Quadratic Programming

10701 Recitations 3

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What is Quadratic Programming

The quadratic programming is formulated as

$$\min_{w} \left\{ \frac{1}{2} w^T Q w + c^T w \right\} \text{ subject to } \begin{cases} Aw \leq b \\ Ew = d \end{cases}$$

where $Q \in \mathbb{R}^{n \times n}$ and is symmetric, $w, c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $E \in \mathbb{R}^{p \times n}$, and $d \in \mathbb{R}^p$,

- constraint optimization: minimize objective + constraints
- w is feasible if satisfying the constraints
- local minimizer: for any feasible u around w, $f(w) \leq f(u)$
- global minimizer: for any feasible u, $f(w) \leq f(u)$

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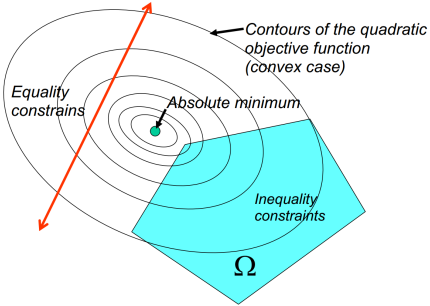
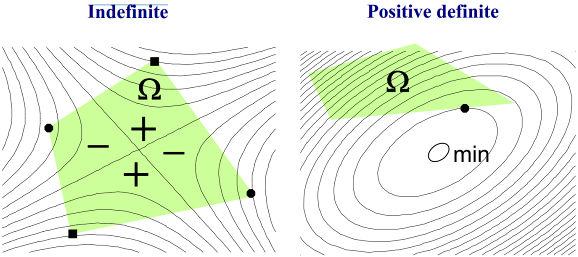


Convex QP

- If Q is positive semidefinite (definite), that is $x^T Qx \geq 0$ (> 0) for any x, the objective function is (strongly) convex. If feasible w exists, any local minimizer is global, and there is at least one (a unique) global minimizer



- If $Q = 0$, QP reduces to linear programming
- Solving general QP is NP-hard, but several algorithms solve convex QP in polynomial time

An Example

Consider the following example:

$$\min_{x,y} \left\{ x^2 + 4(y-4)^2 \right\} \text{ subject to } \begin{cases} x + y \le 7, -x + 2y \le 4 \\ x \ge 0, y \ge 0, y \le 4 \end{cases}$$

We have

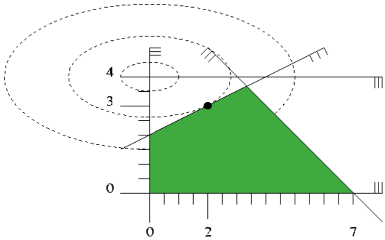
$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \quad c = \begin{bmatrix} 0 \\ -32 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ 4 \\ 0 \\ 0 \\ 4 \end{bmatrix} \quad E = d = 0$$

run Matlab command

```
x = quadprog(Q,c,A,b)
```

and get the results

```
x =
     2.0000
     3.0000
```



QP and SVM

- Recall the Support Vector Machine:

$$\min_{w} \frac{1}{2}\|w\|^2 \text{ subject to } y_i[\langle x_i, w\rangle + b_0] \geq 1 \text{ for any } i,$$

  where $w \in \mathbb{R}^p, x_i \in \mathbb{R}^p$, and $y_i \in \{-1, 1\}$

- It is straightforward to formulate it as QP

$$Q = I, A = \begin{bmatrix} -y_1 x_1 \\ \vdots \\ -y_n x_n \end{bmatrix}, b = \begin{bmatrix} y_1 b_0 - 1 \\ \vdots \\ y_n b_0 - 1 \end{bmatrix}, c = E = d = 0$$

- Solving SVM: `x = quadprog(Q,c,A,b)`, suitable for medium size n

Questions?