Bayes Nets

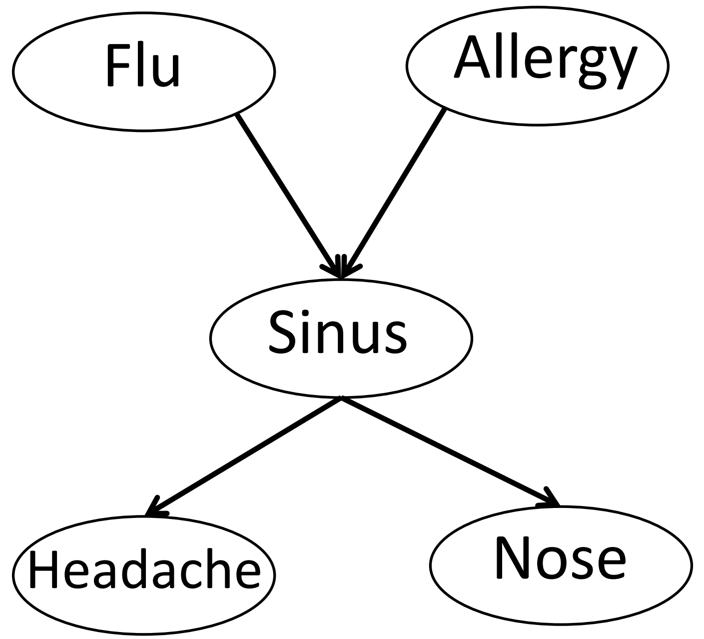
10-701 recitation

04-02-2013

Bayes Nets

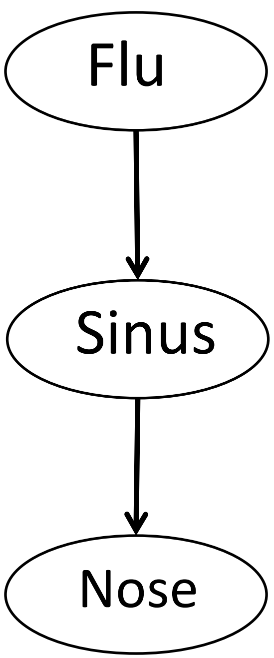
- Represent dependencies between variables
- Compact representation of probability distribution

- Encodes causal relationships



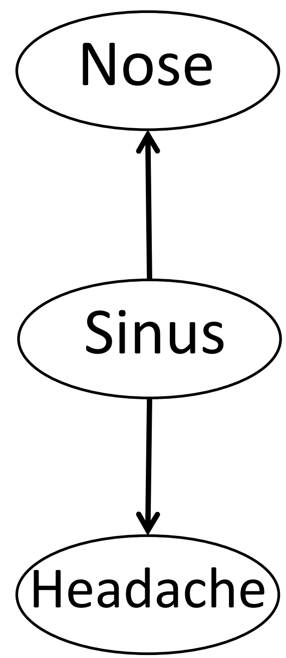
Conditional independence

- $P(X,Y|Z) = P(X|Z) \times P(Y|Z)$



$F \text{ not} \perp N$

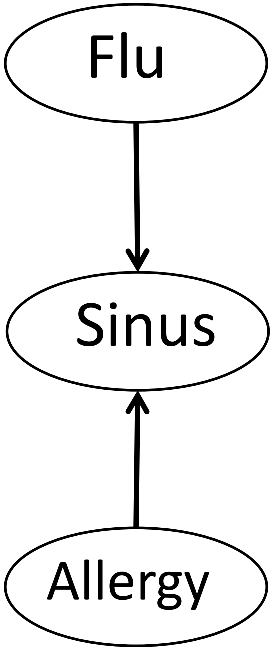
$F \perp N \mid S$



$N \text{ not} \perp H$

$N \perp H \mid S$

Conditional independence



$F \perp A$

$F \text{ not} \perp A \mid S$

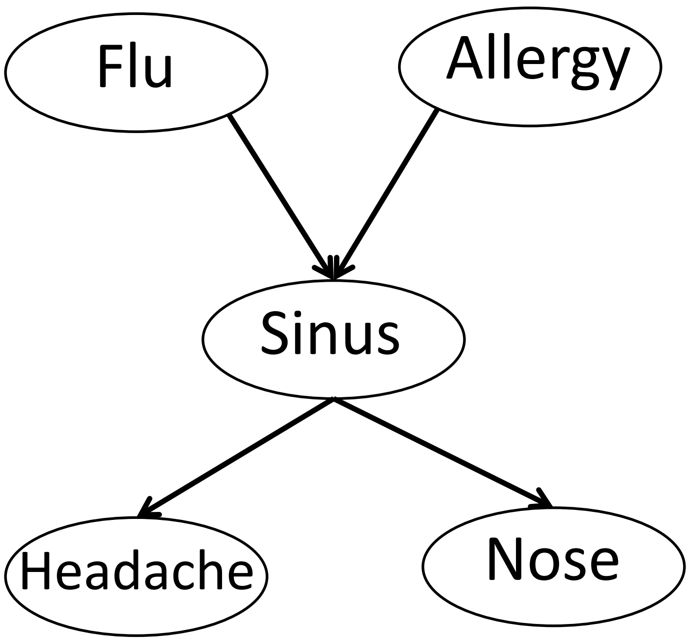
- Explaining away:
  - P(F = t | S = t) is high
  - But P(F = t | S = t , A = t) is lower

Joint probability distribution

- Chain rule of probability:

  $P(X_1, X_{2,} \ldots, X_n) = P(X_1)\, P(X_2|X_1) \ldots P(X_n|X_1,X_2 \ldots X_{n-1})$

Joint probability distribution

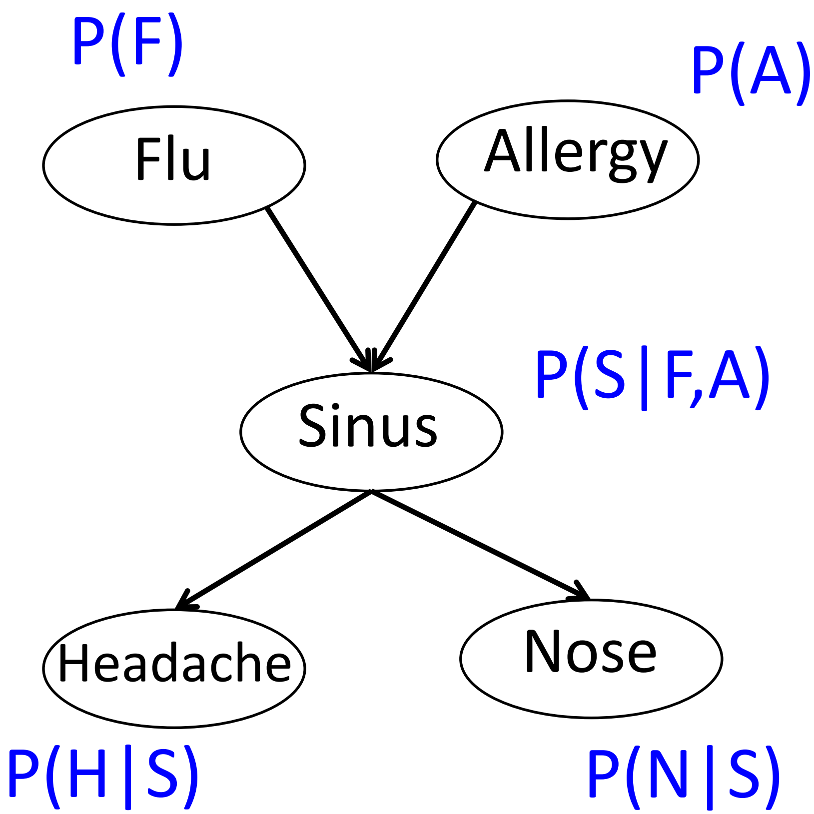


- Chain rule of probability:

$$P(F,A,S,H,N) = P(F)\, P(A|F)\, P(S|A,F)\, P(H|F,A,S)\, P(N|F,A,S,H)$$

Table with 2^5 entries!

Joint probability distribution

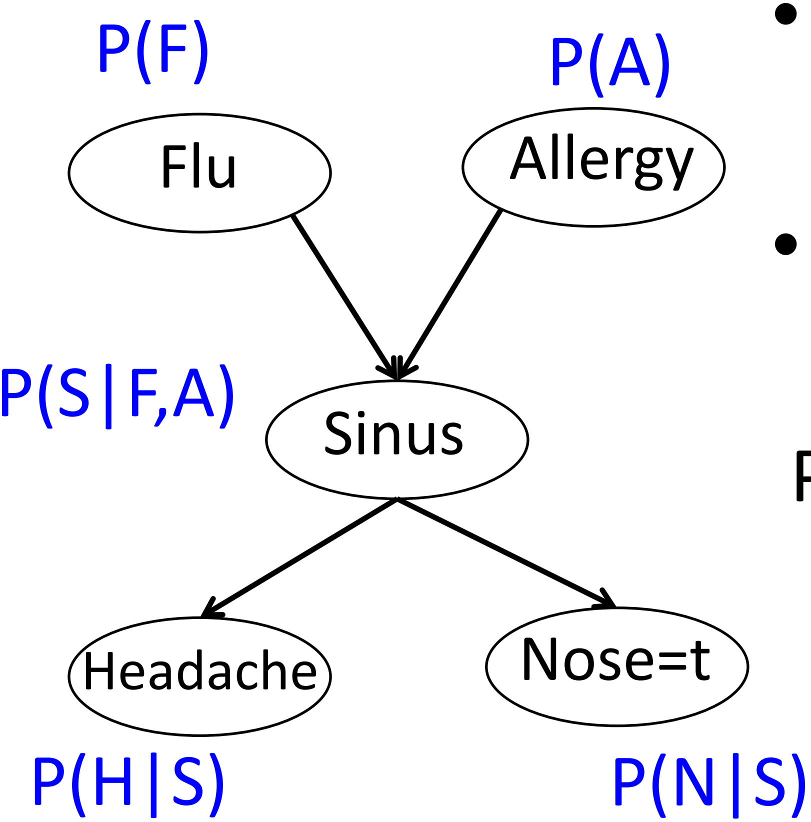


- Local markov assumption: A variable X is independent of it's non-descendants given it's parents

$$P(F,A,S,H,N) = P(F)\,P(A)\,P(S|A,F)\,P(H|S)\,P(N|S)$$

	F = t, A = t	F = t, A = f	F = f, A = t	F = f, A = f
S = t	0.9	0.8	0.7	0.1
S = f	0.1	0.2	0.3	0.9

Queries, Inference

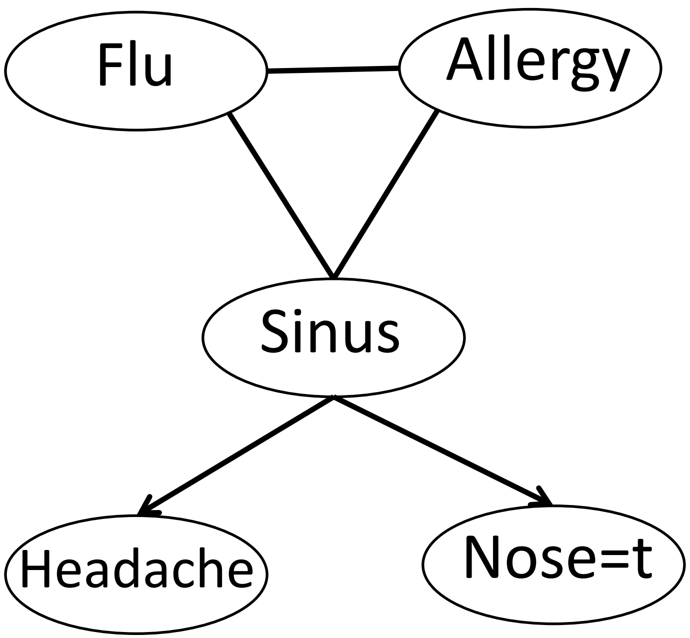


- $P(F = t \mid N = t)$?
- $P(F=t|N=t) = P(F=t,N=t)/P(N=t)$

$$P(F,N=t) = \Sigma_{A,S,H}\, P(F,A,S,H,N=t)$$

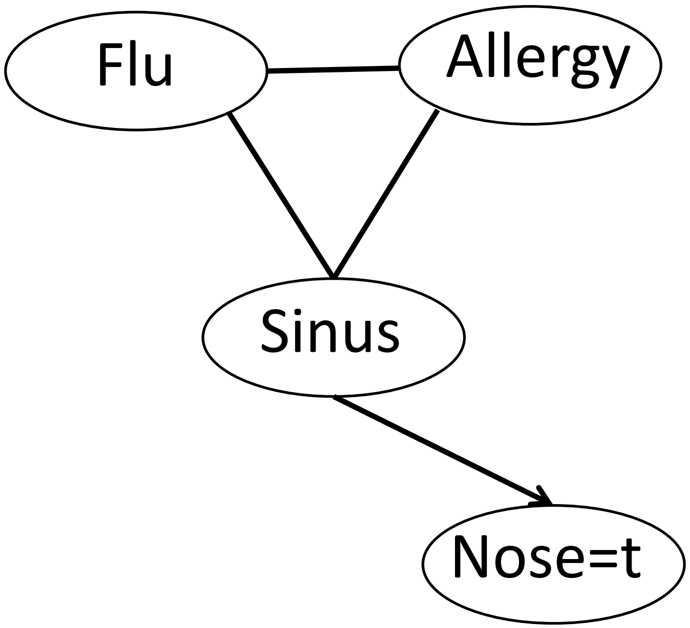
$$= \Sigma_{A,S,H}\; P(F)\, P(A)\, P(S|A,F)\, P(H|S)\, P(N=t|S)$$

Moralizing the graph



- Eliminating A will create a factor with F and S
- To assess complexity we can moralize the graph: connect parents

Chose an optimal order



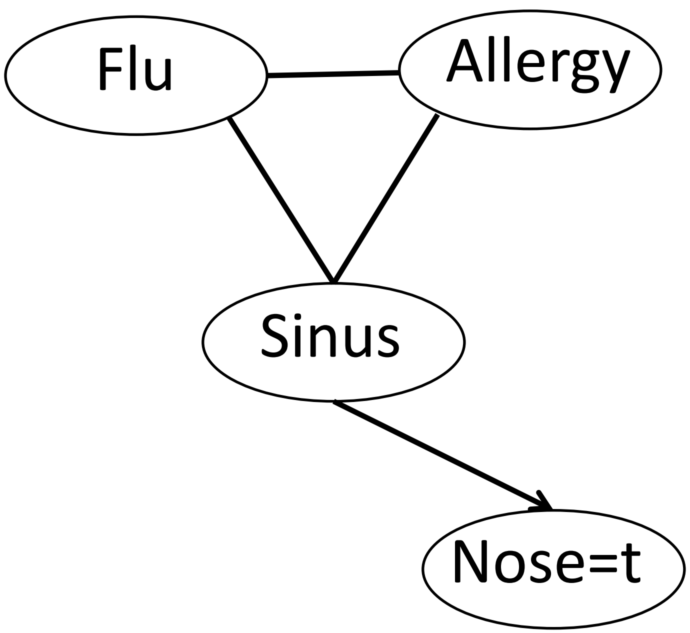
If we start with H:

$$P(F,N=t) = \Sigma_{A,S}\ P(F)\ P(A)\ P(S|A,F)\ P(N=t\ |S)\ \Sigma_H\ P(H|S)$$

=1

$$= \Sigma_{A,S}\ P(F)\ P(A)\ P(S|A,F)\ P(N=t\ |S)$$

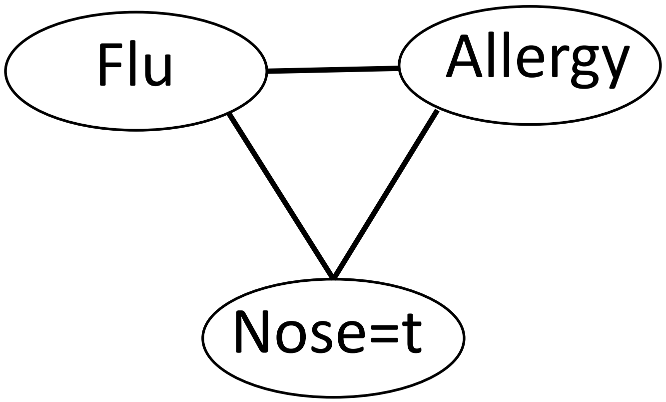
Removing S



$$P(F,N=t) = \Sigma_{A,S}\ P(F)\ P(A)\ P(S|A,F)\ P(N=t\ |S)$$

$$= \Sigma_{A}\ P(F)\ P(A)\ \Sigma_{S}\ P(S|A,F)\ P(N=t\ |S)$$

$$= \Sigma_{A}\ P(F)\ P(A)\ g_1(F,A)$$



Removing A

$$P(F,N=t) = P(F)\, \Sigma_A\, P(A)\, g_1(F,A)$$

$$= P(F)\, g_2(F)$$

$=P(N=t|F)$

$$P(F=t|N=t) = P(F=t,N=t)/P(N=t)$$

$$P(N=t) = \Sigma_F\, P(F,N=t)$$

Independencies and active trails

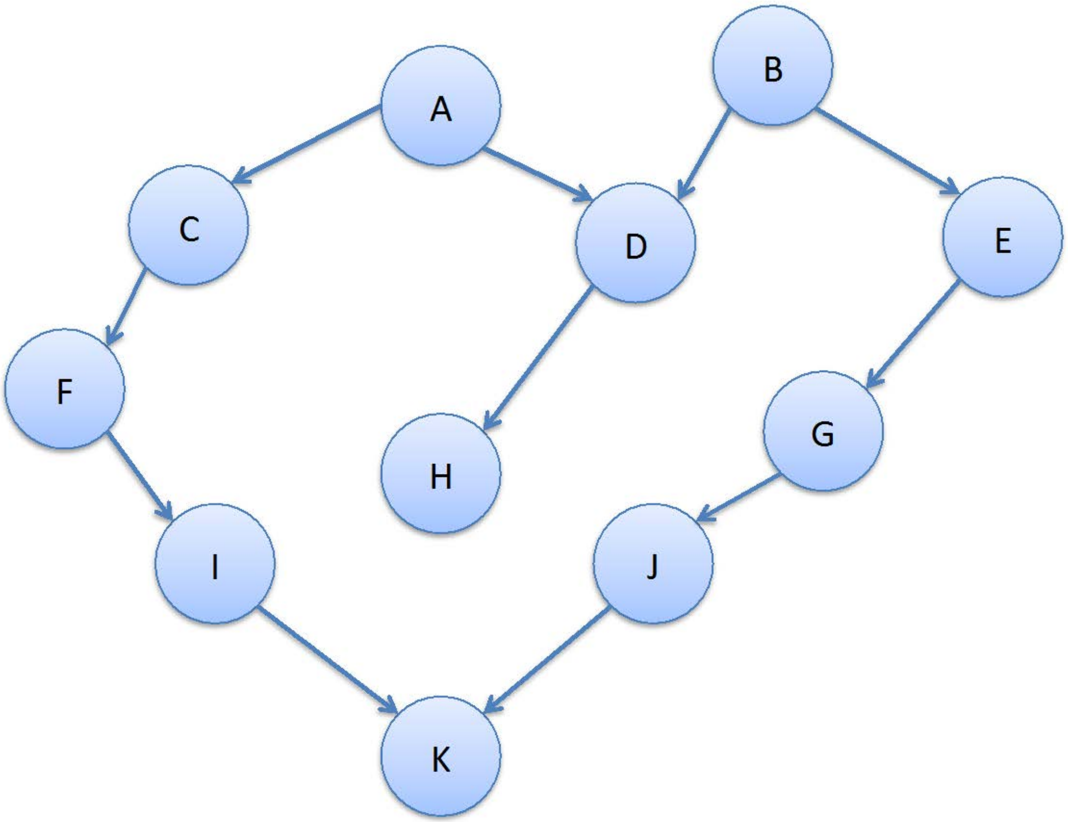
Is A⊥H? When is it not?

A is not⊥H when given C and F or F’ or F’’ and not {B,D,E,G}

Independencies and active trails

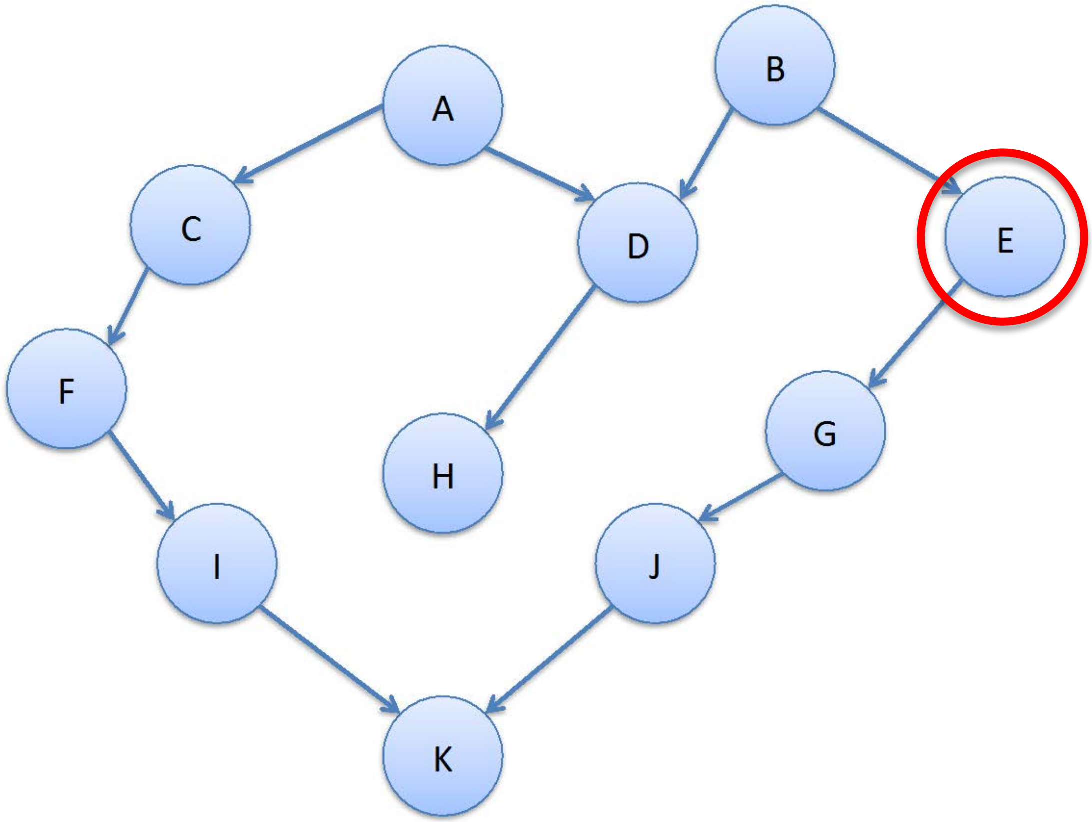
- Active trail between variables $X_1, X_2 \ldots X_{n-1}$ when:
  - $X_{i-1} \rightarrow X_i \rightarrow X_{i+1}$ and X_i not observed
  - $X_{i-1} \leftarrow X_i \leftarrow X_{i+1}$ and X_i not observed
  - $X_{i-1} \leftarrow X_i \rightarrow X_{i+1}$ and X_i not observed
  - $X_{i-1} \rightarrow X_i \leftarrow X_{i+1}$ and X_i or one of its descendants is observed

Independencies and active trails



A⊥B ?

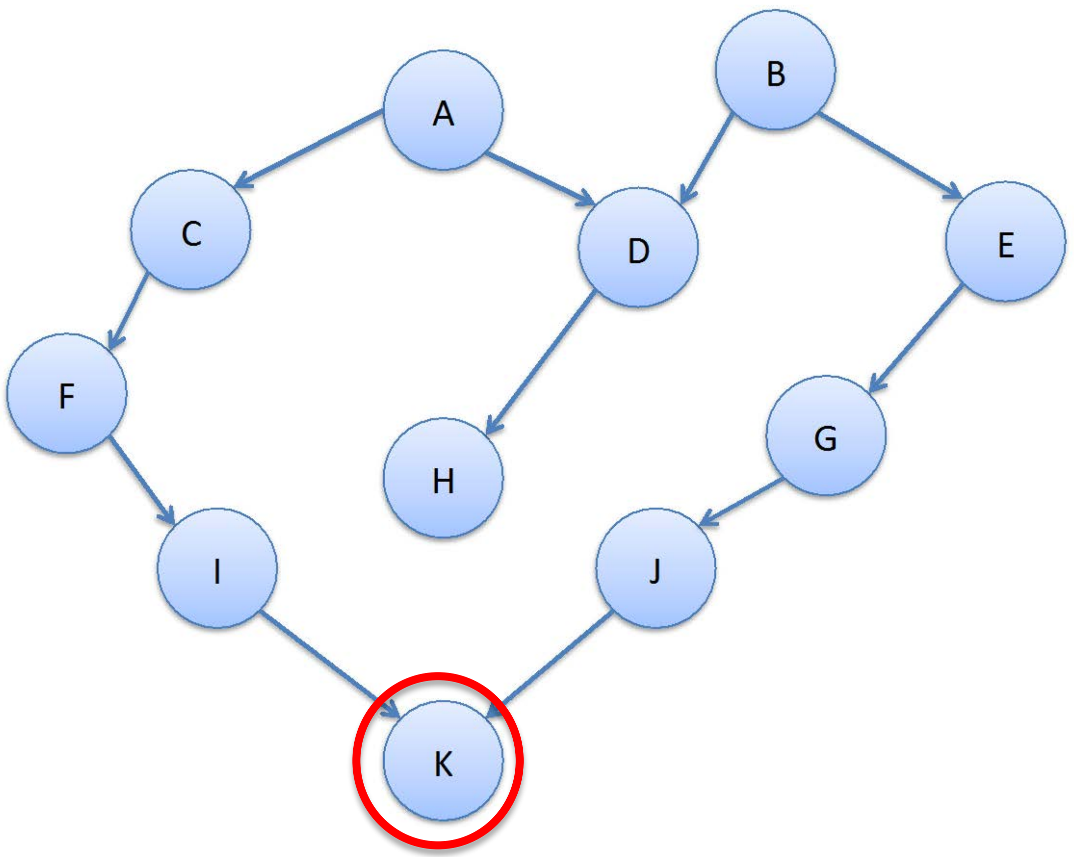
Independencies and active trails



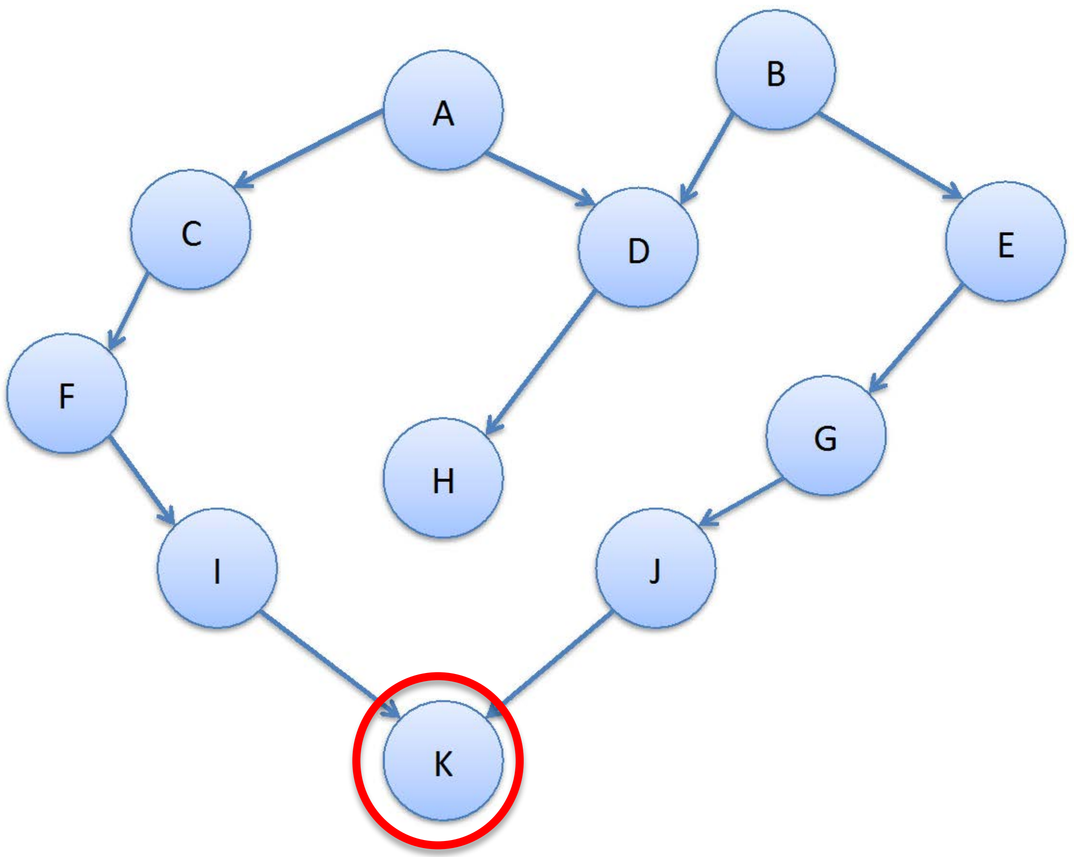
$B \perp G \mid E$?

Independencies and active trails

$I \perp J \mid K$?

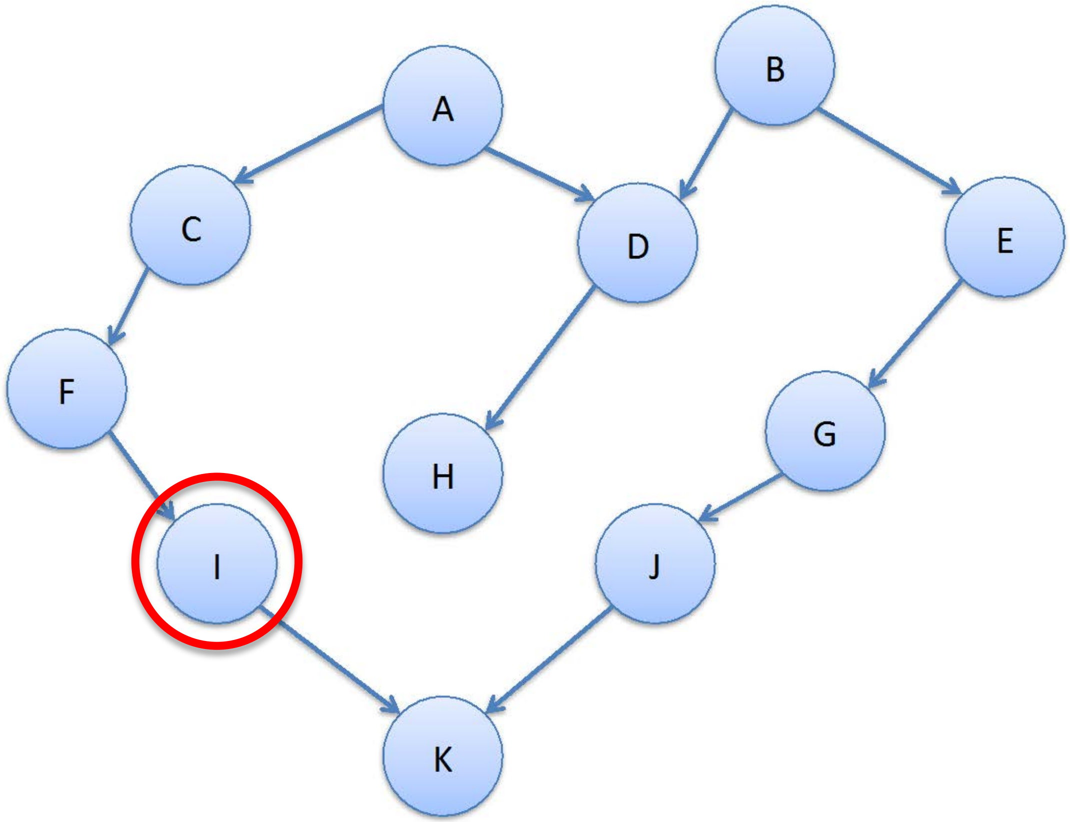


Independencies and active trails



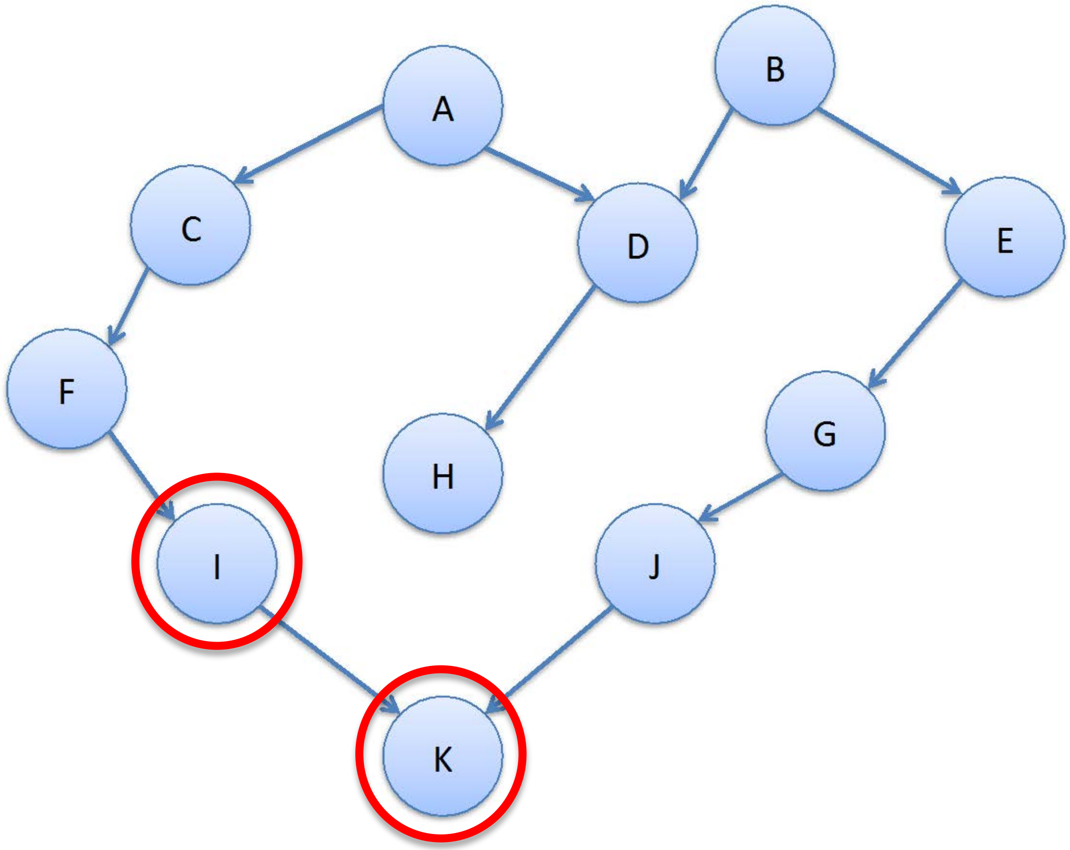
$E \perp F \mid K$?

Independencies and active trails



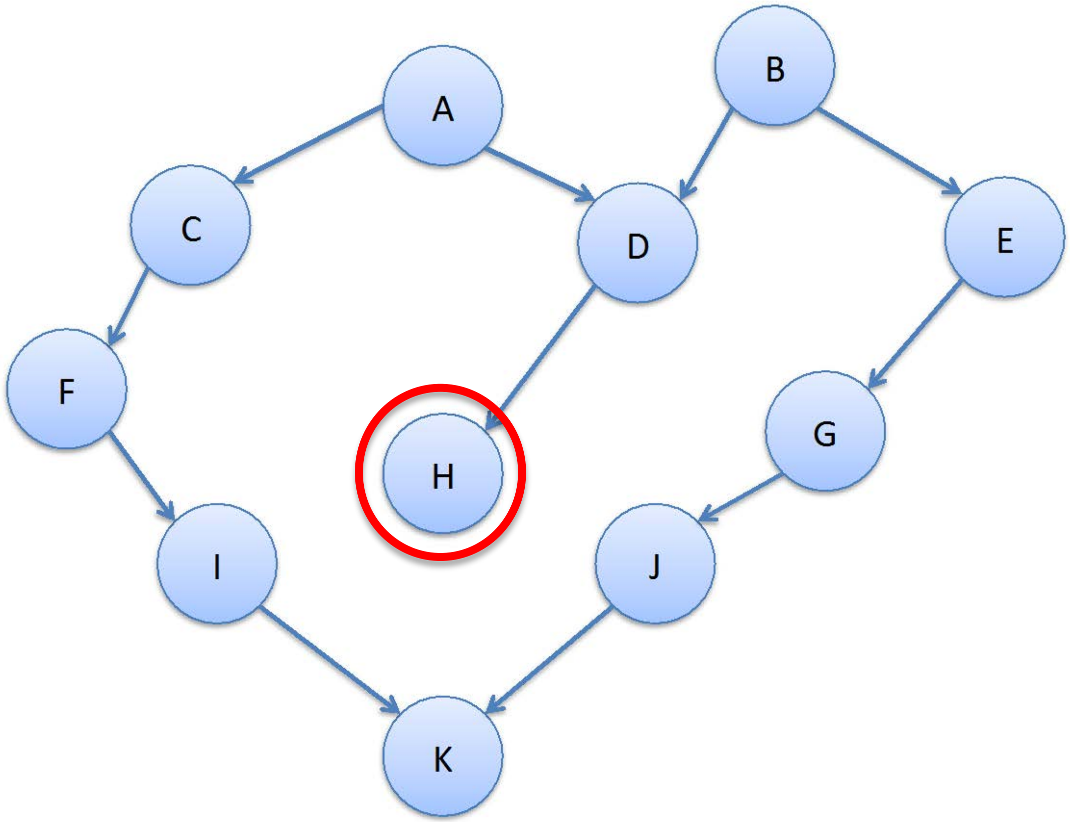
$F \perp K \mid I$?

Independencies and active trails



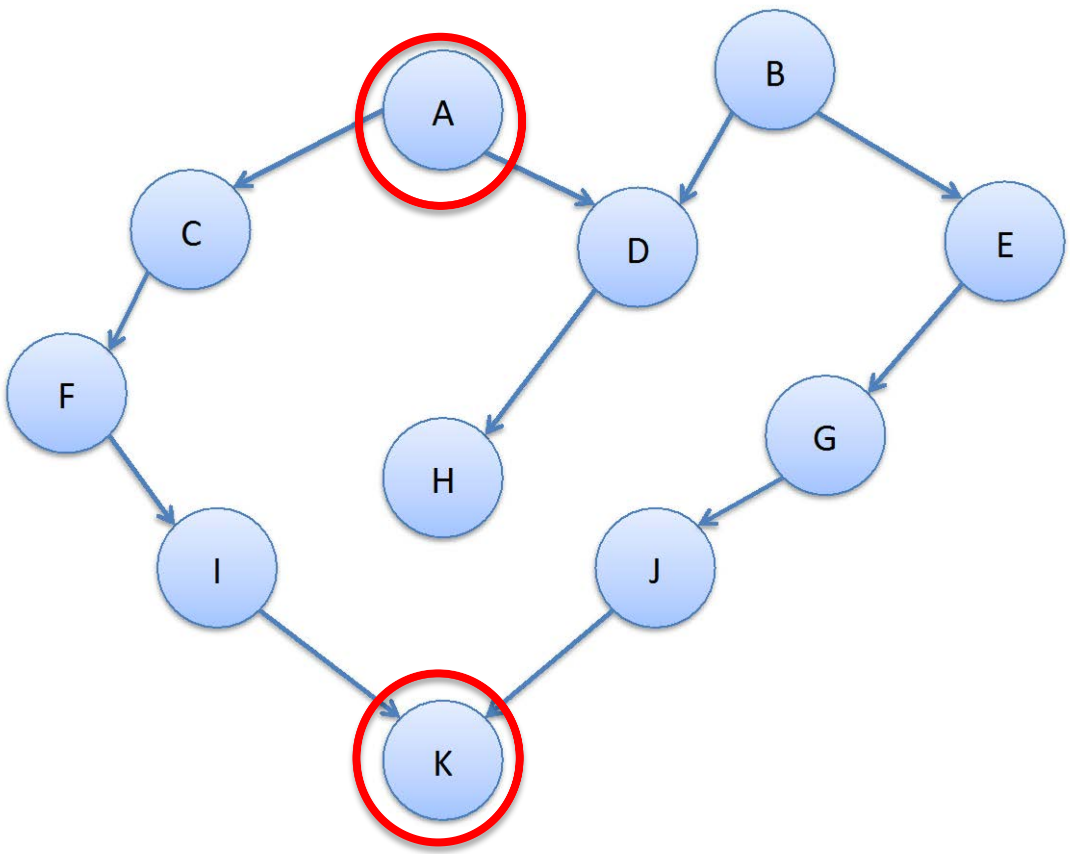
$E \perp F \mid I, K$?

Independencies and active trails



F⊥G |H ?

Independencies and active trails



$F \perp G \mid H, A$?