Recitation

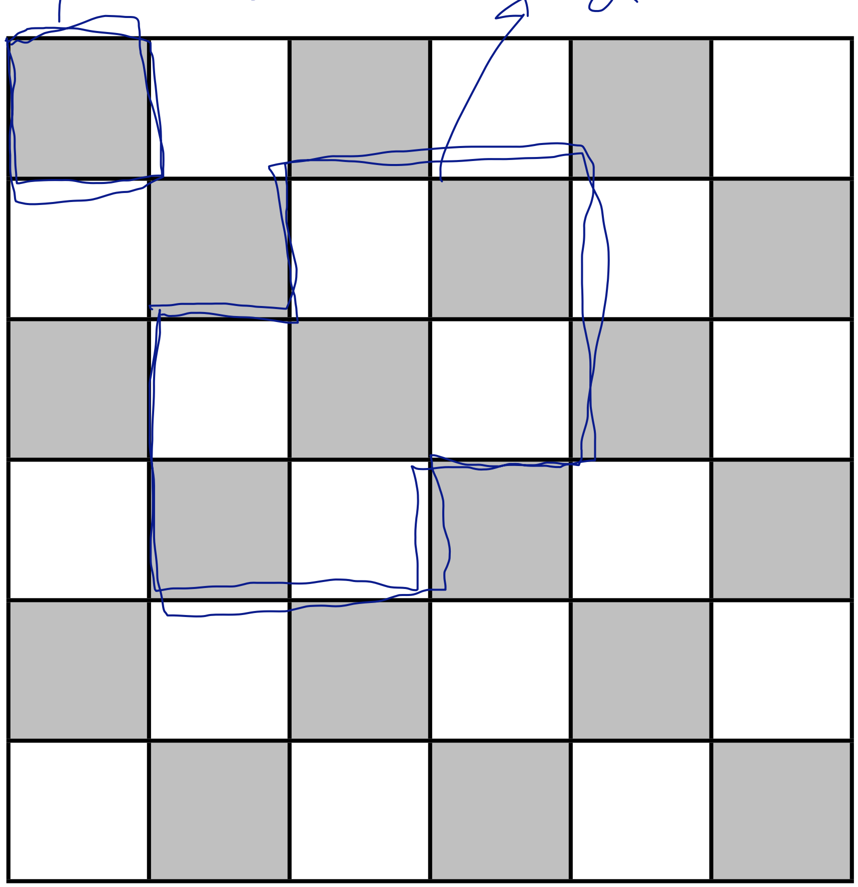
- First recitation tomorrow 5–6:30 here
- Linear algebra

Probability

atomic event

event a

U = universe $\longrightarrow P = 1/|U| = 1/36$



$P(a) = 7/36 = |a|/|U|$

$P(u) = |U|/|U| = 1$

$P(\sim a) = 1 - P(a) = 29/36$

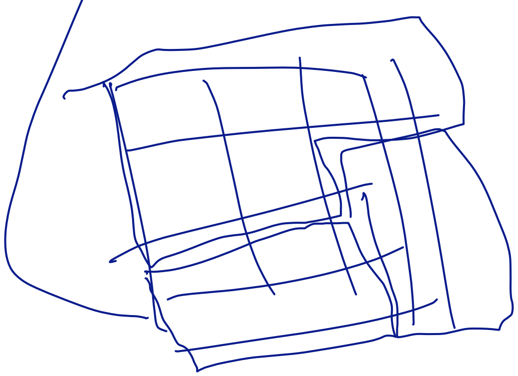
$\neg a = U \setminus a$

Conventions

a $\quad A = \{a, \neg a\}$ $\quad P(a)$ $\quad P(A) = [P(a)\ \ P(\neg a)]$

MEEP $A = \{a_i\}$ $\quad a_i \cap a_j = \phi \quad i \neq j$

a $\quad a_1 \ldots a_n$ $\quad \bigcup_i a_i = U$ $\quad P(A) = [P(a_1) \ldots P(a_n)]$



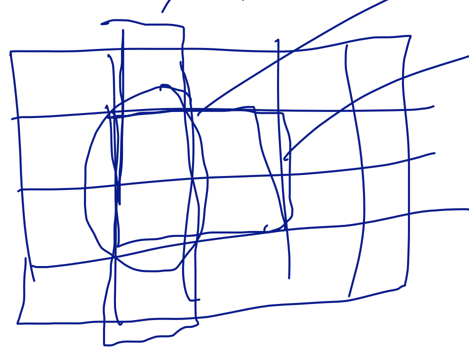
MEEP A, B $\quad P(A,B)\ [P(a_i \wedge b_j)]$

$a_i \cap b_j$

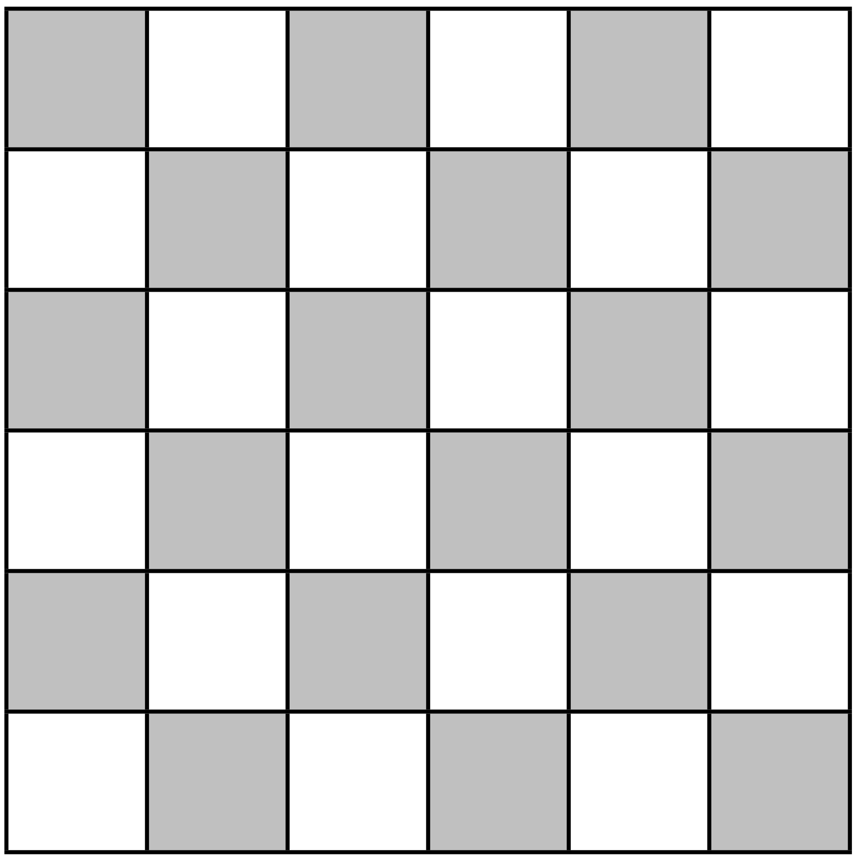
$a_i \wedge b_j$ $\quad$ over i, j

$\neg a_i$

b_j



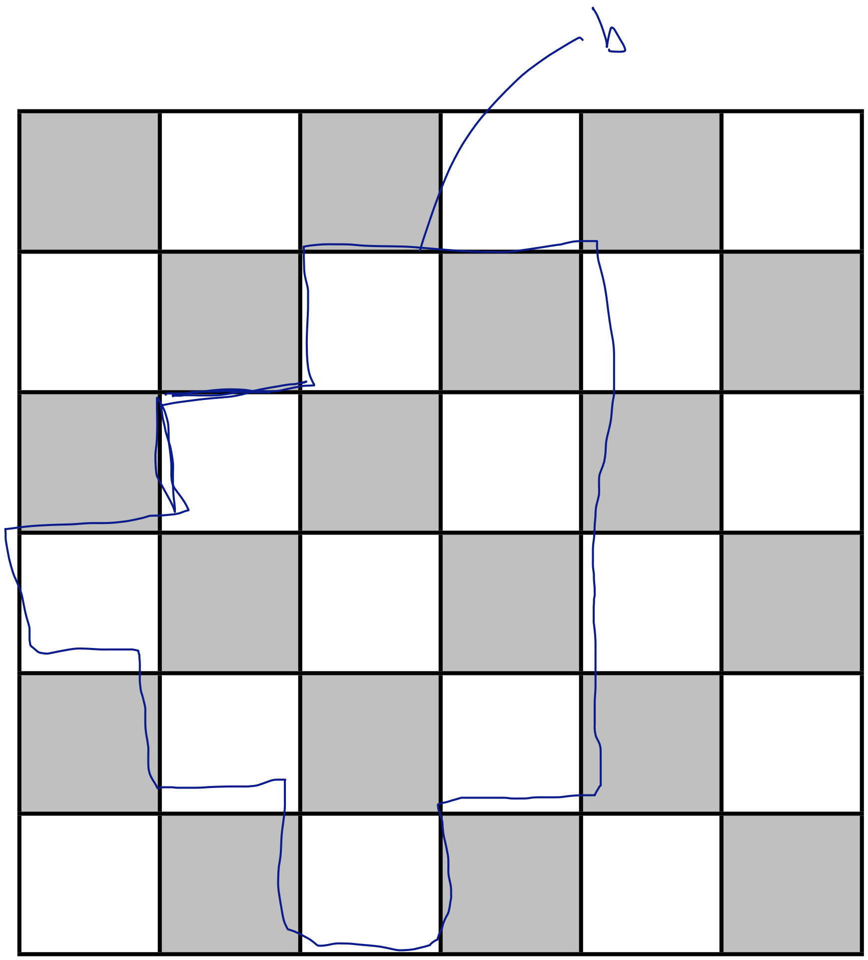
Union, intersection



$a \wedge b$ $\quad a \cap b$

$a \vee b$ $\quad a \cup b$

Conditioning



$b \mid a_1$

$a_1 \rightarrow 0$

$a_2 \rightarrow 2/13 = \dfrac{|a_2 \cap b|}{|b|} = P(a_2 \mid b)$

a_j

a_6

MEEP

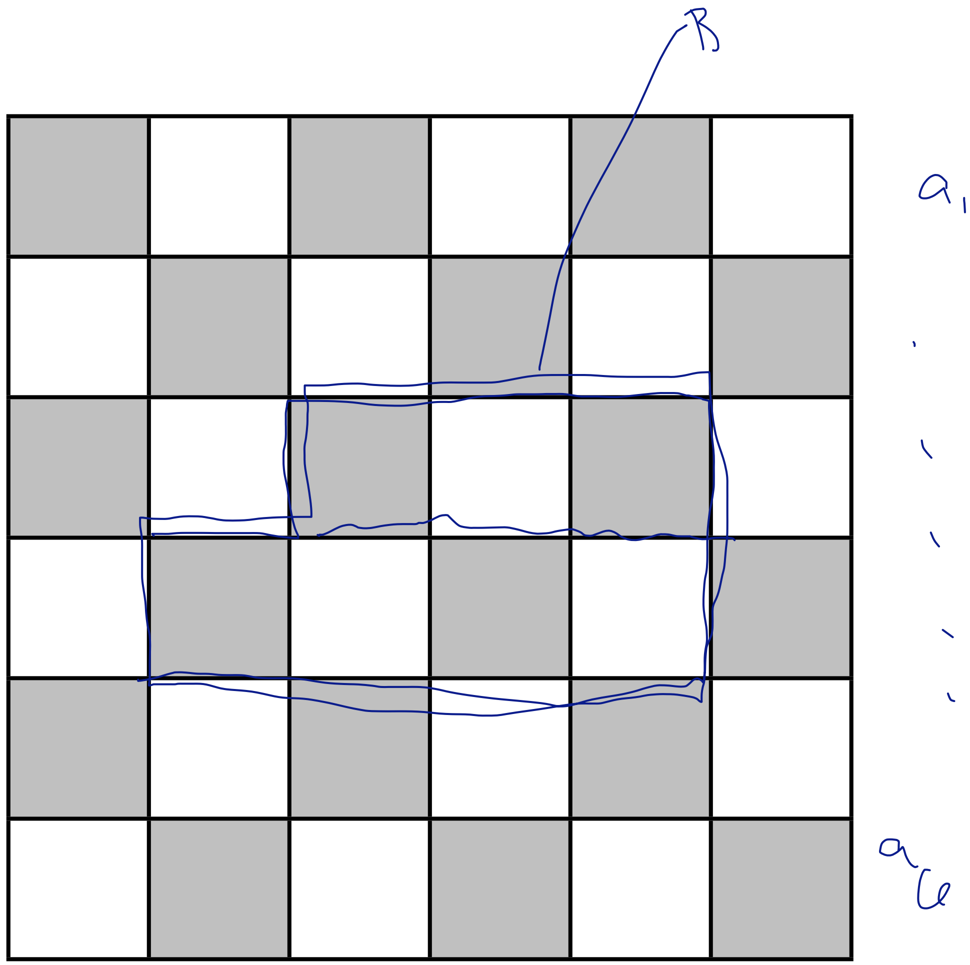
A, B

$P(A \mid B)$

$[P(a_i \mid b_j)]_{ij}$

CPT

Law of total probability



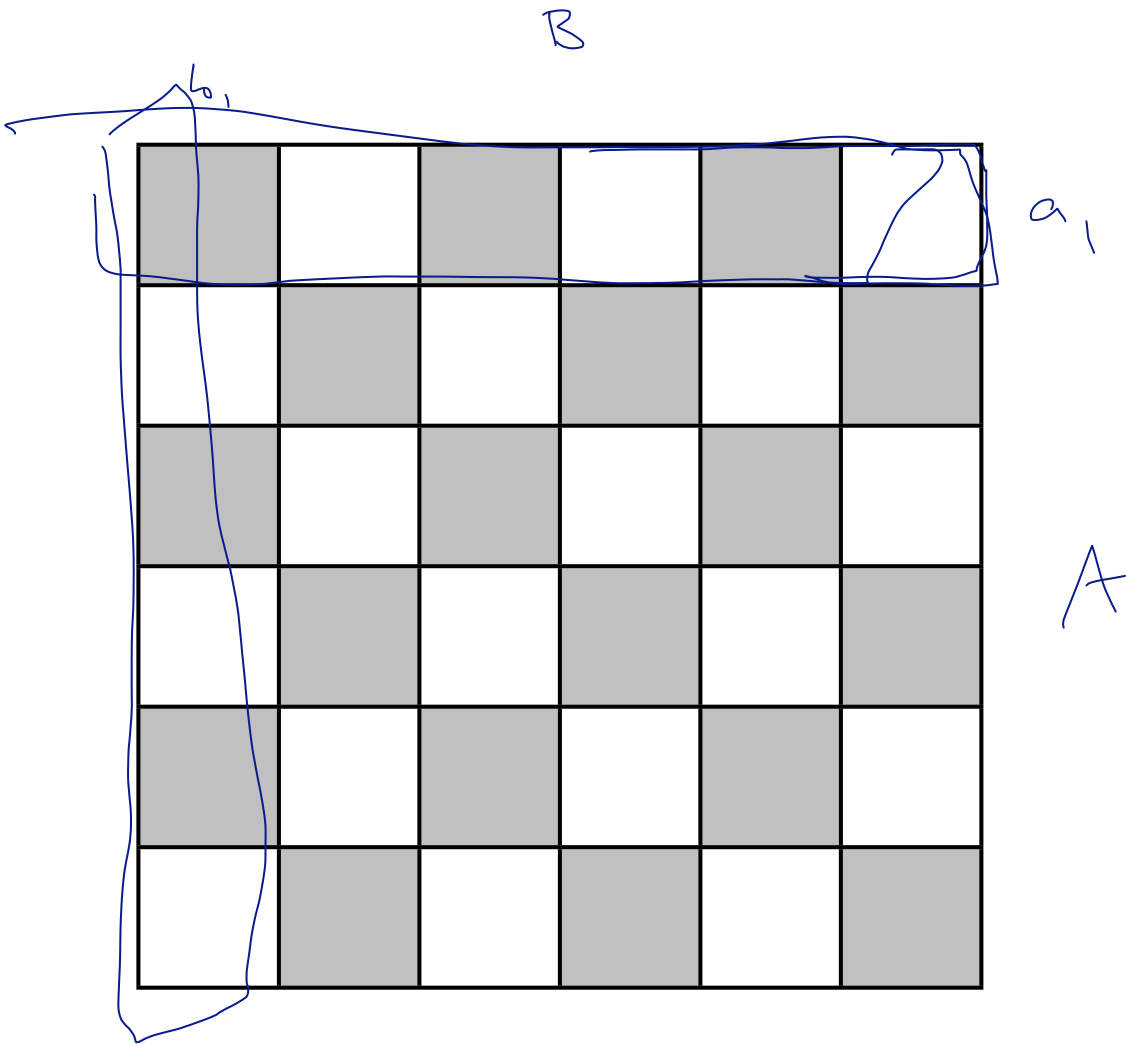
$p(b) = p(a_3 \wedge b)$
$\quad + p(a_4 \wedge b)$

MEEP A

$p(b) = \sum_{a_i \in A} p(b \wedge a_i)$

exercise
prove $\sum_i p(a_i \mid b) = 1$

Marginals



$P(A, B)$

$P(A)$

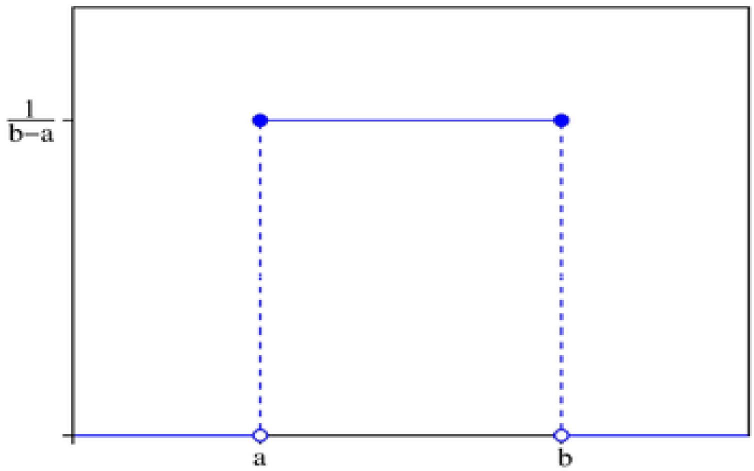
$$p(a_i) = \sum_j P(a_i, b_j)$$

Finite vs. infinite |u|

- http://www.amazon.com/Probability-Measure-Wiley-Series-Statistics/dp/1118122372
- http://en.wikipedia.org/wiki/Regular_conditional_probability
- http://en.wikipedia.org/wiki/Borel%E2%80%93Kolmogorov_paradox

Banach-Tarski theorem

How I learned to stop worrying and love the density function…

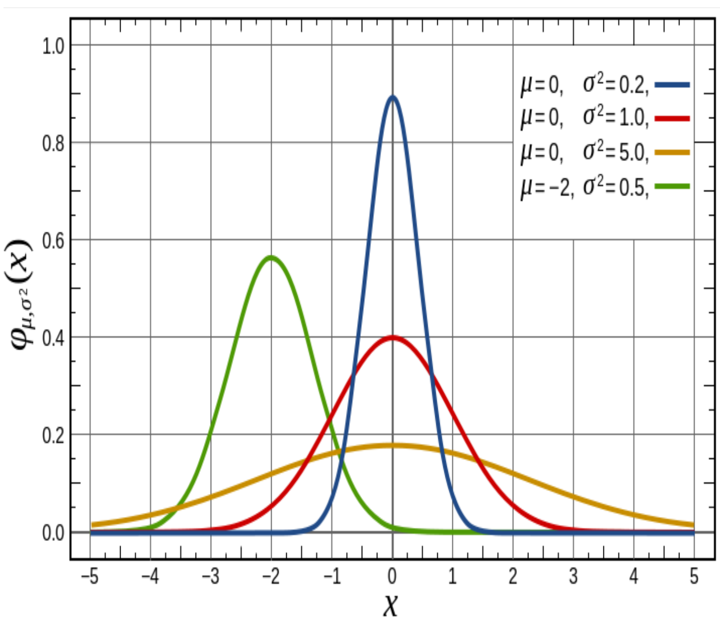


$$\begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{o/w} \end{cases}$$

$p(x)$

st $\int_U p(x)\,dx = 1$

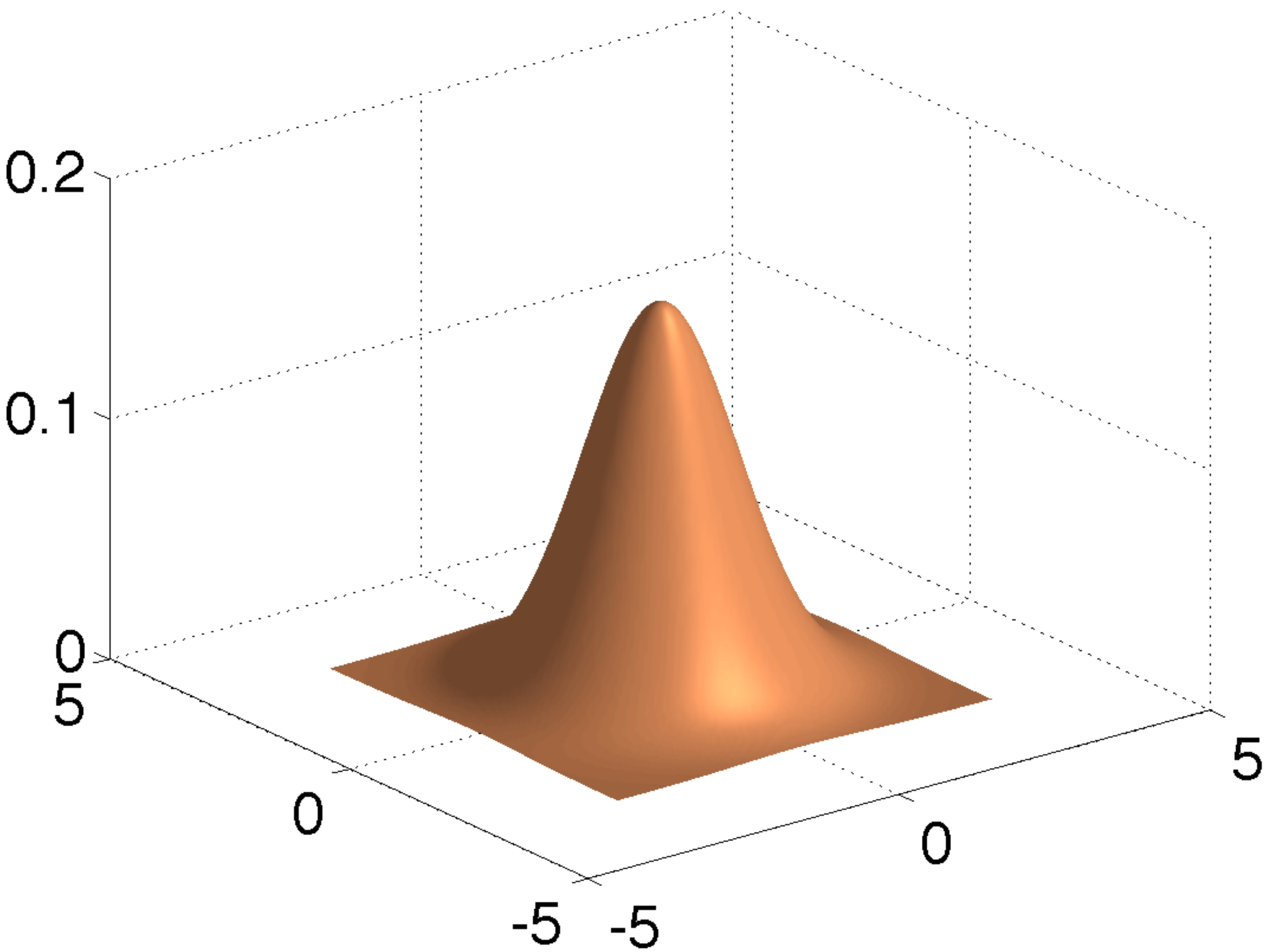
$\frac{p(a \wedge b)}{p(b)} = p(a \mid b)$



$$\frac{1}{\sqrt{2\pi}\sigma}\exp(-\tfrac{1}{2}(x-\mu)^2/\sigma^2)$$

Multivariate densities

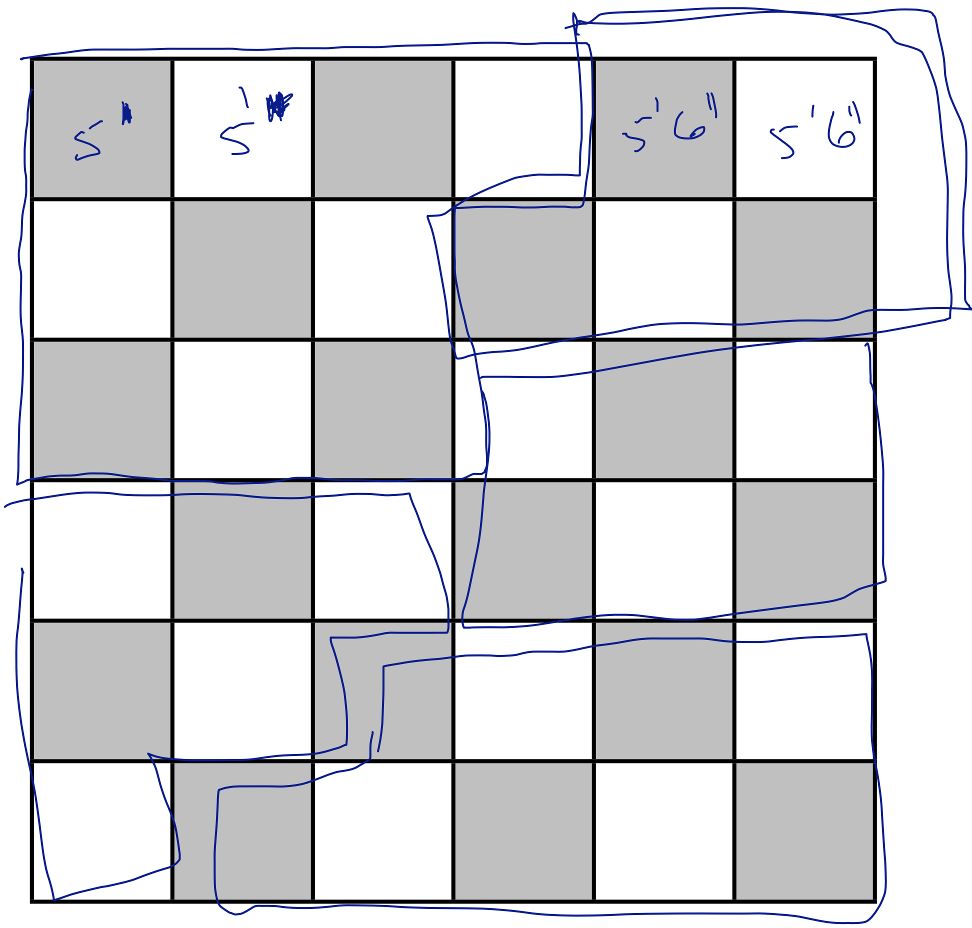
$p(x, y) \quad \mathbb{R}^2 \to \mathbb{R}$



$703 \frac{\text{lbs}}{(\text{inches})^2}$

Random variables

fn of atomic events



X = height

$X = 5'6"$ ← event

$X = x$

Probability space
(σ-algebra)

Bayes rule

- recall def of conditional:
  - P(a|b) = P(a^b) / P(b) if P(b) != 0

a = model
b = data

$P(a|b)P(b) = P(a \wedge b) = P(b|a)P(a)$

$$\boxed{P(a|b) = P(b|a)P(a) / P(b)}$$

$P(b) \neq 0$