondes mature tit galleries vids movie posing hardcore white anal blowjob natural bukkake sluts mpeg fakes showing college breasts pink dildo cocks nice gorgeous chicks suck ebony redhead latina nasty face action legs nurse licking stripping hairy love wife schoolgirls body whore
- iraq war iraqi saddam baghdad military security bush hussein iraqis troops al government gulf forces abu coalition intelligence soldiers weapons conflict international administration middle report east american oil reconstruction ghraib pentagon united world iran kuwait country officials killed regime rumsfeld news afghanistan army attacks president blair general invasion occupation foreign mass british arab civilian reports police defense people wmd council freedom torture destruction soldier minister detainees human official post insurgents americans abuse media democracy civilians nations march fallujah secretary operation political humanitarian casualties washington saudi attack prisoners press april support senior region sanctions syria house bombs central special peace states
- ship ships sea captain crew vessel boat port navy submarine vessels men deck water aboard maritime boats coast hull board voyage built miles feet ocean monitor salvage cargo naval ss commander guns sinking merchant war fleet officer seas rescue wreck atlantic titanic sank passengers depth officers lost shore crevalle hms great small bow sailors lieutenant sunk sail side steam tons speed bottom ports long sailed days marine patrol line waters left heavy command action deep made large disaster sailing passenger squadron young surface mate bridge convoy shipping stern bay south underwater torpedo room gun survivors crews engine iron forward shot

The meanings of Jordan

- **Country** france germany italy spain australia canada united japan sweden china south netherlands mexico switzerland belgium ireland africa brazil kingdom denmark portugal russia austria india usa zealand norway greece korea republic finland country turkey argentina poland singapore england uk countries israel hungary states czech hong kong europe taiwan world thailand malaysia chile romania select international croatia scotland egypt indonesia philippines bulgaria slovenia america luxembourg peru holland iceland britain worldwide rica venezuela asia saudi arabia slovakia costa ukraine pakistan morocco colombia cyprus malta estonia uruguay vietnam ecuador lithuania latvia arab rico yugoslavia iran wales puerto global european choose emirates tunisia jordan panama
- **Girl's names** anna jennifer amy playboy lisa rachel sarah michelle nicole heather amanda melissa christina kelly kate jessica rebecca laura ashley katie kim julie maria marie victoria elizabeth lee karen lauren eva heidi linda jenny stephanie cindy girls playmate angela liz anne catherine smith vanessa girl amber emma patricia emily tina monica taylor shannon claire brooke tiffany donna lynn jones nikki moore anderson pamela julia dawn natalie renee carmen leslie holly samantha jill nina alicia gina ryan jamie sharon claudia denise de lori williams andrea debra danielle tyler jordan erica jane tara mary joan barbara diane megan tanya sophie celebrity courtney brittany
- **Middle East** israel israeli jewish jerusalem palestinian arab palestine east middle lebanon jews herzl zionist world anti land palestinians sharon aviv state peace zionism jordan israelis gaza syria west bank diaries national ariel tel beirut minister arabs fisk semitism political haifa arafat holy ahram hebrew lebanese zion egypt prime jihad el support sea british security semitic david people syrian ben international movement mount strip abbas conflict iran territories muslim al religious united including adl zionists hamas islamic egyptian home eastern leaders organization dead jtf intifada galilee armageddon pollard hadassah nation christian times holocaust michael war mossad plan protocols kibbutz years karaim letter
- **Superstars** michael jackson jordan eugenics twins scottie pippen family pca www gavin living twin eugenic rowe neverland shine brought comments ken coverage spit trial brand boy janet sterilization galton michaeljacksontrials kenfrost molestation shape accuser part arvizo ranch shoe eugenicists buck chandler barbara man grand prosecution polish children stand associates oneida melville noyes hereditary boys mike told davenport testimony documentary mother singer santa tsg defence team book imes falls laughlin lin mr child traits star included chicago made shapeup past yesterday mj families heredity bashir security alleged feeble bell philosophical brother pearson jackpot francis jordie unfit twinning pop hygiene leno eugenicist mip
- **Basketball** nba bulls game basketball lakers team draft season bryant pistons fg rebounds stats kobe players pacers def spurs nets pts ast wizards pf celtics tot ft mavericks rockets player kings nuggets coach playoffs playoff sports star knicks lebron blk heat ers teams stl year magic round clippers min suns news aba apg mavs yao grizzlies league finals bucks college pick fgm ppg guard hornets ftm mapct warriors points jazz pm jersey hawks detroit aldridge scoring cavaliers assists indiana court shaq neal totals raptors trail postseason jordan timberwolves iverson bpg spg blazers supersonics averages forward antonio state dallas ming career houston
- **Islamic World** islamic islam al muslim iran ali arabic muslims turkish arabia pakistan arab shi saudi persian muhammad iranian el imam mosque middle turkey east shia sufi abdul abu iraq egypt ibn sunni ahmad halal world khan arabian ite syria mohammed ahmed kuwait ottoman shah sultan ism religious omar qatar bahrain persia sufism arabs ites gulf english mohammad dubai caliph lebanon religion caliphate sheikh law abdullah imams msa tribal tehran hasan amila yemen ibrahim prophet mosques elite mecca india cairo hassan west din emirates ce baghdad qur morocco oman mohamed istanbul abd rumi countries western wahhabi language scholars jordan turks crescent abbas
- **TV Show**s ff aa bd fw ba fc fb af nbc fd ac bf bc order fe law ab ad night tt pk news fs factor bm home featured specials late eb bb apprentice dmf daytime wh ef dc fear lm days meet select online mlc special call west daly video grace intent features fa aaa lax dateline access kd watch unit las brien press live lp ccf sports vegas joey discovery games wing er ea cg criminal office primetime dreams casting medium show medical victims crossing contender american biggest scrubs winners jordan nightly web committed trial kids treasure movies music today
- **Country** islands republic cellular guinea united island india st south korea french mexico netherlands africa virgin kenya indonesia egypt pakistan congo china colombia bangladesh samoa arab arabia lanka thailand australia nigeria saudi sri turkey guatemala saint ghana lebanon morocco uganda trinidad jordan bolivia jamaica romania peru ukraine ecuador ireland georgia malaysia philippines iran armenia tanzania bulgaria venezuela japan salvador nicaragua israel cambodia poland country panama kuwait states nepal costa chile russia cyprus senegal zimbabwe dominican ethiopia el brazil rica taiwan honduras greece algeria argentina hungary haiti bahamas zealand zambia estonia mauritius barbados yemen cameroon czech france albania vietnam fiji oman croatia

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view our exclusive day playability guarantee golf savings with a monster attitude clubs bags balls shoes nbsp gloves apparel accessories site features mens woods mens irons womens woods womens irons discount book store video store golf tip of the month latest golf news your golf weather golf links home proshop provided by tgw privacy policy guarantee contact customer service legal notices security all other inquiries please contact at copyright rights reserved your online golf store for discounts on brand name golf equipment apparel golf clubs golf shoes and more to speak to a customer service representative directly call toll free international thank you and we appreciate your patronage updated weekly bookmark now free wedge with purchase cleveland mens tour action irons click for price great new low price on cleveland irons cleveland gunmetal low bounce wedge value free with purchase proshop powered by tgw prices starting at only taylormade titanium woods sale click for price great price on taylormade ti t miss out proshop powered by tgw free rescue wood w set purchase taylormade rac irons with free rescue mid wood click for price taylormade rac feel forgiveness free taylormade rescue mid wood with purchase of rac irons set click for full offer details proshop powered by tgw today s top sellers gift certificates taylormade r quad drivers callaway big bertha x irons odyssey white steel putters taylormade mens rac os irons more items proshop powered by tgw browse by manufacturer adams ashworth callaway cleveland cobra datrek dexter dunlop etonic footjoy hogan izzo lajolla maxfli mizuno nike never compromise norman collection odyssey orlimar ping powerbilt precept seemore sun mountain taylor made titleist tommy armour wilson proshop powered by tgw deal of the week free adams stand bag value with idea combo irons set purchase click here for prices adams golf idea accuracy easy proshop powered by tgw pants shorts sale save big on top of the line name brands click here for prices who cares how good you putt hitting it far is what counts proshop powered by tgw brand name golf ball sale save big on titleist nike callaway top flite and more click here for price save up to on selected golf ball purchases book of the week golf digest s ultimate drill book over drills that are guaranteed to improve every aspect of your game and lower your handicap one of america s leading

0.53: golf, clubs, club, swing, ball, game, balls, irons, putting, woods, pro, bags, bag, driver, handicap, custom, tour, equipment, putter, shaft,
0.18: price, sale, click, cart, store, info, list, items, search, view, shopping, buy, shipping, privacy, shop, policy, specials, products, retail, save,
0.03: shoes, clothing, skateboard, ski, lbs, bags, shirt, shirts, accessories, black, shop, snowboard, shoe, apparel, boots, men, white, skate, footwear, women,

Some applications

- Synchronize multilingual database via topics
- Webspam/Mailspam detection
- Coverage / diversity in search
- Features for MLR / integration with tags
- Automatic ontology construction
- PSOX information extraction / resolution
- Personalization of sessions / users
- Collaborative filtering (COKE, sponsored search)

Outlook

undirected graphical models

- Hammersley Clifford Theorem
- Junction Trees
- Message Passing
- Generalized Distributive Law
- Conditional Random Fields
- Connections to Classification and Regression
- Annotation, tagging, structured estimation
- Max-Margin-Markov Networks