# Templates

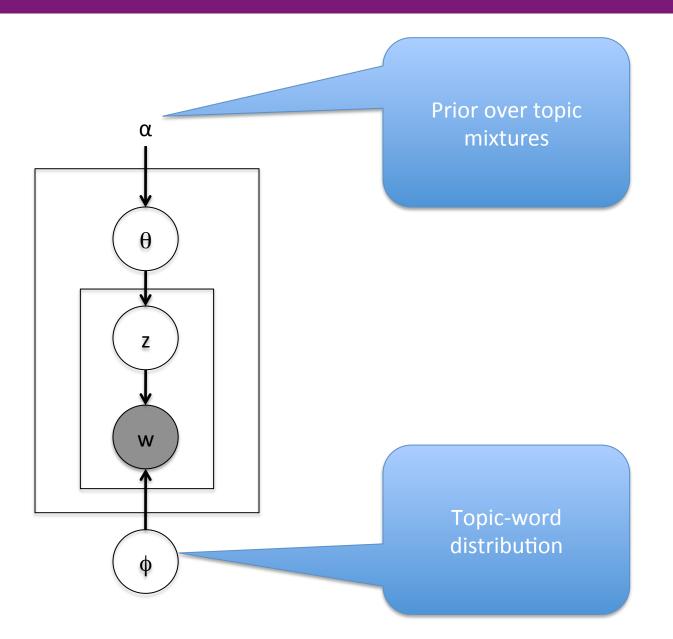
for scalable data analysis

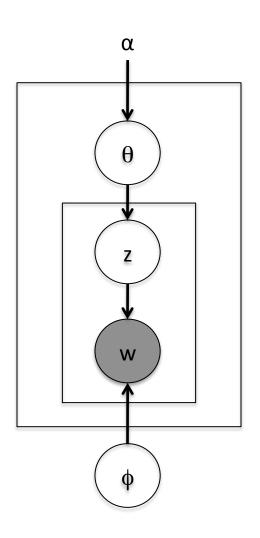
4 Applications: User Modeling and Graph Factorization

Amr Ahmed, Alexander J Smola, Markus Weimer Yahoo! Research & UC Berkeley & ANU

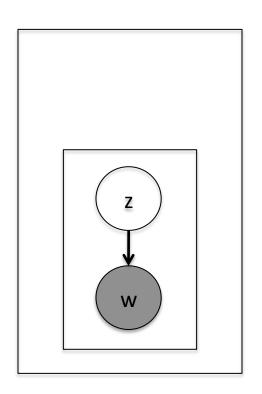
## Wrapping up

- Distributed inference in latent variable models
  - Star Synchronization
  - Delta aggregation





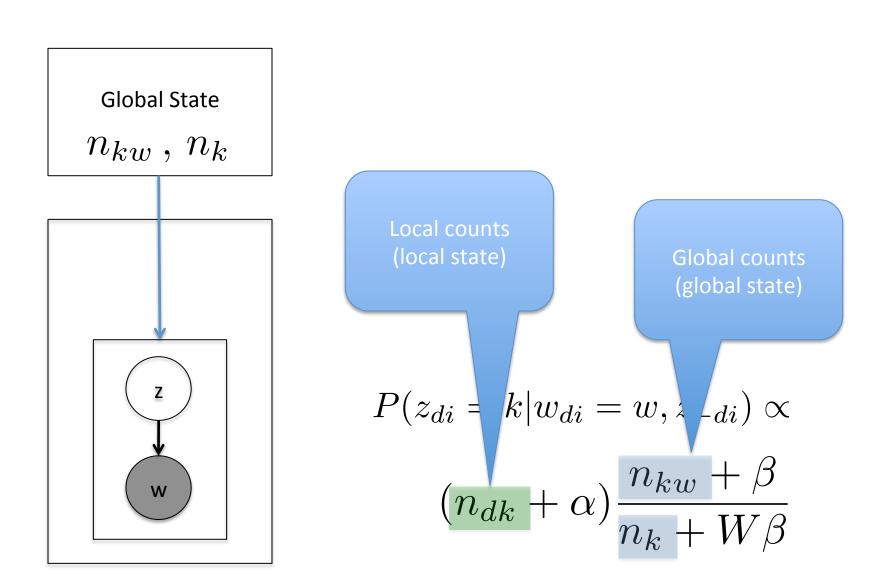
- Global variables
  - Ф: Topic distribution over words
- Local variables
  - $-\theta$ : topic mixing vector
  - Z: topic indicator

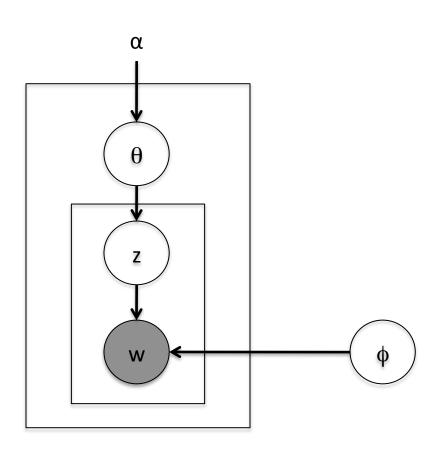


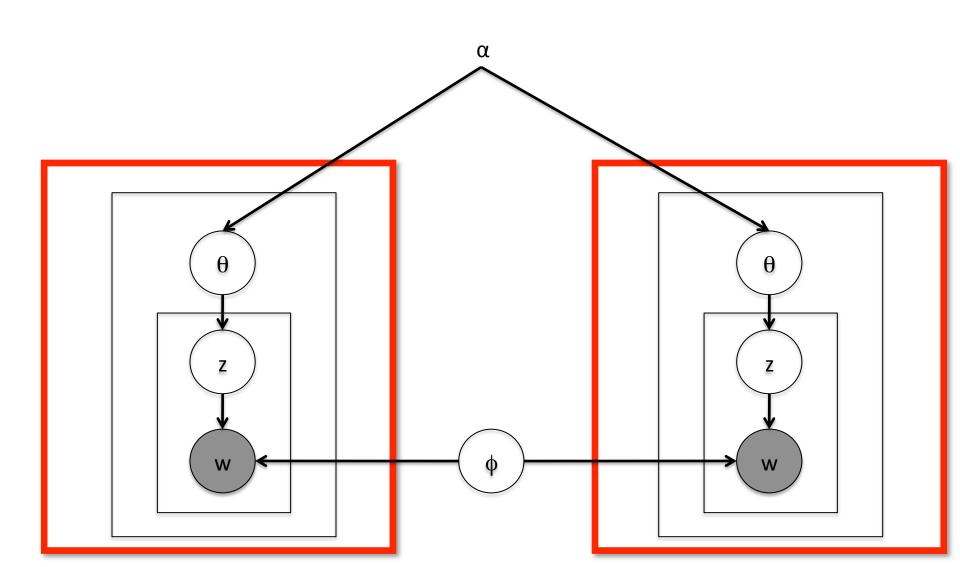
- Collapse global variables
  - Ф
- Collapse local variables
  - -e
- Couples all Zs
- Run collapsed sampler

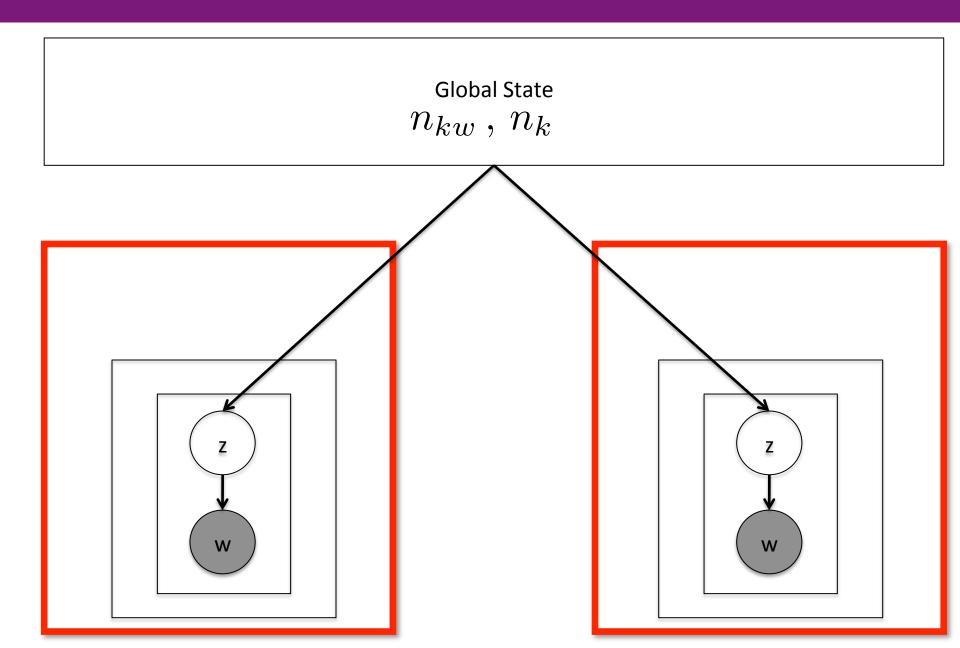
$$P(z_{di} = k | w_{di} = w, z_{-di}) \propto$$

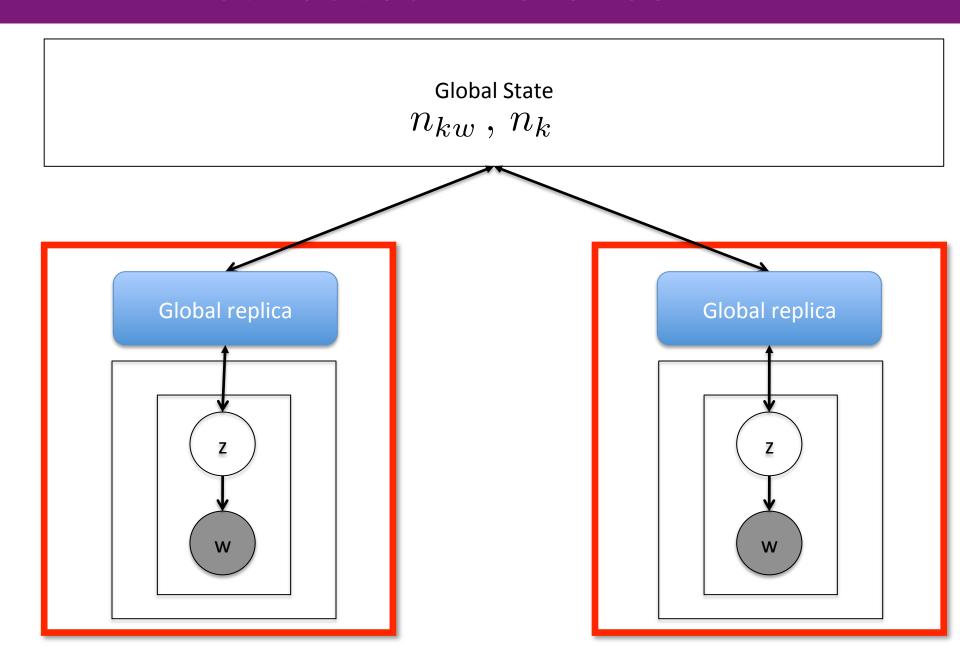
$$(n_{dk} + \alpha) \frac{n_{kw} + \beta}{n_k + W\beta}$$





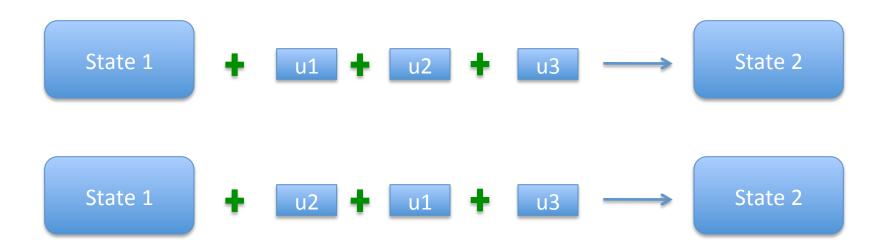






#### General Architecture

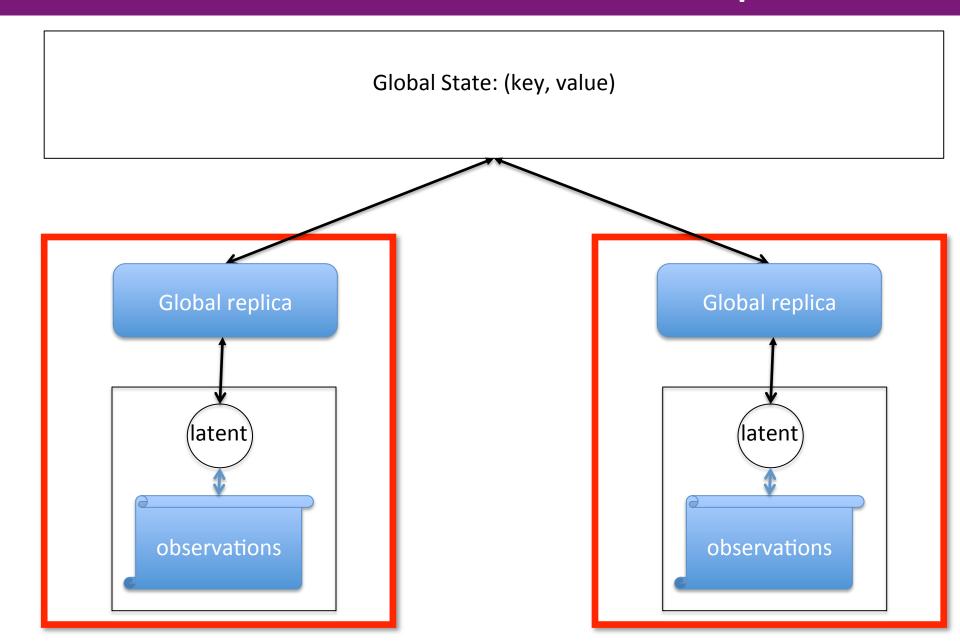
- Star synchronization
  - Works when variables depend on each other via aggregates
    - Counts, sums, etc.
  - When state objects form an Abelian group



#### Template

- Fit most topic models in collapsed representation
  - Define the state (key, value) pairs
    - Mostly counts, sums, lists, hash tables
  - Define the +,- operations on a state object
  - Write your sampler
    - Input: document, state
    - Output:
      - Update document local variables
      - Update the global state
- Our API will take care of the rest
  - Synchronization, threading, distribution, etc

## Distributed Inference: template



#### State Example: LDA

- Alternative 1
  - Key: (topic, word)
  - value: count
  - Operators:
    - +,- are trivially defines
- Alternative 2
  - Key: word
  - value: list of (topic, count)
    - Allows efficient samplers
  - Operators: sparse vector operations
    - Might need to delete and merge

$$P(z_{di} = k | w_{di} = w, z_{-di}) \propto$$

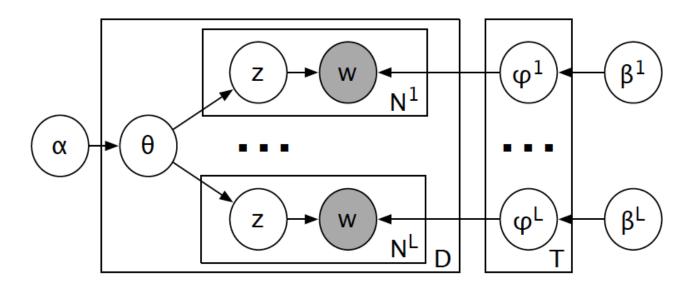
$$(n_{dk} + \alpha) \frac{n_{kw} + \beta}{n_k + W\beta}$$

#### State Example: LDA

- You get the idea?
- Define the state to work with your sampler
- Define +,- for synchronization
- All details are abstracted form the synchronization logic
  - It just uses the +,- operators your just defined
  - Requires an iterator over state objects

## Example 2: Multilingual LDA

- Each topic has a distribution over words
- Fits parallel documents
  - Example: Wikipedia

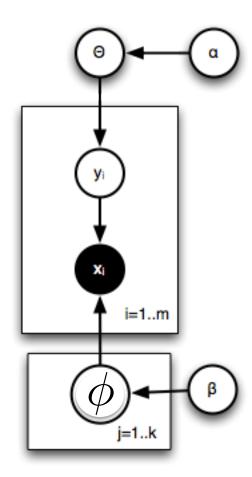


#### State Example: Multilingual-LDA

- Alternative 1
  - Key: (topic, language, word)
  - value: count
  - Operators: +,- are trivially defines
- Alternative 2
  - Key: word
  - value: list of (topic, language, count)
    - Allows writing efficient samplers
  - Operators: Sparse vector operations
    - Might need to delete and merge

#### State Example: Clustering

- Alternative 1
  - Key: Cluster ID
  - value:
    - Document counts
    - Parameter representation
      - Hash table: (word, count)
  - Operations
    - Define +,- over each field
    - You write this code
    - Part of the application logic
    - You have to do it anyhow when:
      - Remove or add a document to a cluster



#### **API Summary**

- Template for distributed inference in latent variables models
- Two basic components
  - Document representation
    - You take care of that via Protocol Buffer
  - State representation
    - Key-value pairs
    - Value can be any object
      - Define +,- over that object
    - Provide an iterator over objects for the synchronizer

#### Code Snippet: object

```
class stats{
public:
    virtual ~stats() { };
    virtual void from_str(const string& serialized_stats) = 0;
    virtual void to_str(string& serialized_stats) = 0;
    virtual void operator+=(stats& inp) = 0;
    virtual void operator-=(stats& inp) = 0;
    virtual int get_id() { return 0; }
    virtual void set_id(int) { }
    virtual void print() { }
};
typedef auto_ptr<stats> stats_ptr;
```

#### Code Snippet: Container

```
class stats_container{
public:
   virtual ~stats_container() { };
   // copy operator
   virtual void from_stats_container(stats_container&) = 0;
   // lock up operator, get stat object with a given id
   virtual stats_ptr get_stats(int id) = 0;
   // update a state object with a give id
   virtual void update(int id, stats& delta) = 0;
   virtual int size() = 0;
   // iterator
   virtual bool has_next() = 0;
   virtual stats_ptr next() = 0;
   virtual void reset_iter() = 0;
   virtual void print() = 0;
};
```

#### Code Snippet: LDA Document

```
message LDA_document {
    optional string docID = 1;
    repeated uint32 body = 3 [packed=true]; // w|
    repeated uint32 topic_assignment = 4 [packed=true]; //Z
    repeated uint32 topic_counts = 5 [packed=true]; // n_dk
}

message clustering_document {
    optional string docID = 1;
    repeated uint32 words = 2; // w
    repeated uint32 label = 3; // cluster assignment
}
```

#### Code Snippet: Sampler

```
class Model_Trainer {
public:
   virtual ~Model_Trainer() { };
   // read a document from disk
    virtual void* read(google::protobuf::Message&) = 0;
   //That is where you write your logic
   virtual void* sample(void* document) = 0;
   // Call in inference mode
    virtual void* test(void* document) = 0;
   // fold an update into the state
    virtual void* update(void* document) = 0;
   // time for synchronous operations
    virtual void* optimize(void*) = 0;
   // diagnostic
   virtual void* eval(void*,double&) = 0;
   //save
   virtual void write(void*) = 0;
   //need more iterations?
    virtual void iteration_done() = 0;
};
```

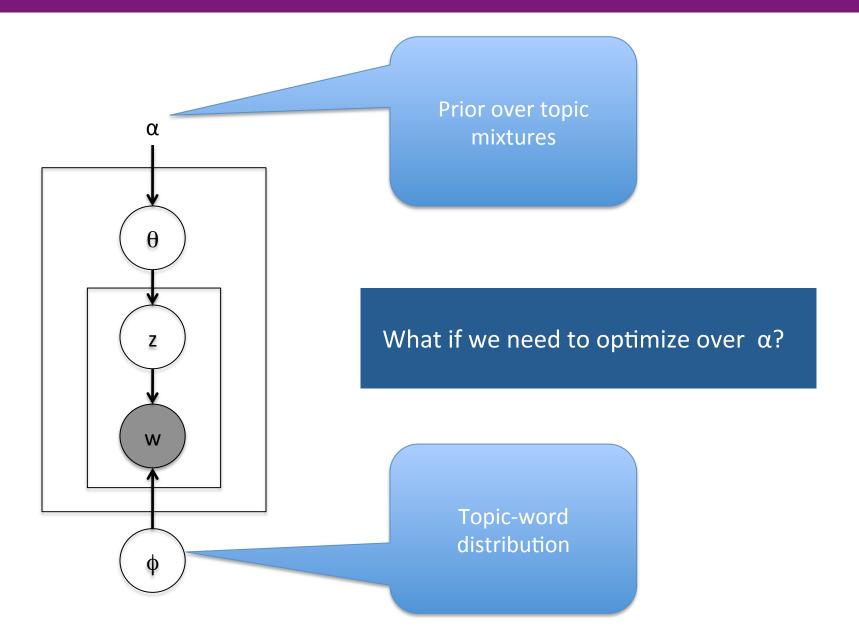
## API Summary

- Current Yahoo! LDA release
  - Tightly integrates state, sampler and synchronization
  - Stay tuned for a new release with the new APIs

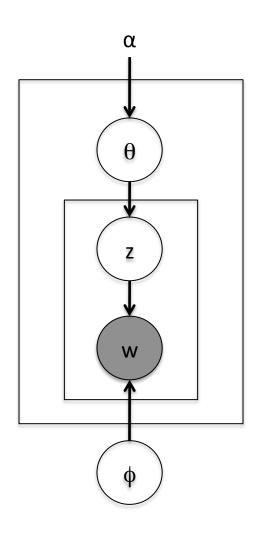
#### What Is next?

- Can we fit any model only with those asynchronous primitives?
  - No
- We need synchronous operations
  - Parameter optimization
    - EM style algorithm
  - Non-collapsed global variables

# The Need for Synchronous Processing

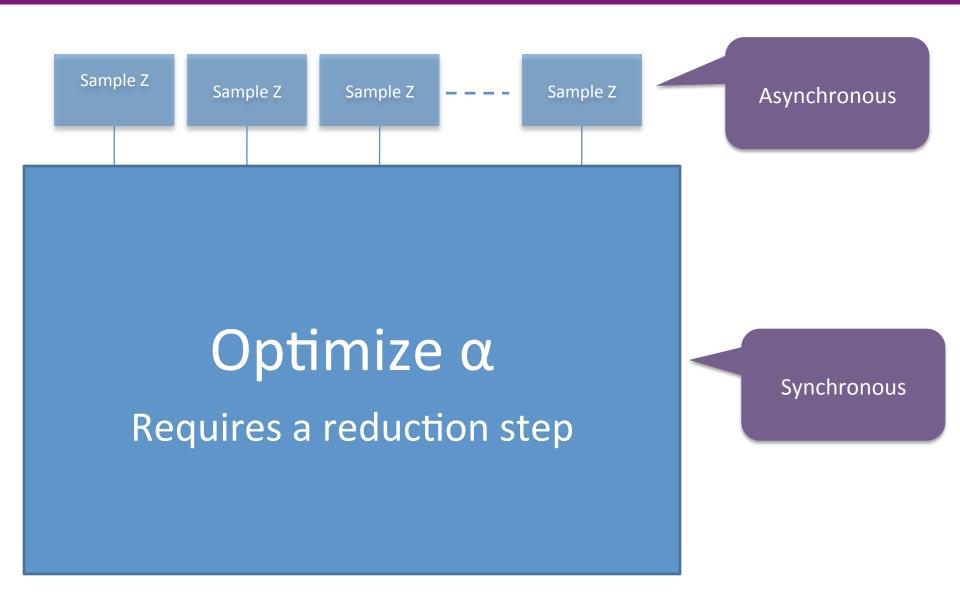


# The Need for Synchronous Processing

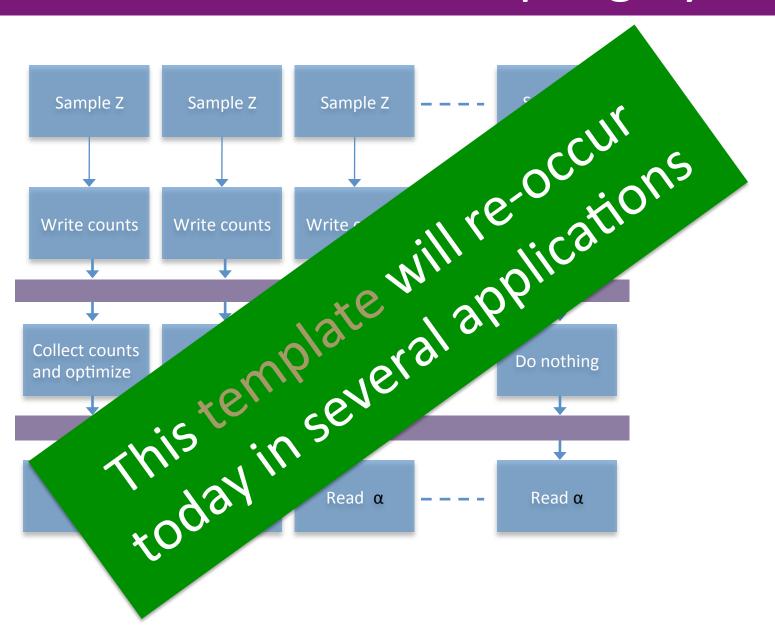


- E-Step
  - Run asynchronous collapsed sampler as before
- M-step
  - Reach a barrier
  - Collect values needed to optimize α
  - One machine optimizes  $\alpha$
  - Broadcast value back

## Distributed Sampling Cycle



#### Distributed Sampling Cycle

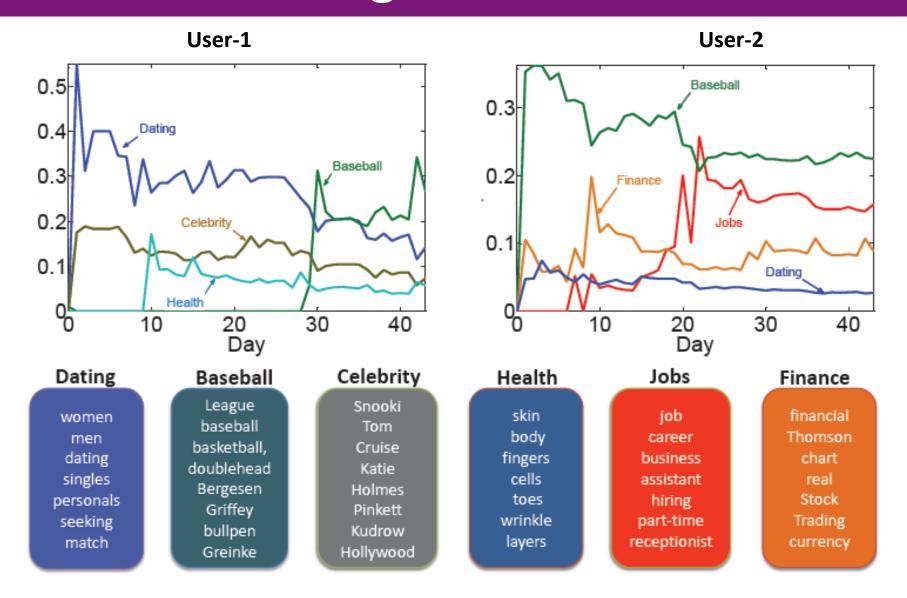


#### Up next

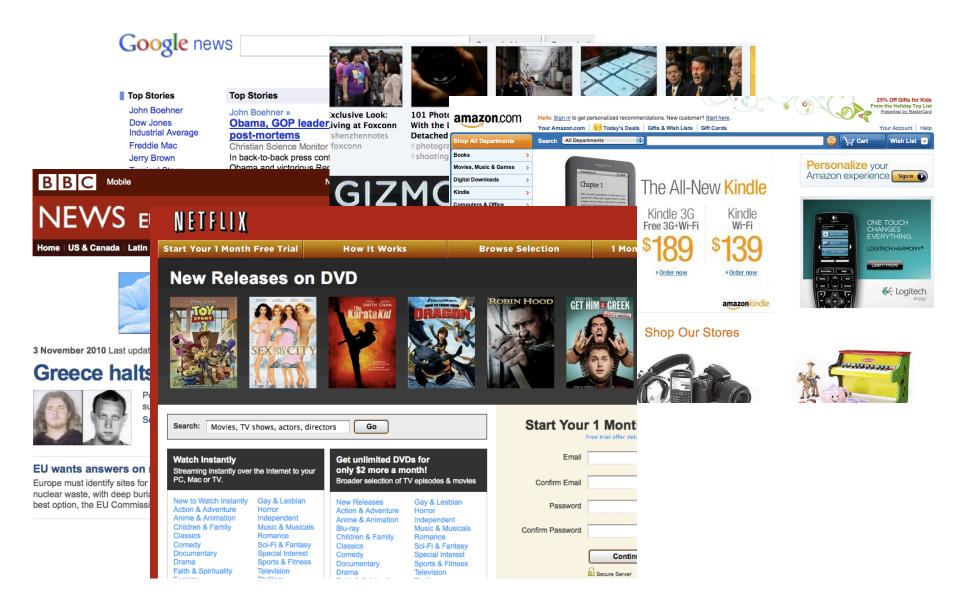
- Application
  - Temporal Modeling of user interests
  - Multi-domain user personalization

- Asynchronous Distributed Optimization
  - Can we get rid of the synchronous step?
  - Asynchronous consensus
  - Factorizing Y!M graph
    - 200 Million users and 10 Billion edges
    - The largest published work on graph factorization

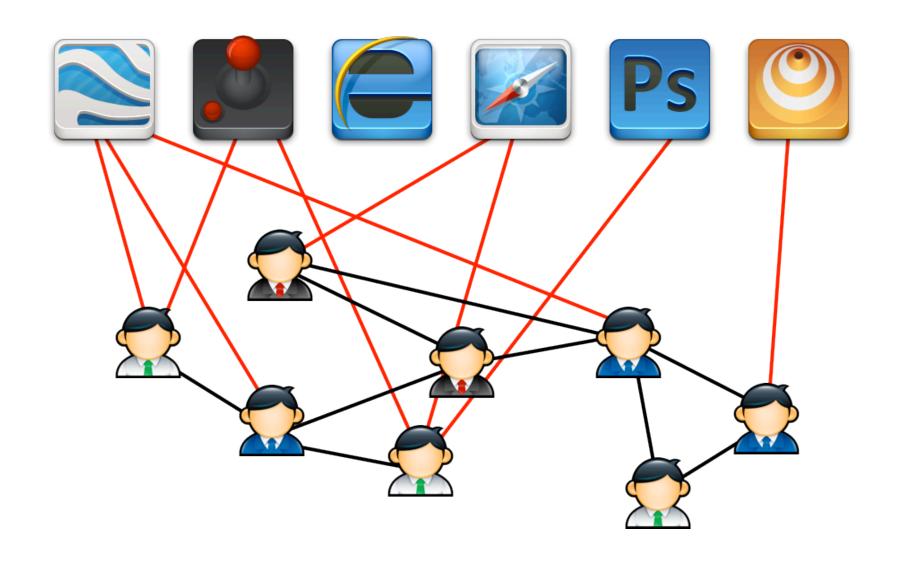
#### **Modeling User Interests**



#### Multi-domain Personalization



## Graph Factorization: Social Network

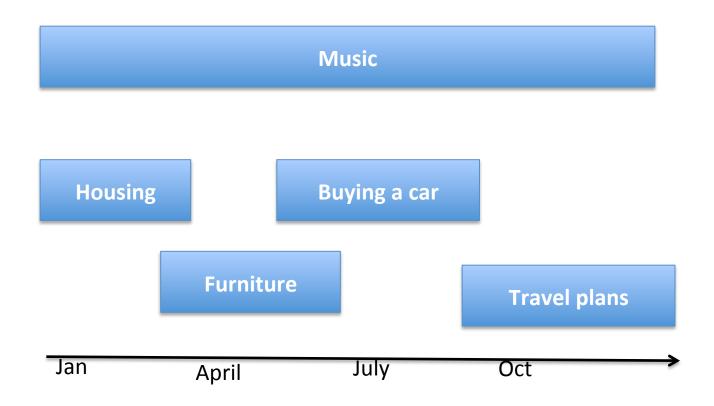


## **Application**

Tracking Users Interest

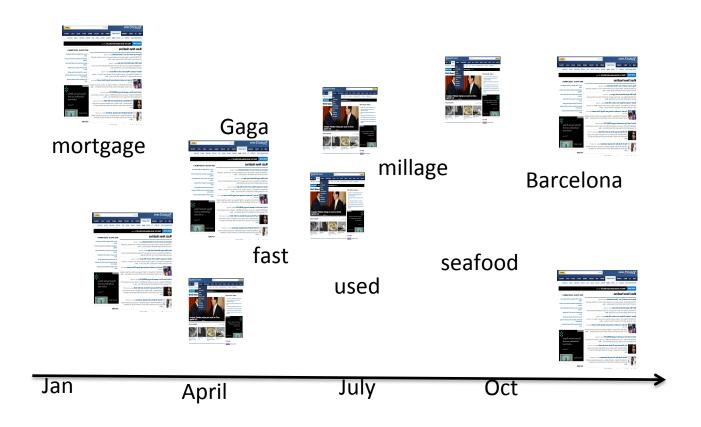
#### Characterizing User Interests

Short term vs long-term



## **Characterizing User Interests**

- Short term vs long-term
- Latent



#### Input

- Queries issued by the user or tags of watched content
- Snippet of page examined by user
- Time stamp of each action (day resolution)

#### Output

- Users' daily distribution over interests
- Dynamic interest representation
- Online and scalable inference
- Language independent



Flight London Hotel weather classes registration housing rent School Supplies Loan semester

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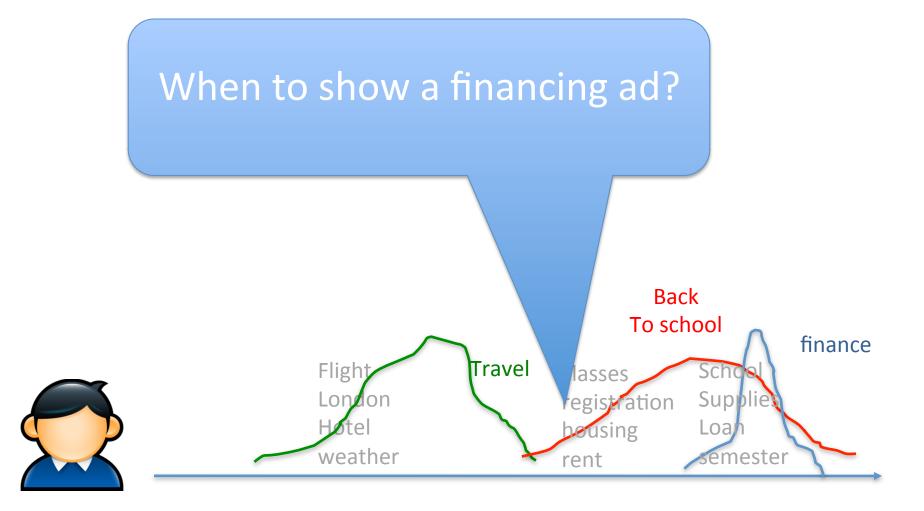


Back

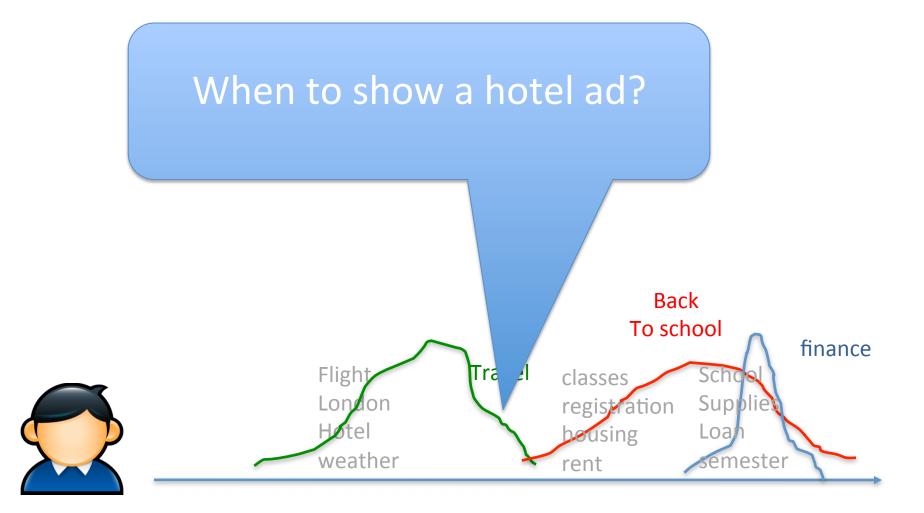
When to show a financing ad?

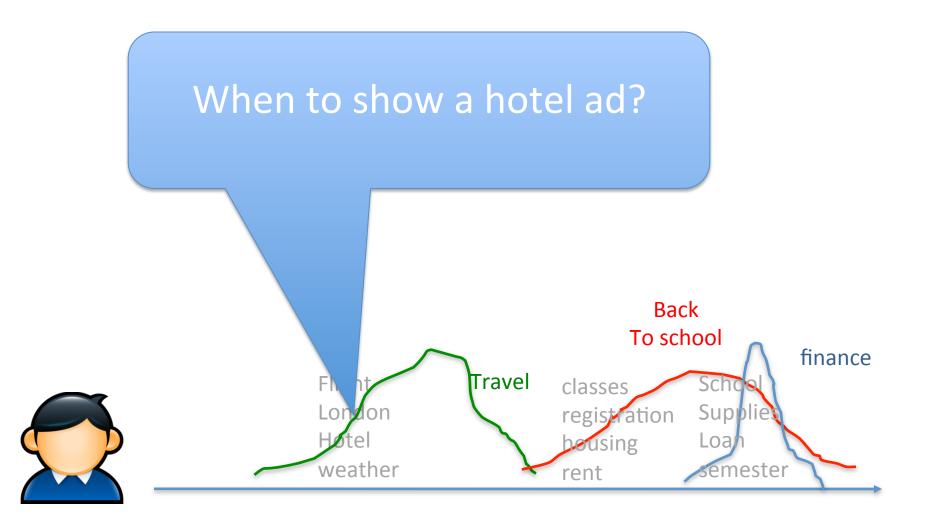






When to show a financing ad? ack hool finance Travel Fligh classes housing weather semester rent





#### Input

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- Snippet of page examined by user
- Time stamp of each action (day resolution)

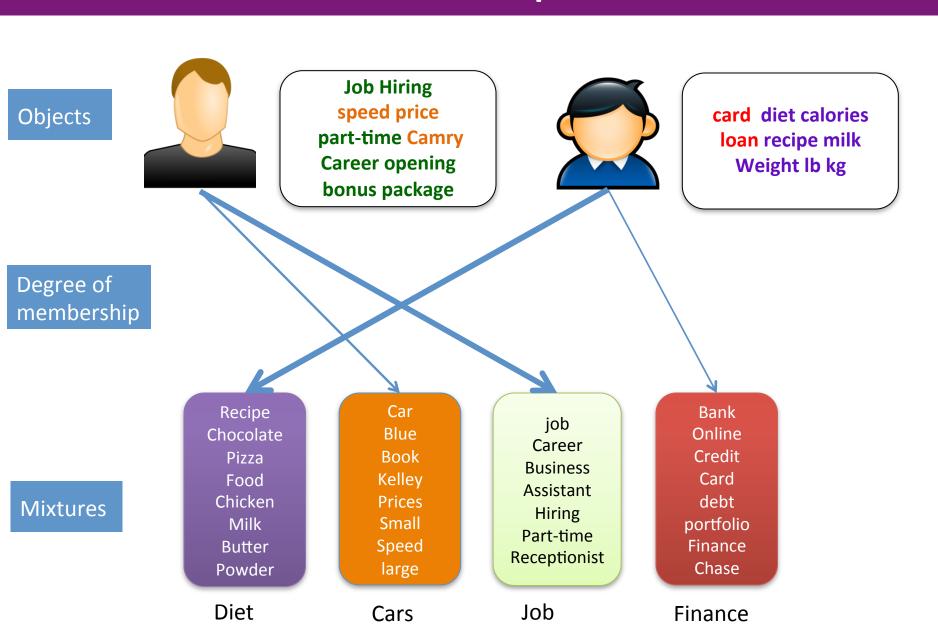
#### Output

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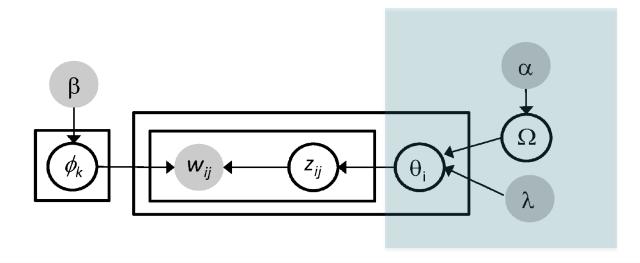


Back

# Mixed-Membership Formulation

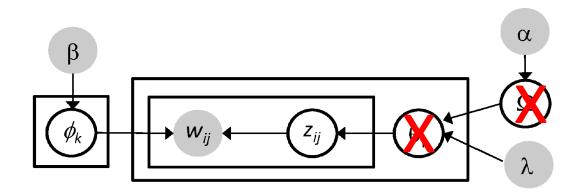


# In Graphical Notation

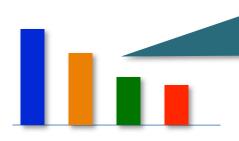


- 1. Draw once  $\Omega | \alpha \sim \text{Dir}(\alpha/K)$ .
- 2. Draw each topic  $\phi_k | \beta \sim \text{Dir}(\beta)$ .
- 3. For each user i:
  - (a) Draw topic proportions  $\theta_i | \lambda, \Omega \sim \text{Dir}(\lambda \Omega)$ .
  - (b) For each word
    - (a) Draw a topic  $z_{ij}|\theta_d \sim \text{Mult}(\theta_i)$ .
    - (b) Draw a word  $w_{ij}|z_{ij}, \phi \sim \text{Multi}(\phi_{z_{ij}})$ .

# In Polya-Urn Representation



- Collapse multinomial variables:  $\theta, \Omega$
- Fixed-dimensional Hierarchal Polya-Urn representation
  - Chinese restaurant franchise

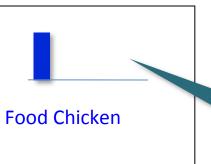


Global topics trends

Recipe Chocolate Pizza Food Chicken Milk Butter Powder Car Blue Book Kelley Prices Small Speed large

job Career Business Assistant Hiring Part-time Receptio nist Bank
Online
Credit
Card
debt
portfolio
Finance
Chase





Topic word-distributions





Car speed offer camry accord career

User-specific topics trends (mixing-vector)

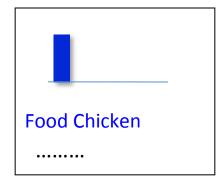
User interactions: queries, keyword from pages viewed





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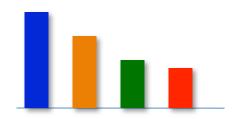








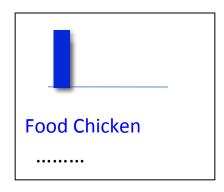
- For each user interaction
  - Choose an intent from local distribution
    - Sample word from the topic's word-distribution
  - •Choose a new intent  $\propto \lambda$ 
    - Sample a new intent from the global distribution
      - Sample word from the new topic word-distribution





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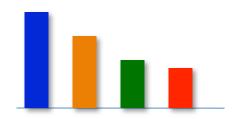








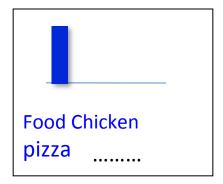
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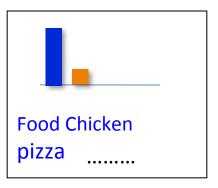


Recipe Chocolate Pizza Food Chicken Milk Butter Powder

Car Blue Book Kelley Prices Small Speed large

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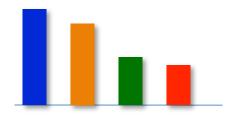








- For each user interaction
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      - Sample from word the new topic word-distribution





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#### **Problems**

- Static Model
- Does not evolve user's interests
- Does not evolve the global trend of interests
- Does not evolve interest's distribution over terms

# At time t



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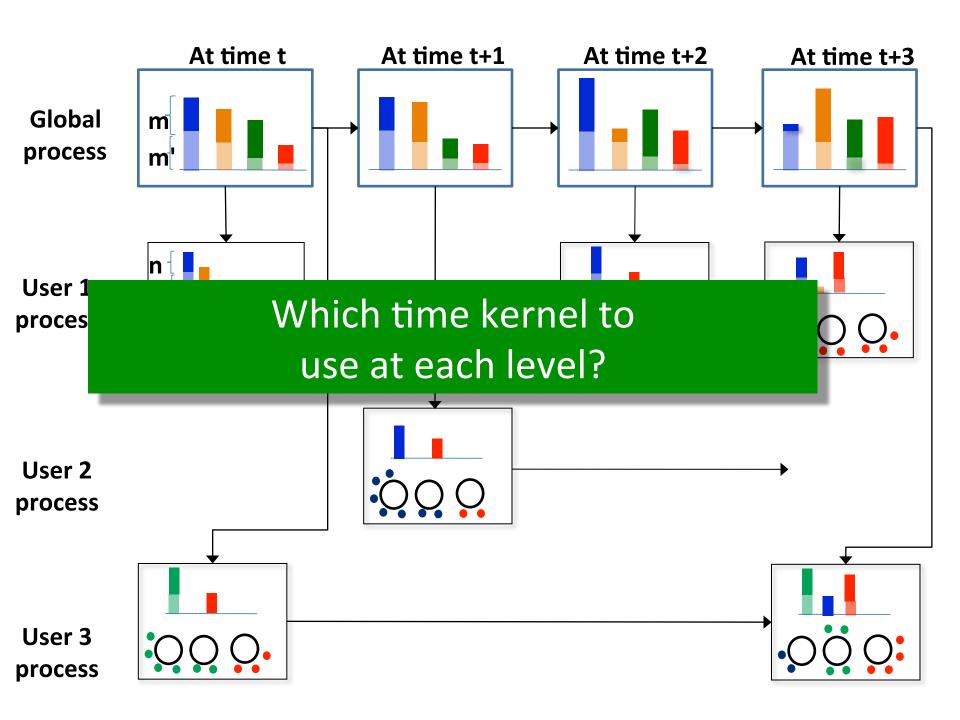


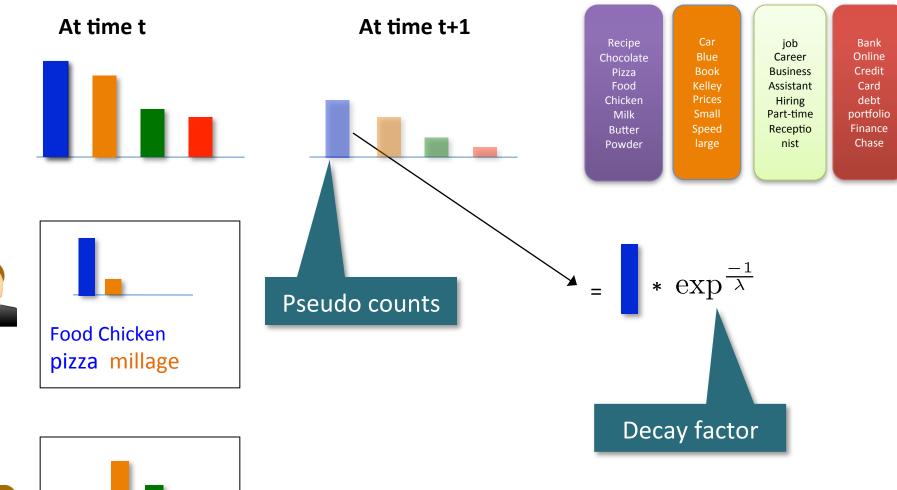
Build a dynamic model



camry accord career

Connect each level using a RCRP









#### Observation 1

-Popular topics at time t are likely to be popular at time t+1  $-\phi_{k,t+1}$  is likely to smoothly evolve from  $\phi_{k,t}$ 

# At time t



#### At time t+1



Recipe Chocolate Pizza Food Chicken Milk Butter Powder

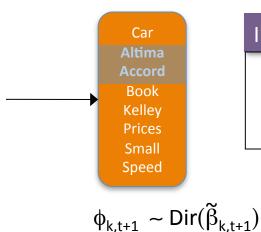
Blue Book Kelley Prices Small Speed large

Car

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Chase







#### Intuition

Captures current trend of the car industry (new release for e.g.)

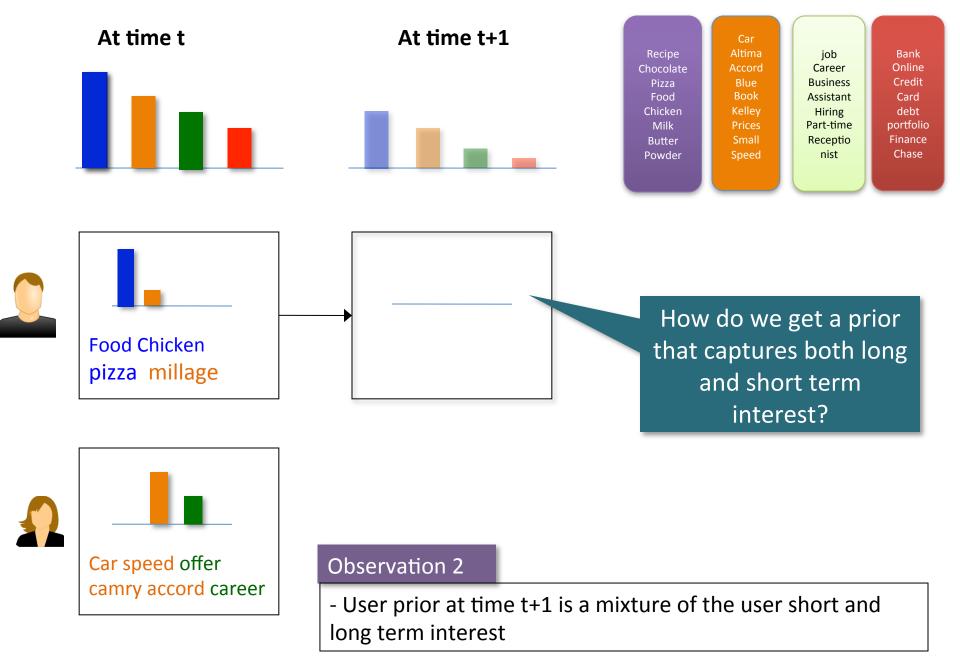


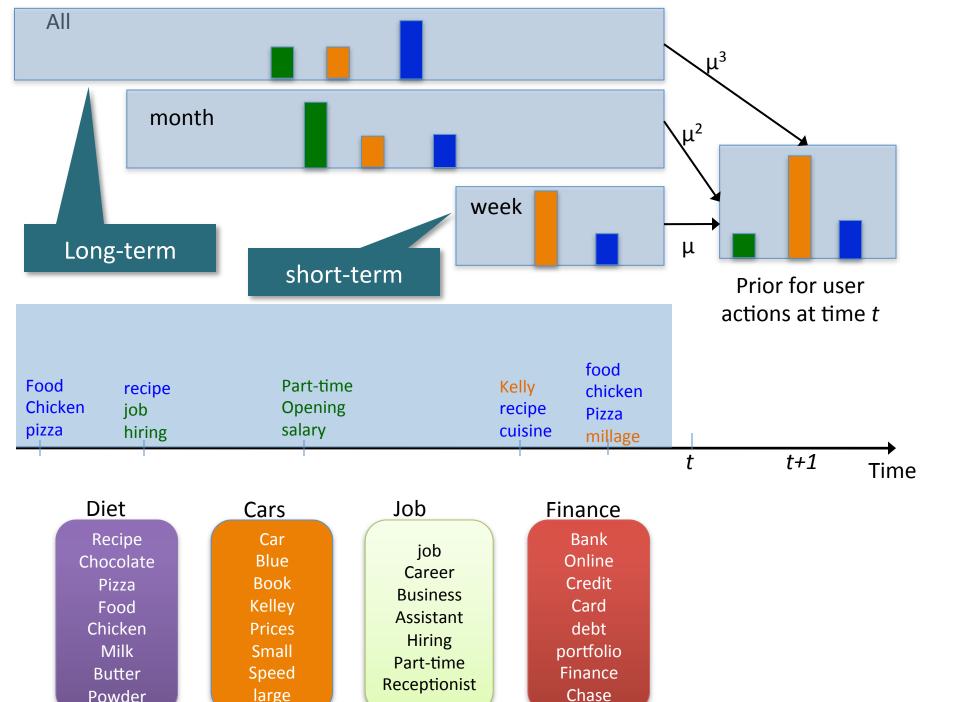


#### Observation 1

 $\phi_{k,t}$ 

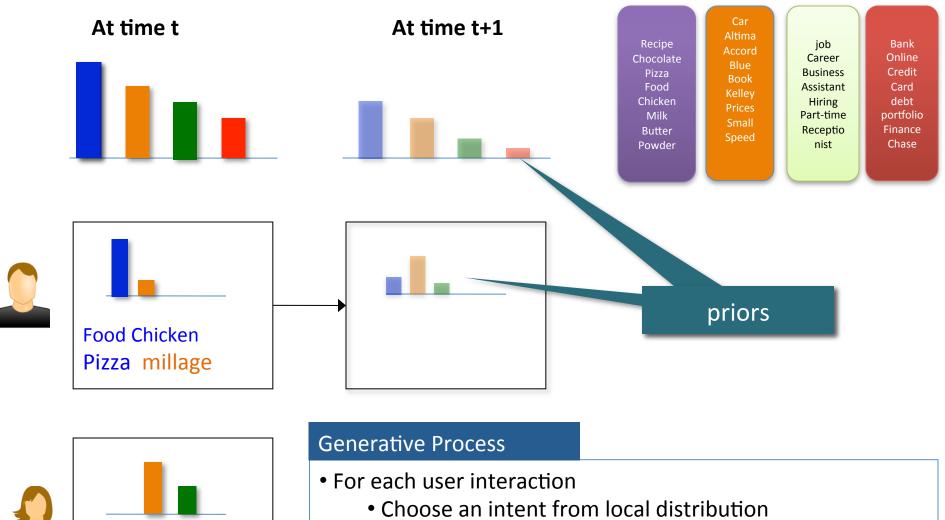
- -Popular topics at time t are likely to be popular at time t+1
- $\, \varphi_{k,t+1}$  is likely to smoothly evolve from  $\, \, \varphi_{k,t} \,$





Powder

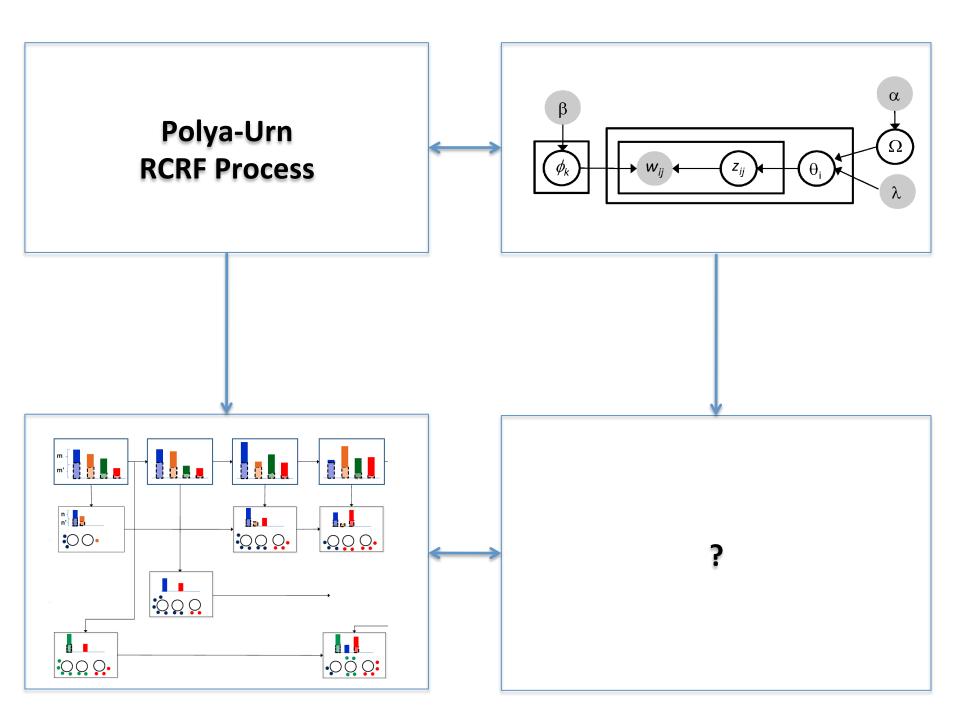
large



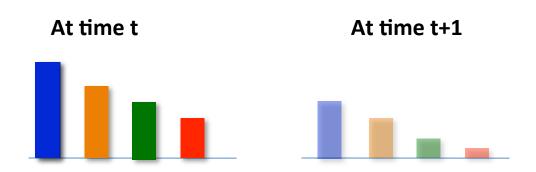


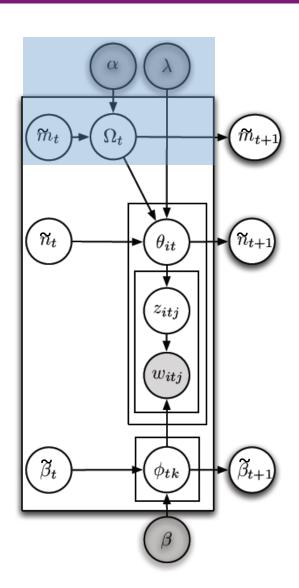


- Sample word from the topic's word-distribution
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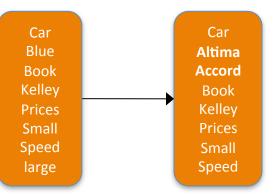
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- 2. Draw each topic,  $\phi_k^t | \beta, \tilde{\beta}_k^t \sim \text{Dir}(\tilde{\beta}_k^t + \beta)$ .
- 3. For each user i:
  - (a) Draw topic proportions  $\theta_i^t | \lambda, \Omega^t, \tilde{\mathbf{n}}_i^t \sim \text{Dir}(\lambda \Omega^t + \tilde{\mathbf{n}}_i^t)$ .
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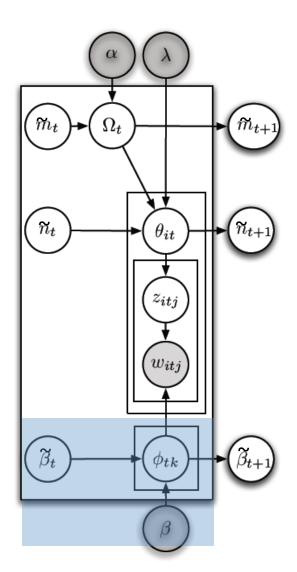




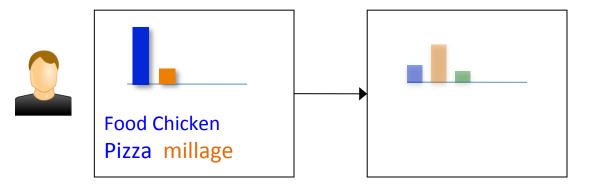
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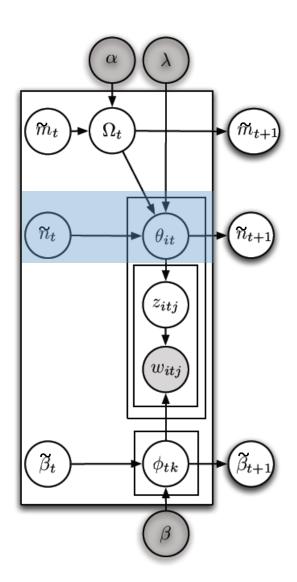
$$\tilde{\beta}_{kw}^t = \sum_{k=1}^{t-1} \exp^{\frac{h-t}{\kappa_0}} n_{kw}^h$$



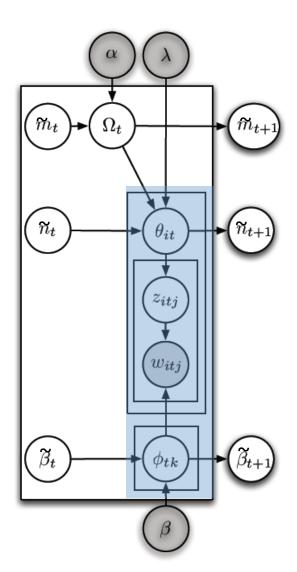


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    - (b) Draw a word  $w_{in}^t | z_{ij}^t, \phi^t \sim \text{Multi}(\phi_{z_{ij}^t}^t)$ .





- 1. Draw once  $\Omega^t | \alpha, \tilde{m}^t \sim \text{Dir}(\tilde{\mathbf{m}}^t + \alpha/K)$ .
- 2. Draw each topic,  $\phi_k^t | \beta, \tilde{\beta}_k^t \sim \text{Dir}(\tilde{\beta}_k^t + \beta)$ .
- 3. For each user i:
  - (a) Draw topic proportions  $\theta_i^t | \lambda, \Omega^t, \tilde{\mathbf{n}}_i^t \sim \text{Dir}(\lambda \Omega^t + \tilde{\mathbf{n}}_i^t)$ .
  - (b) For each word
    - (a) Draw a topic  $z_{in}^t | \theta_i^t \sim \text{Mult}(\theta_i^t)$ .
    - (b) Draw a word  $w_{in}^t | z_{ij}^t, \phi^t \sim \text{Multi}(\phi_{z_{ij}^t}^t)$ .



- 1. Draw once  $\Omega^t | \alpha, \tilde{m}^t \sim \text{Dir}(\tilde{\mathbf{m}}^t + \alpha/K)$ .
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    - (b) Draw a word  $w_{in}^t | z_{ij}^t, \phi^t \sim \text{Multi}(\phi_{z_{ij}^t}^t)$ .

#### Topics evolve over time?

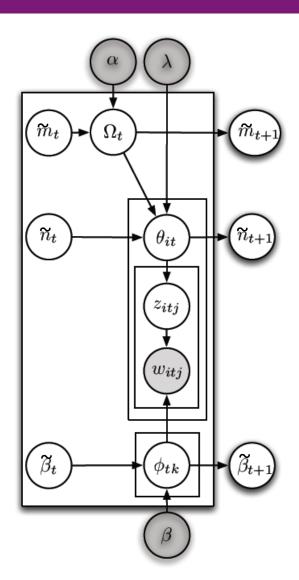


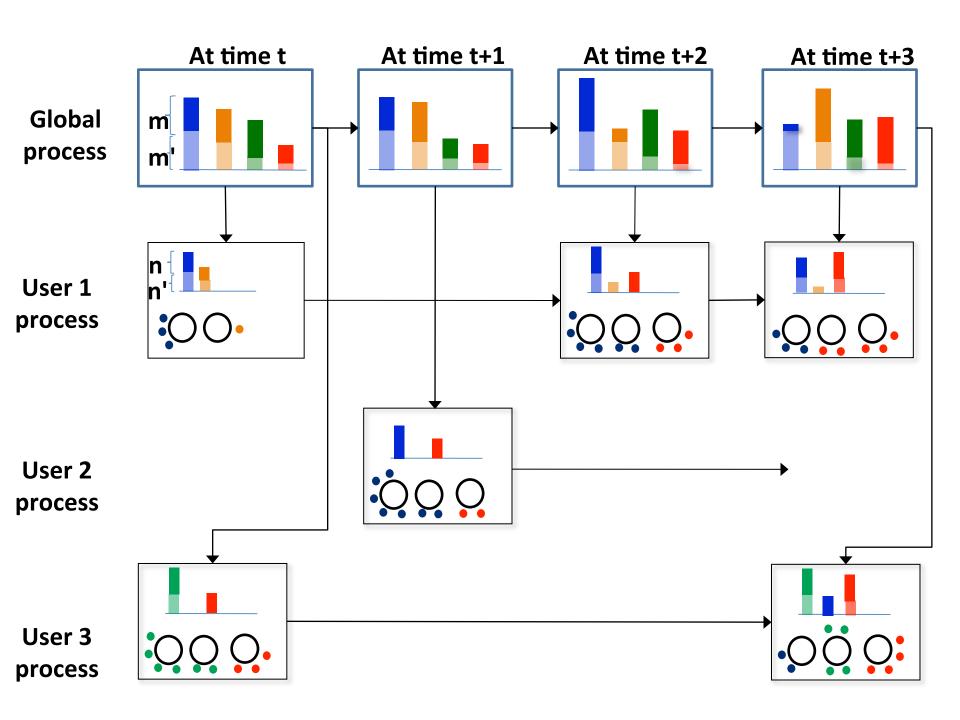
User's intent evolve over time?



Capture long and term interests of users?



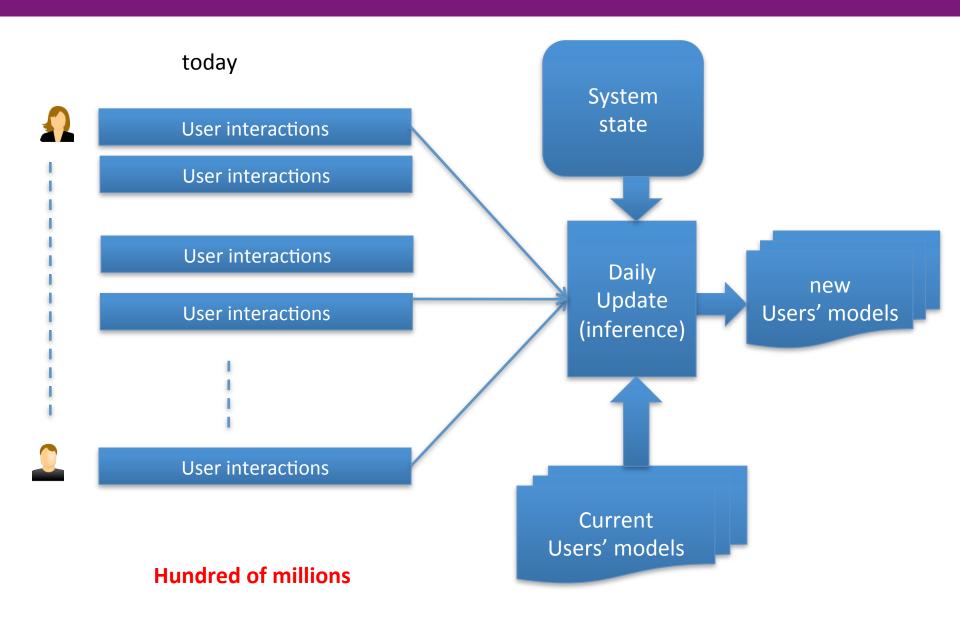




# Online Distributed Inference

**Work Flow** 

# **Work Flow**

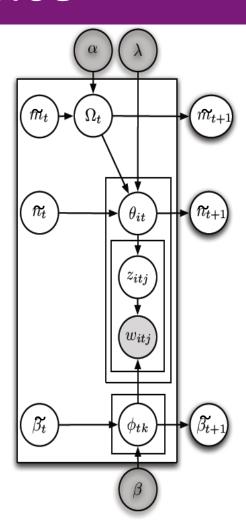


# Online Scalable Inference

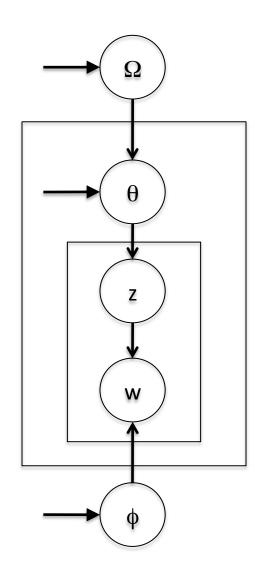
- Online algorithm
  - Greedy 1-particle filtering algorithm
  - Works well in practice
  - Collapse all multinomials except  $\Omega_t$ 
    - This makes distributed inference easier
  - At each time t:

$$P(\Omega^t, \mathbf{z}^t | \tilde{\mathbf{n}}^t, \tilde{\beta}^t, \tilde{\mathbf{m}}^t)$$

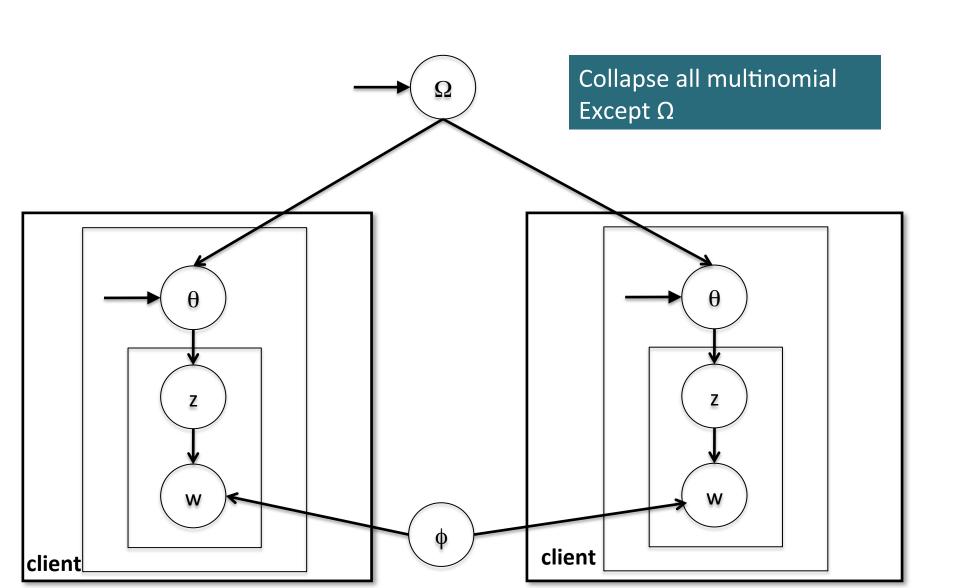
- Distributed scalable implementation
  - Used first part architecture as a subroutine
  - Added synchronous sampling capabilities



# Distributed Inference (at time t)



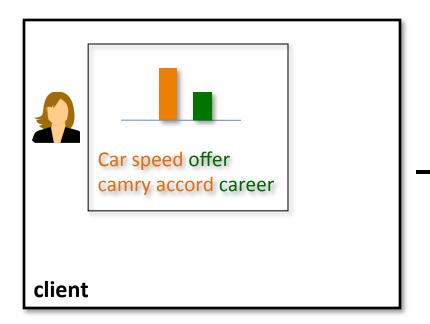
## Distributed Inference (at time t)

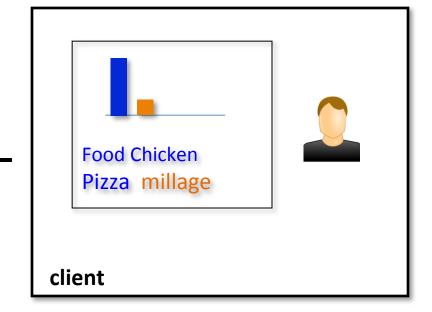


# After collapsing

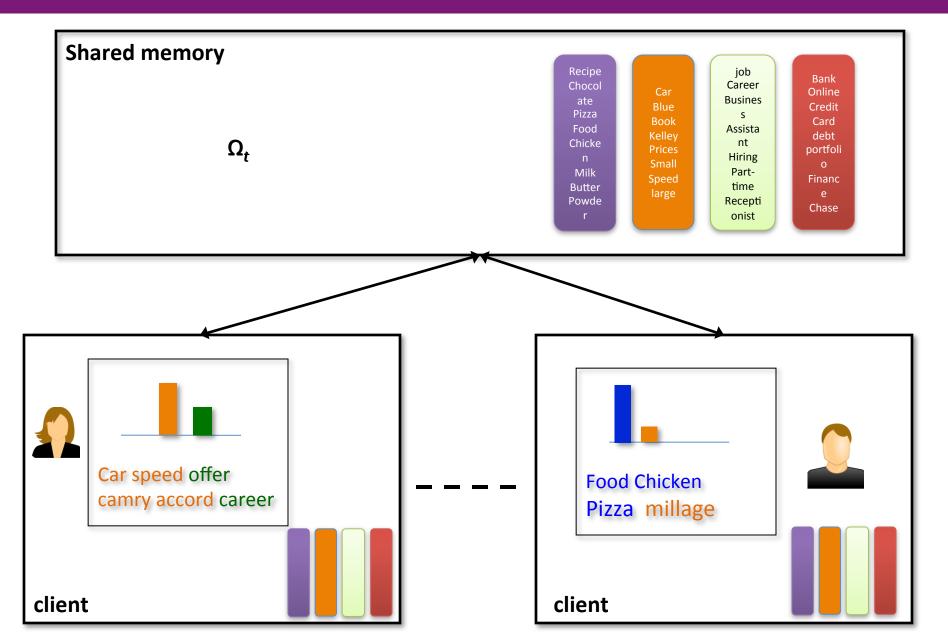


## Use Star-Synchronization





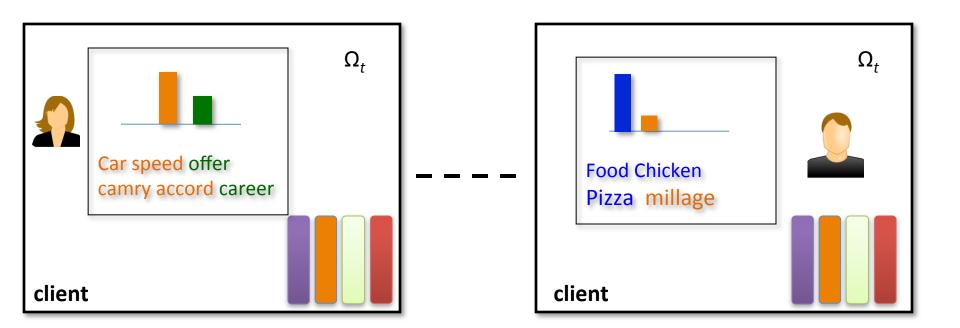
## **Fully Collapsed**



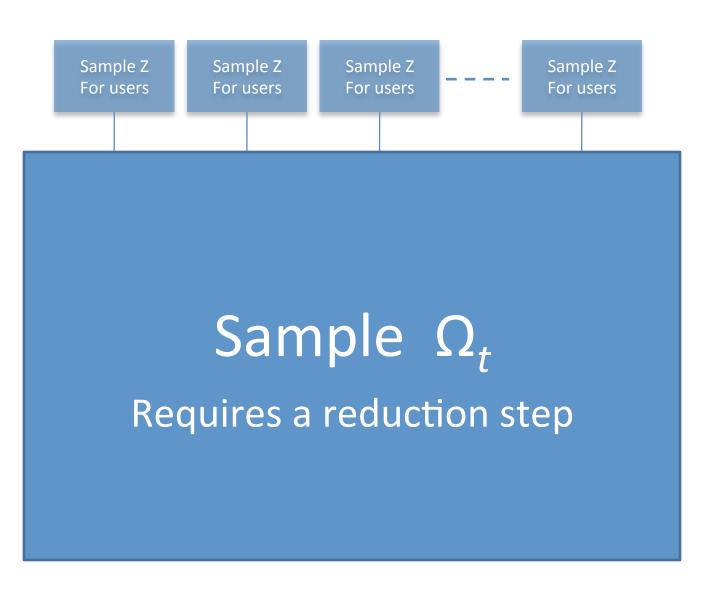
## Semi-Collapsed

$$P(z_{ij}^t = k | w_{ij}^t = w, \Omega^t, \tilde{\mathbf{n}}_i^t)$$

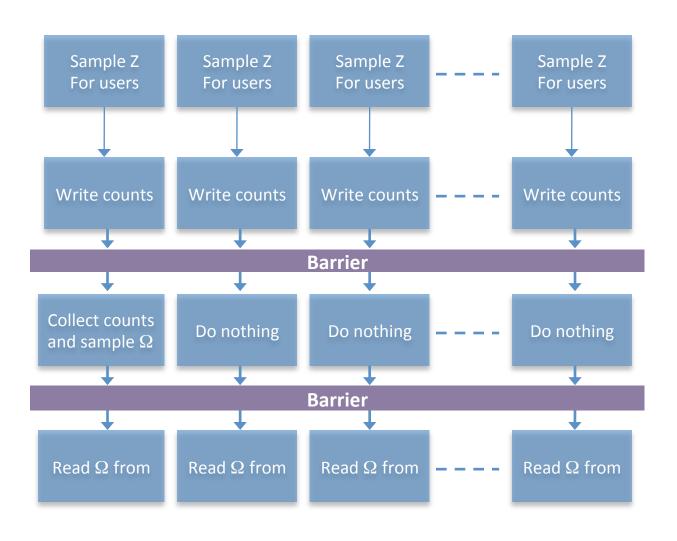
$$\propto \left( n_{ik}^{t,-j} + \tilde{n}_{ik}^t + \lambda \Omega^t \right) \frac{n_{kw}^{t,-j} + \tilde{\beta}_{kw}^t + \beta}{\sum_l n_{kl}^{t,-j} + \tilde{\beta}_{kl}^t + \beta}$$



# Distributed Sampling Cycle



## Distributed Sampling Cycle

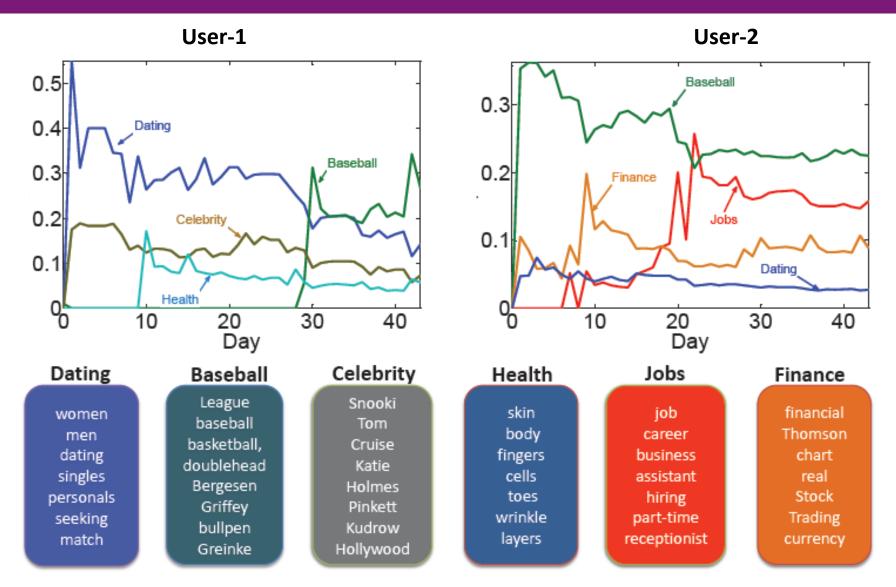


## **Experimental Results**

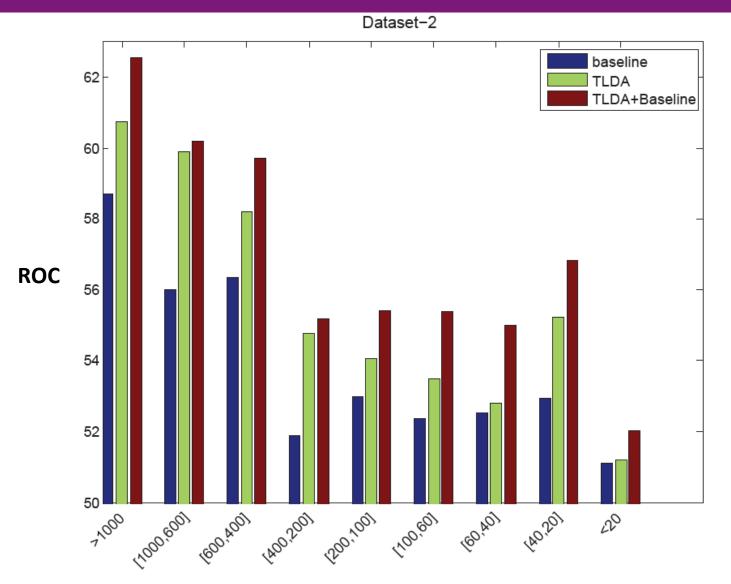
- Tasks is predicting convergence in display advertising
- Use two datasets
  - 6 weeks of user history
  - Last week responses to Ads are used for testing
- Baseline:
  - User raw data as features
  - Static topic model

dataset	# days	# users	# campaigns	size
1	56	13.34M	241	242GB
2	44	33.5M	216	435GB

## Interpretability



## Performance in Display Advertising



**Number of conversions** 

## Performance in Display Advertising

#### Weighted ROC measure

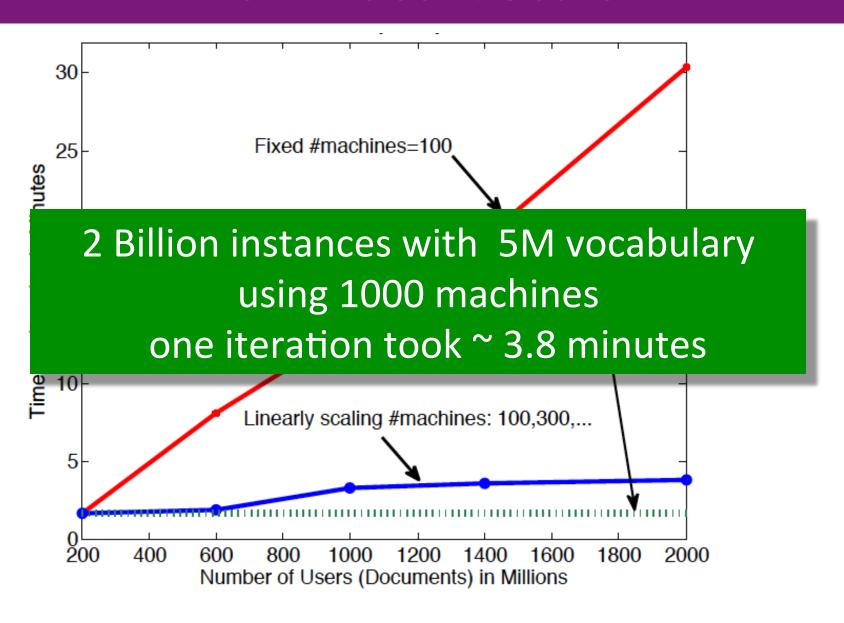
	base	TLDA	TLDA+base	LDA+base
dataset 1	54.40	55.78	56.94	55.80
dataset 2	57.03	57.70	60.38	58.54

#### Effect of number of topics

	topics	TLDA	TLDA + base
dataset 1	50	55.32	56.01
	100	55.5	56.56
	200	55.8	$\boldsymbol{56.94}$
dataset 2	50	59.10	60.40
	100	<b>59.14</b>	60.60
	200	58.7	60.38

Static Batch models

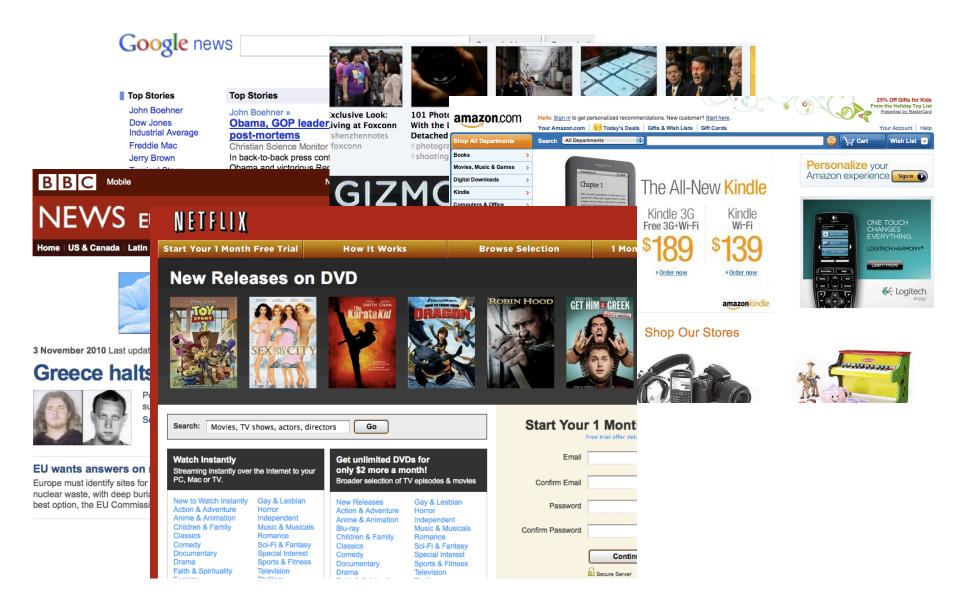
#### How Does It Scale?



## **Application**

# Multi-Domain Personalization

#### Problem



### Multi-domain Personalization

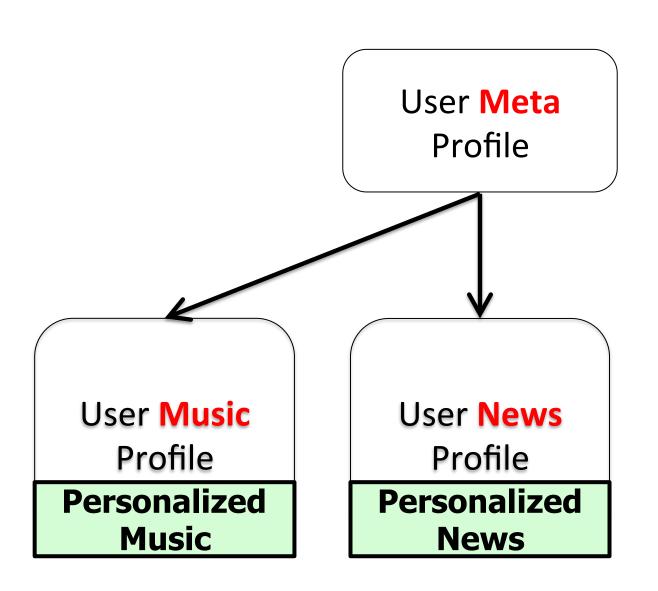
#### Intuition

- We observe user interaction with news and movies
- Can we predict his music taste?

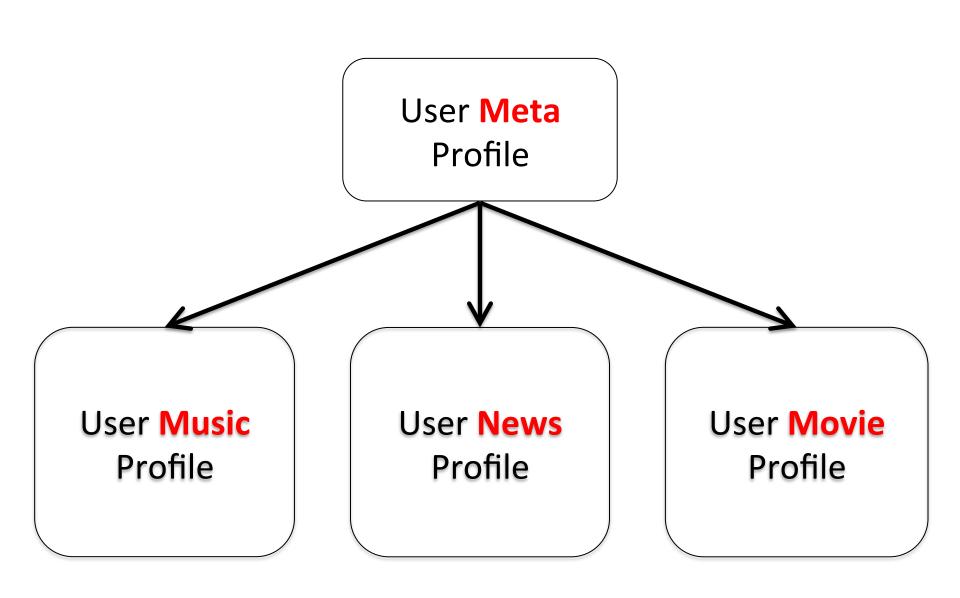
#### Interaction definition

 A bag of words describing objects user interacts with in a given domain

## Example



# Example



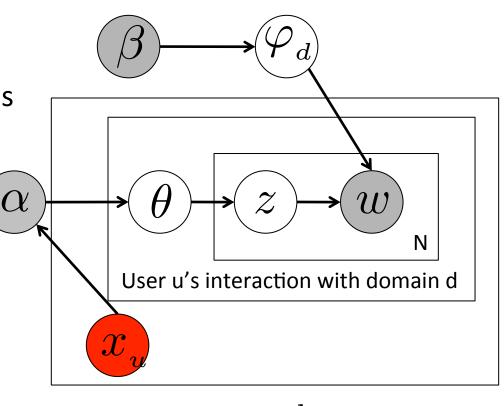
## The Model

A user's interaction with a domain is a bag of words.



User's **prior** interest in a domain is

$$\alpha = \log(1 + \exp(\lambda_d x_u))$$

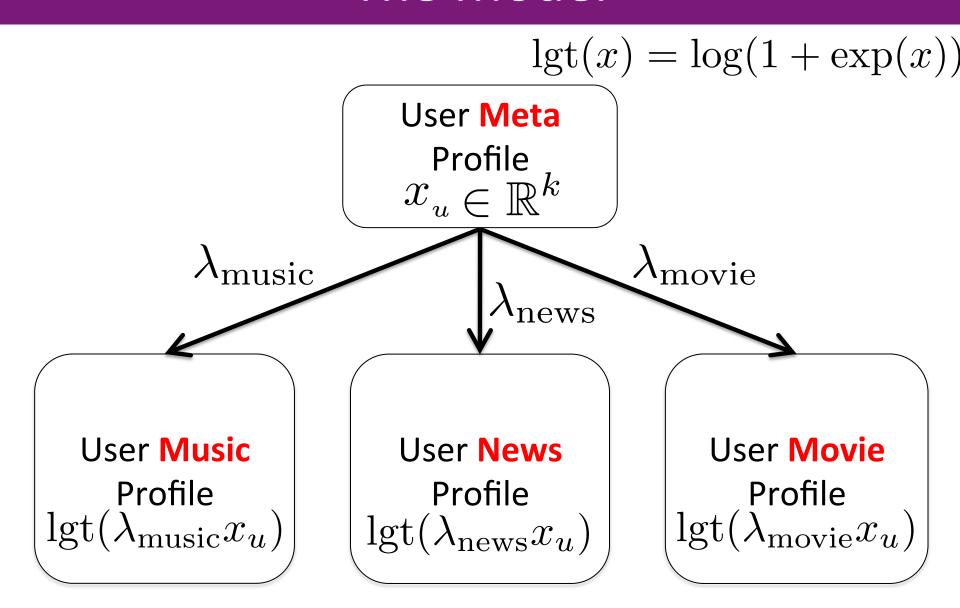


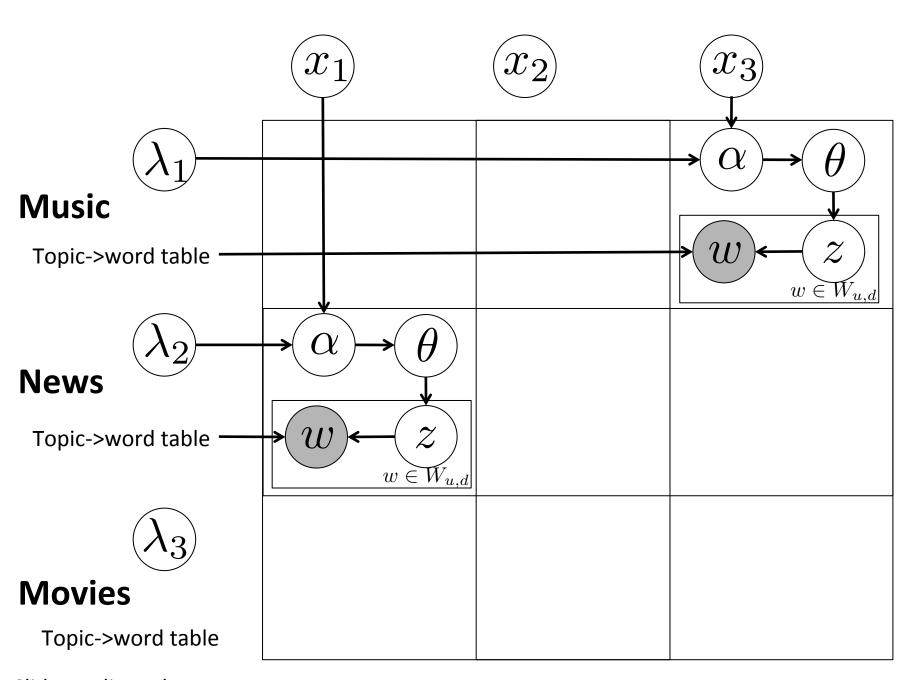
Each user has a meta-profile:

Each domain has a latent matrix:

$$x_u \in \mathbb{R}^k$$
$$\lambda_d \in \mathbb{R}^{k \times t_d}$$

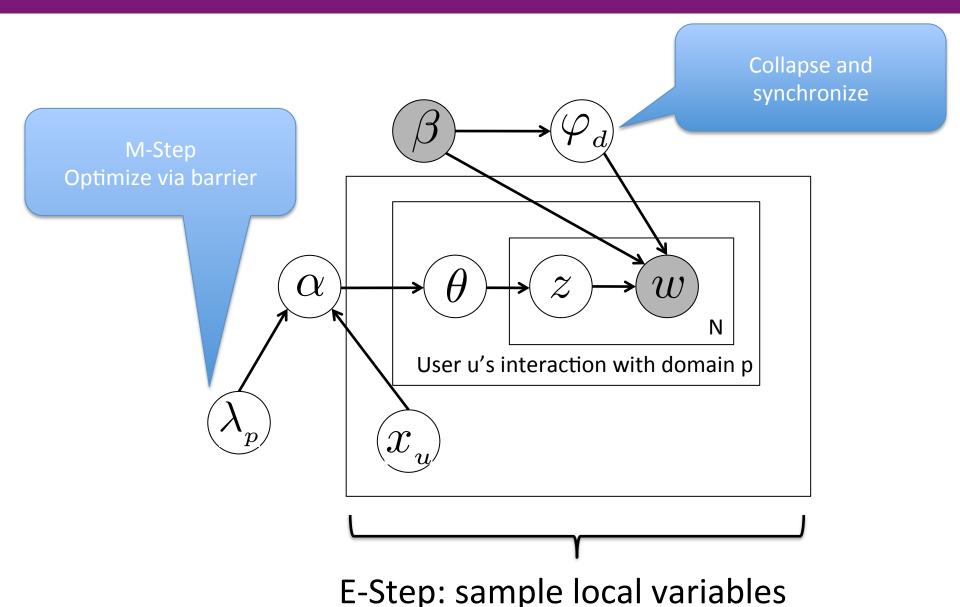
## The Model



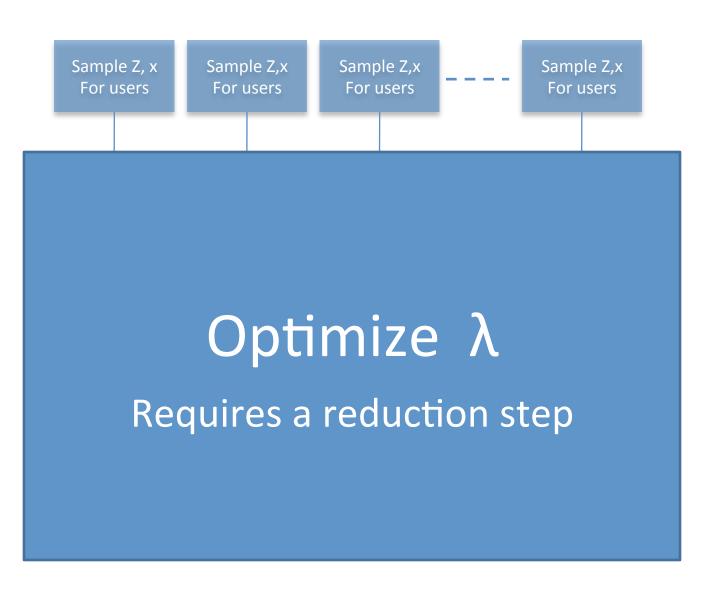


Slide credit Yucheng Low

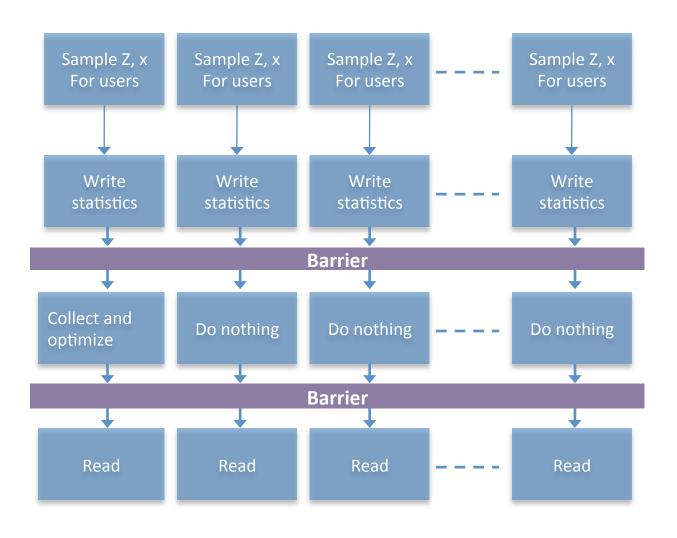
# Inference and Learning



## Distributed Sampling Cycle



## Distributed Sampling Cycle



#### Results

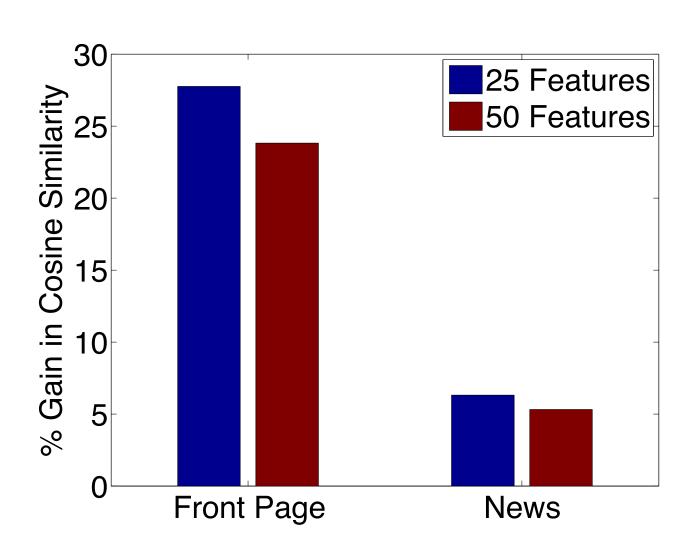
2 domain dataset.

Frontpage and News clicks of **5.6 million users.** 

Frontpage/News: Article text for each click.

 Measure gain relative to independent models on each domain

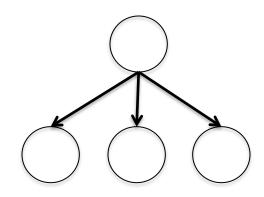
## Results



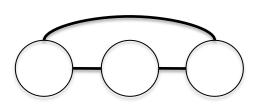
# Distributed Inference Revisited

## To collapse or not to collapse?

- Not collapsing
  - Keeps conditional independence
    - Good for parallelization
    - Requires synchronous sampling
  - Might mix slowly



- Collapsing
  - Mixes faster
  - Hinder parallelism
  - Use star-synchronization
    - Works well if sibling depends on each others via aggregates
    - Requires asynchronous communication



#### Inference Primitive

- Collapse a variable
  - Star synchronization for the sufficient statistics
- Sampling a variable
  - Local
    - Sample it locally (possibly using the synchronized statistics)
  - Shared
    - Synchronous sampling using a barrier
- Optimizing a variable
  - Same as in the shared variable case
  - Ex. Conditional topic models

# Asynchronous Optimization

## **Asynchronous Processing**

- Needed when
  - Ex: Optimizing a global variable
- Mostly requires a barrier
- Advantages
  - Easy to program
  - Well-understood reusable templates
- Disadvantages
  - The curse of the last reducer
  - You are as fast as the slowest machine!

## **Asynchronous Processing**

- Needed when
  - Ex: Optimize a global varial
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- an me do better. - Well-und
- Disady
  - the last reducer The cult
  - You are as fast as the slowest machine!

## **Asynchronous Optimization**

**Graph Factorization** 

## **Graph Factorization Problem**

- Factor a graph into low rank components
- Assign a latent vector  $Z_i \in \mathcal{R}^k$  with each node
- Optimize:

$$f(Y, Z, \lambda) = \frac{1}{2} \sum_{(i,j) \in E} (Y_{ij} - \langle Z_i, Z_j \rangle)^2 + \frac{\lambda}{2} \sum_i n_i ||Z_i||^2$$

Observed value over edges

Predicted value

Regularization

## Single-Machine Algorithm

Just use stochastic gradient decent (SGD)

$$\frac{\partial f}{\partial Z_i} = -\sum_{j \in \mathcal{N}(i)} (Y_{ij} - \langle Z_i, Z_j \rangle) Z_j + \lambda n_i Z_i$$

- Cycle until convergence
  - Read a node, i
  - Update its latent factor

$$Z_i \leftarrow Z_i - \eta \left(\frac{\partial f}{\partial Z_i}\right)$$

### Problem Scale

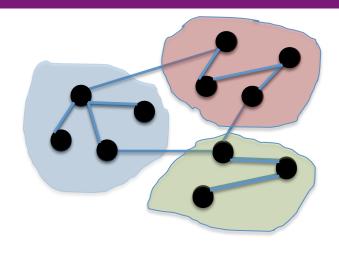
- Yahoo IM and Mail graphs
- Nodes are users
- Edges represent (log) number of messages
- 200 Million vertices
- 10 Billion edges

## Challenges

- Parameter storage
  - Too much for a single machine
- Approach
  - Distribute the graph over machines
    - How to partition the nodes?
  - Synchronization
    - How to synchronize replicated nodes
  - Communication
    - How to accommodate network topology

## Challenges

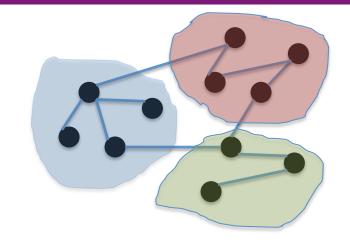
Can we solve the problem with similar ideas to what we have covered?

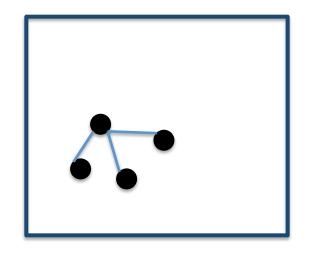


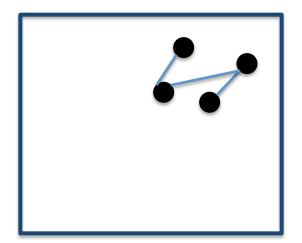


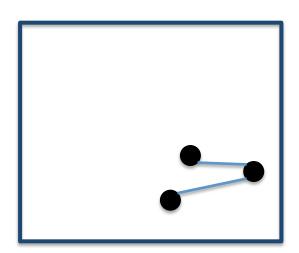
- Cycle until convergence
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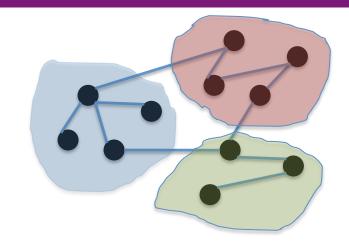


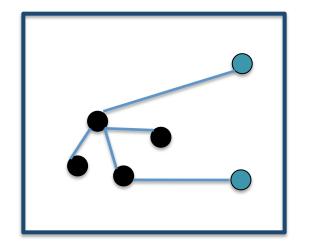


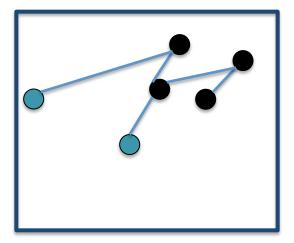


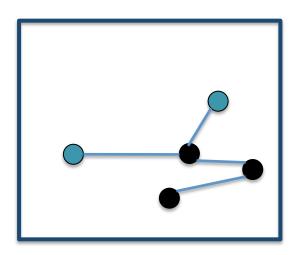


- Problem
  - Some neighbors are missing
- Solution
  - Replicate and synchronize
  - Borrowed vs. owned nodes

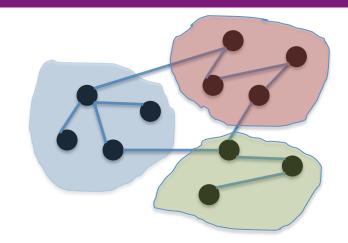


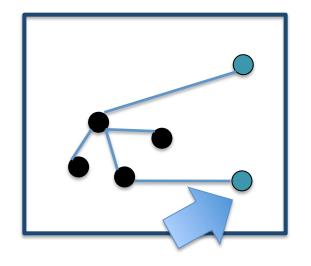


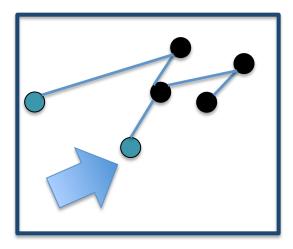


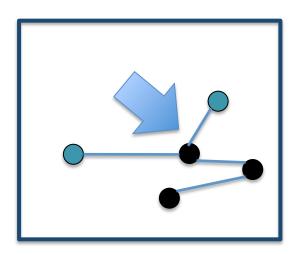


- Formulation
  - Introduce local copies
    - A factor per node X
  - Tie across machines
    - Introduce global factor Z
    - Penalizes deviations









#### **Formulation**

Original problem

$$f(Y, Z, \lambda) = \frac{1}{2} \sum_{(i,j) \in E} (Y_{ij} - \langle Z_i, Z_j \rangle)^2 + \frac{\lambda}{2} \sum_{i} n_i ||Z_i||^2$$

Relaxed problem

$$\sum_{k=1}^{K} f_k(Y, X^{(k)}, \lambda) + \frac{1}{2} \sum_{k=1}^{K} \left[ \mu \sum_{i \in V_k} ||Z_i - X_i^{(k)}||^2 \right]$$

**Local factors** 

Deviation

Global factor

Local problem

$$f_k(Y, X^{(k)}, \lambda) = \frac{1}{2} \left[ \sum_{\substack{(i,j) \in E, \\ i \in V}} \left( Y_{ij} - \langle X_i^{(k)}, X_j^{(k)} \rangle \right)^2 + \lambda \sum_{i \in V_k} n_i ||X_i^{(k)}||^2 \right]$$

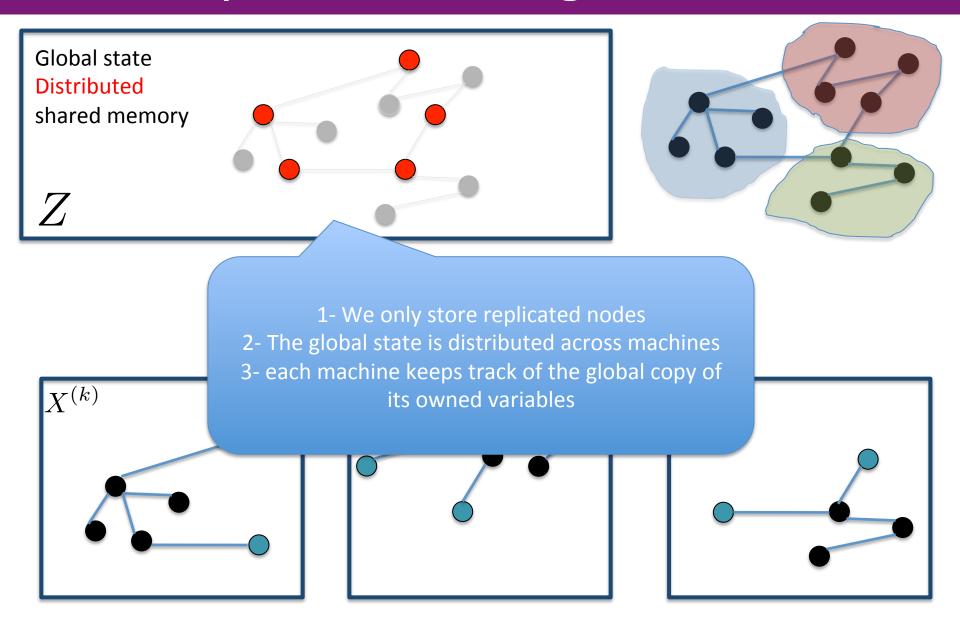
### Synchronous Algorithms

- Optimize joint objective over X,Z
- Local parameter updates
  - Run SGD until convergence

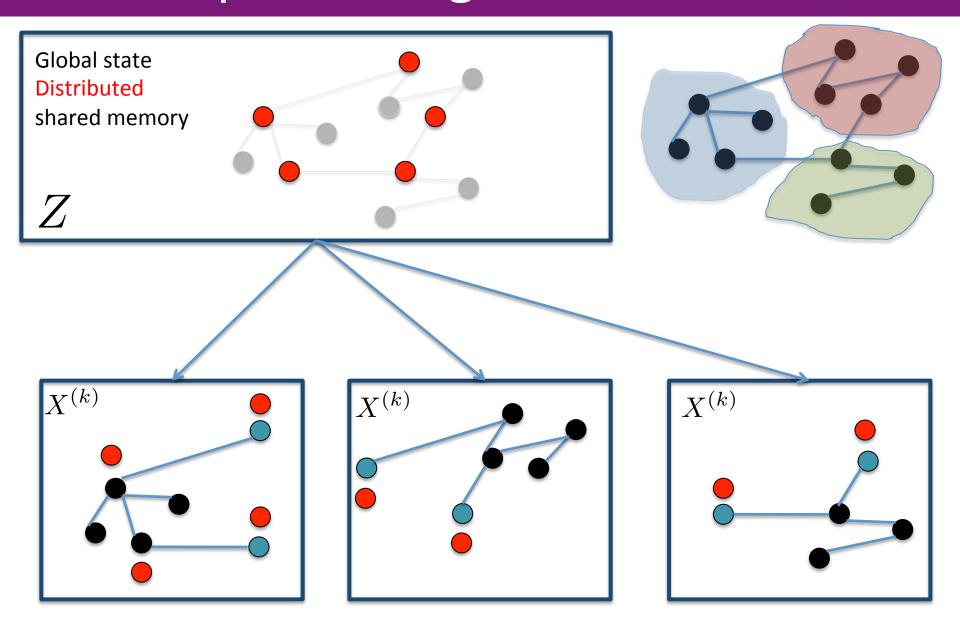
Global parameter updates

minimize<sub>Z</sub> 
$$\frac{1}{2} \sum_{k=1}^{K} \left[ \mu \sum_{i \in V_k} \|Z_i - X_i^{(k)}\|^2 \right]$$

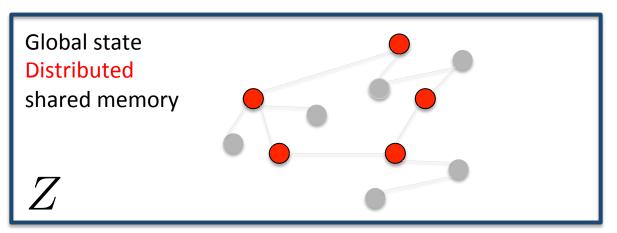
# Synchronous Algorithms

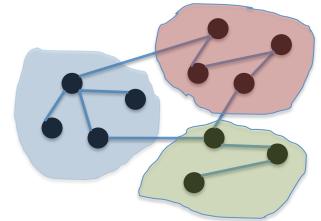


# Step 1: Push global variables

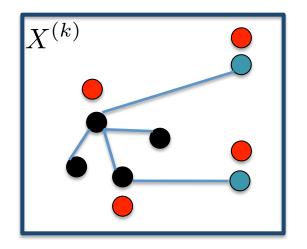


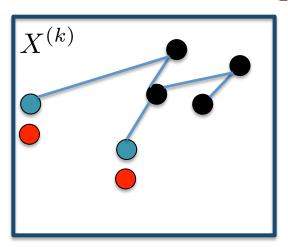
#### Step 2: Local Optimization

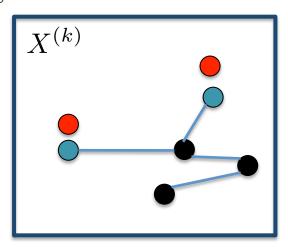




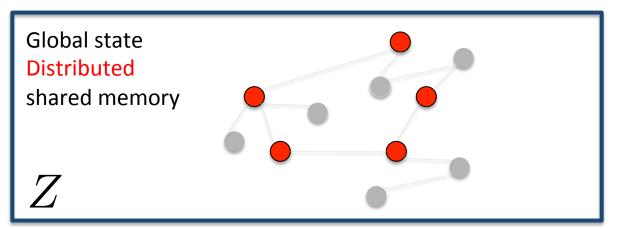
minimize<sub>X(k)</sub> 
$$f_k(Y, X^{(k)}, \lambda) + \frac{1}{2} \mu \sum_{i \in V_k} ||Z_i - X_i^{(k)}||^2$$

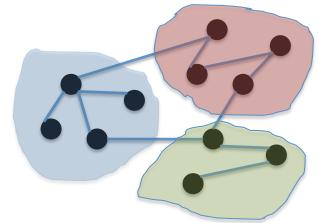






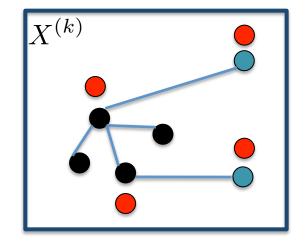
#### Step 3: Push and average

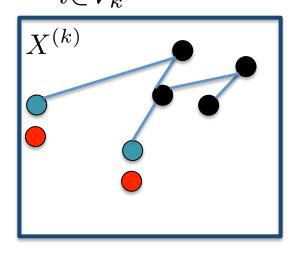


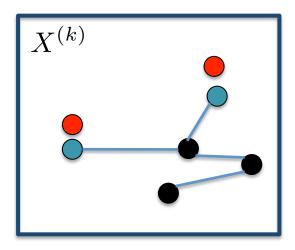


$$\operatorname{minimize}_Z$$

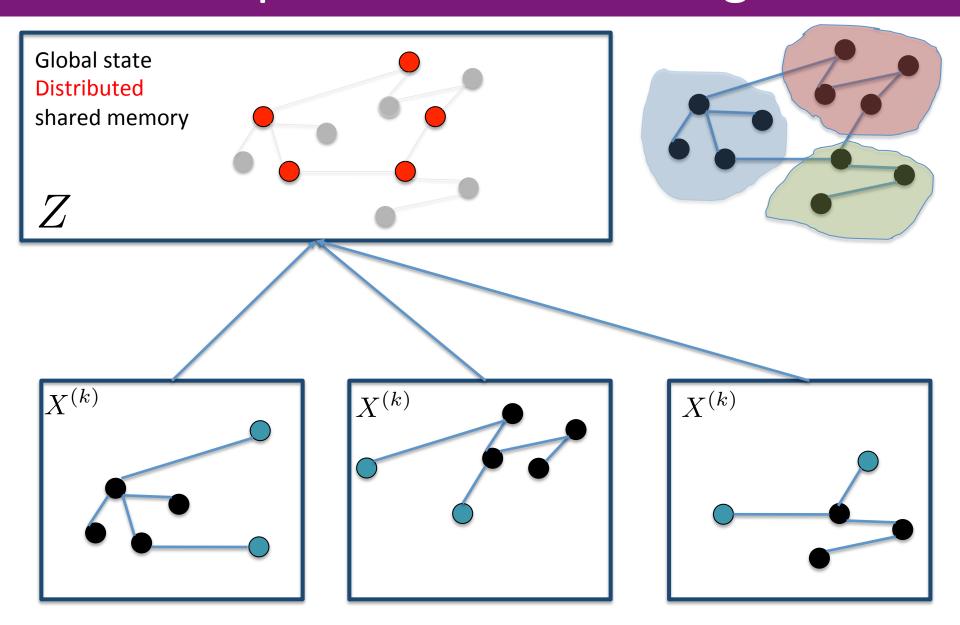
$$\frac{1}{2} \sum_{k=1}^{K} \left[ \mu \sum_{i \in V_k} \|Z_i - X_i^{(k)}\|^2 \right]$$







# Step 3: Push and average



# Summary of Asynchronous Algorithms

- An improvement over standard Map-Reduce
- Curse of the last reducer
- You are as fast as the slowest machine
  - Optimize local variables
  - Barrier
  - Optimize global variables
  - Barrier
- Can we do better?

### An Asynchronous Algorithm

- Conceptual idea
  - Optimize X and Z jointly

$$\sum_{k=1}^{K} f_k(Y, X^{(k)}, \lambda) + \frac{1}{2} \sum_{k=1}^{K} \left[ \mu \sum_{i \in V_k} ||Z_i - X_i^{(k)}||^2 \right]$$

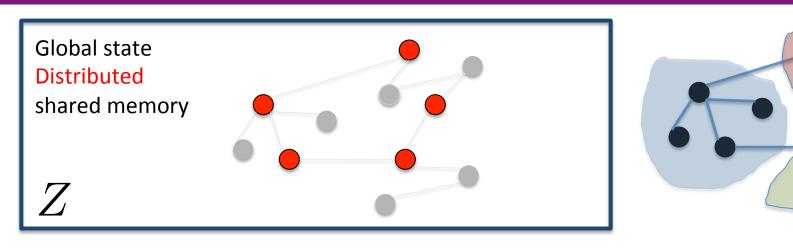
- User SGD over (X,Z)
- Pick a local node
- Do a gradient step over corresponding X,Z!

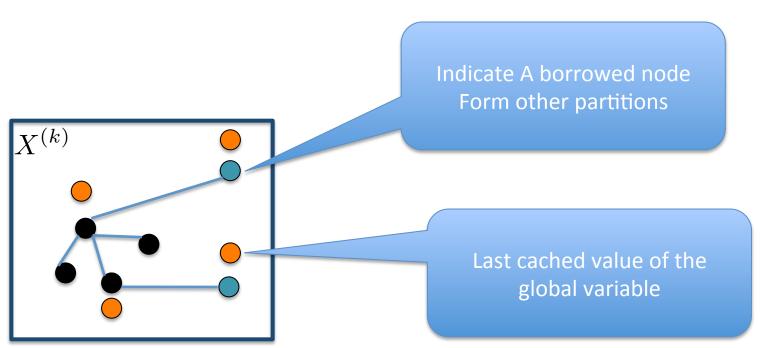
#### Conceptual Idea

$$\sum_{k=1}^{K} f_k(Y, X^{(k)}, \lambda) + \frac{1}{2} \sum_{k=1}^{K} \left[ \mu \sum_{i \in V_k} \|Z_i - X_i^{(k)}\|^2 \right]$$

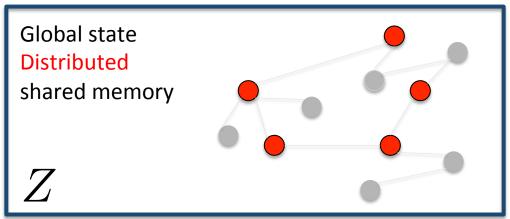
$$\frac{\partial f}{\partial Z_i} \left[ X_i^{(k)} \right] = \mu(Z_i - X_i^{(k)}).$$
Cache the global variables variables Locally (Asynchronous updates) 
$$+ \lambda n_i X_i + \mu(X_i^{(k)} - Z_i).$$

# Parallel Updates

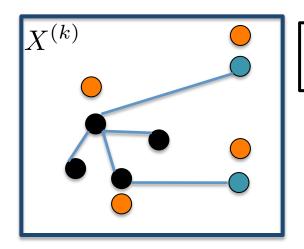




## Parallel Asynchronous Updates



$$\frac{\partial f}{\partial X_i^{(k)}} = -\sum_{j \in N(i)} (Y_{ij} - \langle X_i^{(k)}, X_j^{(k)} \rangle) X_j^{(k)} + \lambda n_i X_i^{(k)} + \mu (X_i^{(k)} - Z_i^{(k)}).$$



-Cycle through nodes-Update local copies

Computation thread

- -Receive local copy X\_i from k
  - -Update Z i
  - -Send back new Z\_i to k

$$\frac{\partial f}{\partial Z_i} \left[ X_i^{(k)} \right] = \mu(Z_i - X_i^{(k)}).$$

#### Synchronization thread Send

- -Cycle through nodes
  - Send local copy to DSM
- -Received Z\_i from DSM
  - update cached copy

Synchronization thread receive

#### Convergence

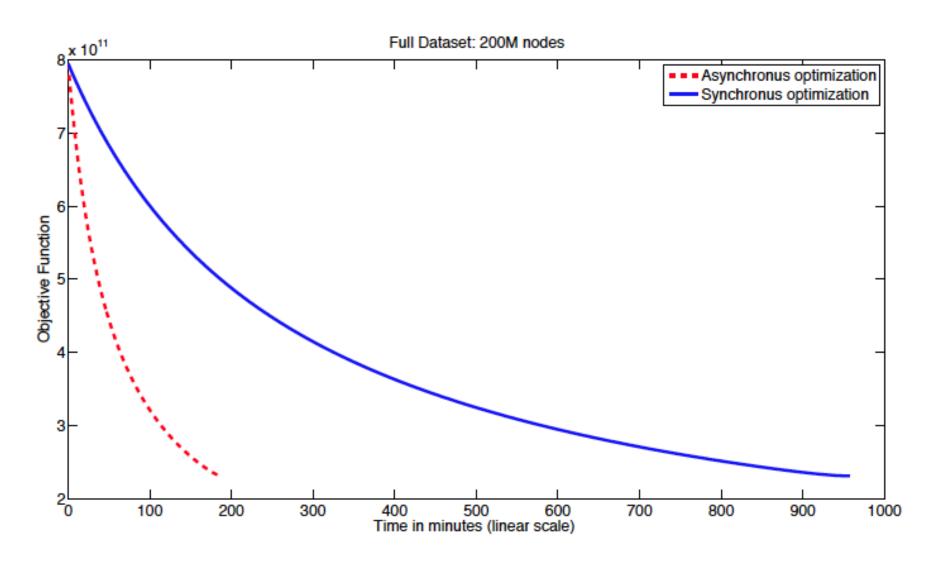
- Can be reduced to lock-free parallel SGD [Hogwild]
- Convergence is affected by
  - Synchronization rate
    - Time needed to refresh the local version of the global variable
    - Number of replicated nodes

## Summary of Asynchronous

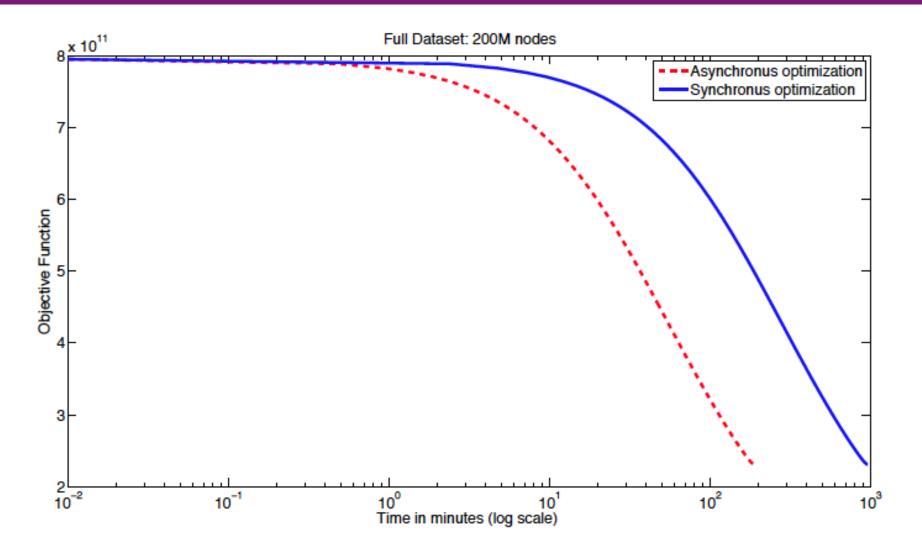
- Continuously update local variables X (via SGD)
- Continuously send local variables to global
- Continuously update global variable Z (via SGD)
- Continuously send & overwrite global variables to local

$$\sum_{k=1}^{K} f_k(Y, X^{(k)}, \lambda) + \frac{1}{2} \sum_{k=1}^{K} \left[ \mu \sum_{i \in V_k} \|Z_i - X_i^{(k)}\|^2 \right]$$

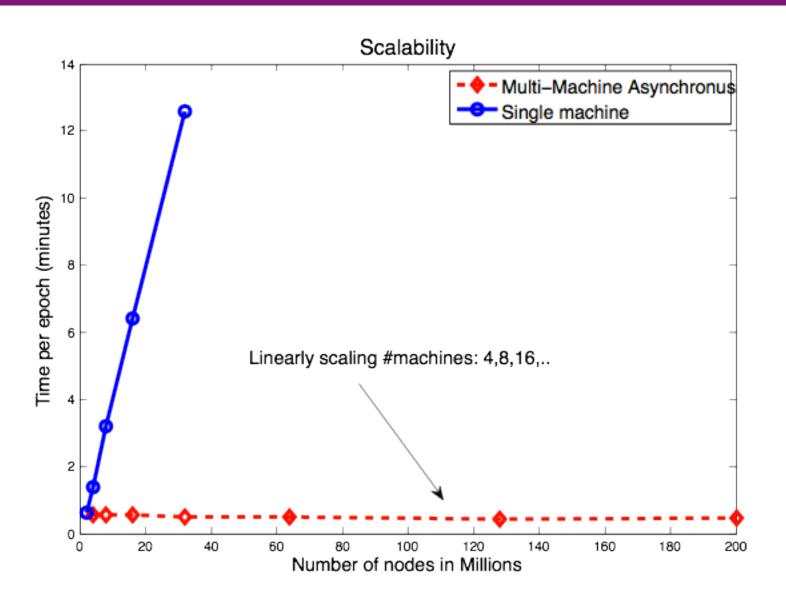
# Convergence



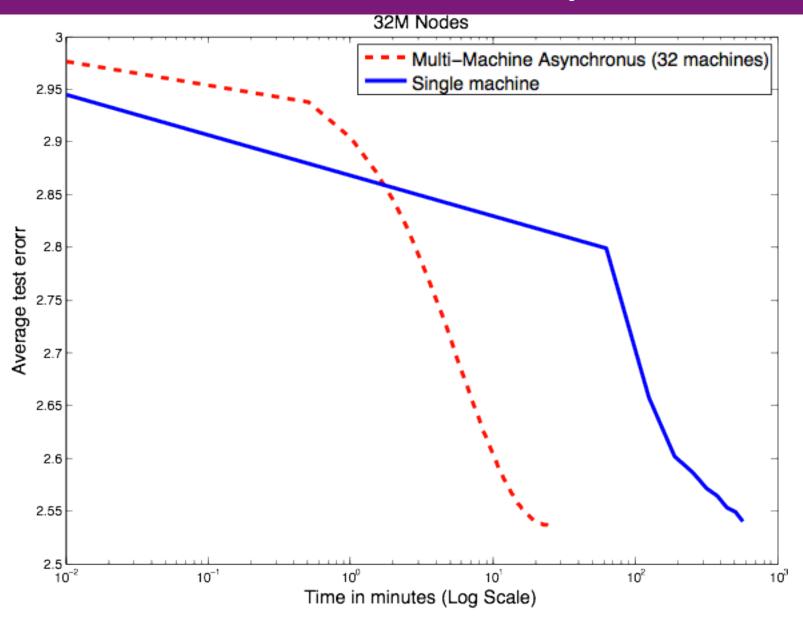
# Convergence



# Scalability



# **Solution Quality**



#### **Practical Considerations**

- How to partition the graph?
  - We want to minimize the number of borrowed nodes
    - Affect convergence
    - Increases the number of deviation penalties
  - Take each machine capacity into consideration
    - Store owned nodes
    - Borrowed nodes
    - Cached copies of relevant global variables
- Network Optimization
  - Take network topology into account

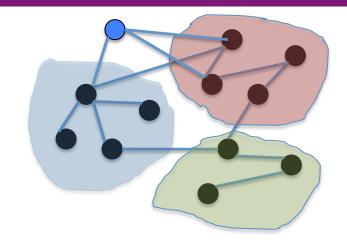
#### **Graph Partition**

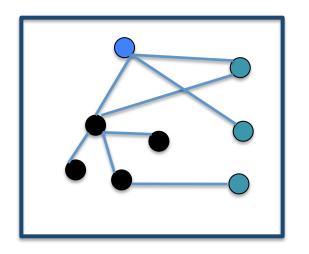
- Find a set of minimally overlapped partitions
   "Decompose the graph to minimize number of vertices + neighbors per partition"
  - NP hard problem by itself [WSDM 2012]
- Under capacity constraints
- We just scratched the surface here
  - Simple greedy algorithm
  - Hierarchal extension
  - LSH and random baselines

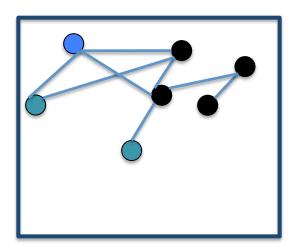
# Single Pass Greedy Algorithm

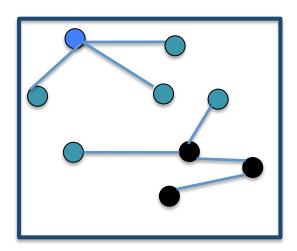
- Intuitively
  - Add each node to where its neighbors are!
- Maintain a set of open partitions
  - Store the borrowed and owned nodes in each partition
- For each vertex v
  - For each partition p
    - We want to make sure that N(v) are in the same partition
    - Add N(v) / Owned(p) to borrowed of p
  - Select p with minimum number of borrowed nodes

- •For each vertex v
  - •For each partition *p* 
    - •We want to make sure that N(v) are in the same partition
    - •Add N(v) / Owned(p) to borrowed of p
  - •Select p with minimum number of borrowed nodes









#### Hierarchical Extension

- Two step approach
  - First run greedy with small number of partitions
  - Second, run greedy over the first level partitions
- Time is proportional to number of open partitions
  - Divide and conquer

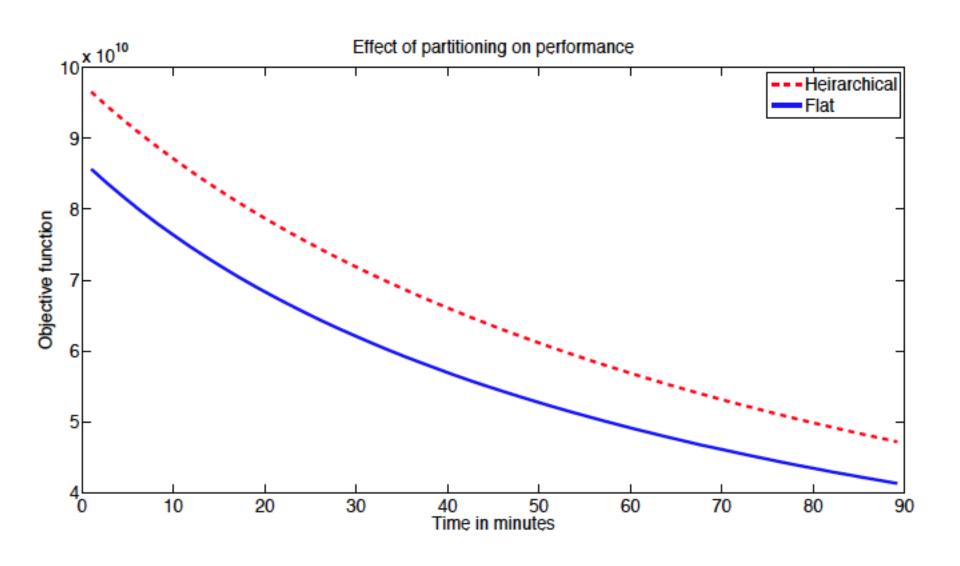
#### Baselines

- Radom
- LSH-based
  - LSH over adjacency matrix
  - Related to shingle-based graph compression approaches
- Metrics
  - Time to perform partitioning
  - Quality of partitions
    - Number of borrowed nodes
    - Time to perform a full synchronization cycle

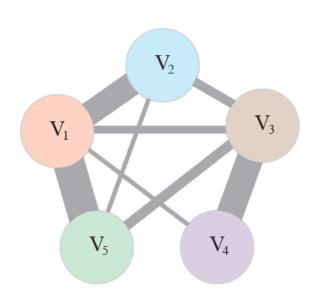
# The Effect of Partitioning Quality

Method	Total borrowed	Partitioning time	Sync time
	nodes (millions)	(minutes)	(seconds)
Flat	252.31	166	71.5
Hierarchical	392.33	48.67	85.9
Hier-LSH	640.67	17.8	136.1
Hier-Random	720.88	11.6	145.2

# The Effect of Partitioning Quality



# **Network Optimization**



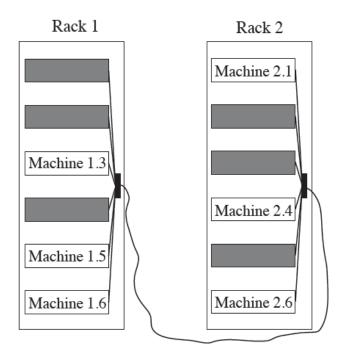
V<sub>1</sub> — Machine 1.6

V<sub>2</sub> — Machine 1.3

V<sub>3</sub> — Machine 2.4

V<sub>4</sub> — Machine 2.1

 $V_5$  — Machine 1.5

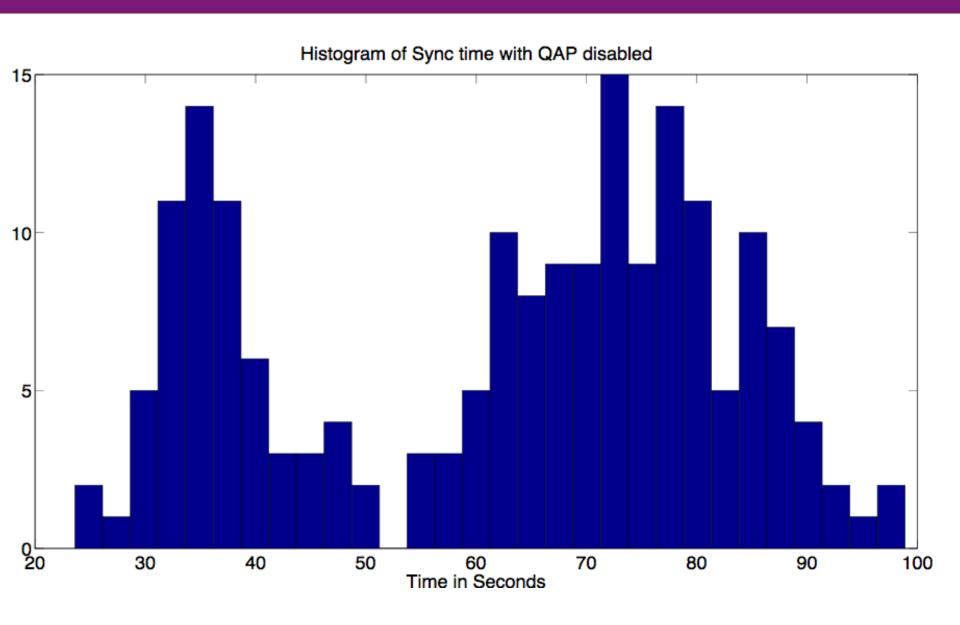


#### **Network Optimization**

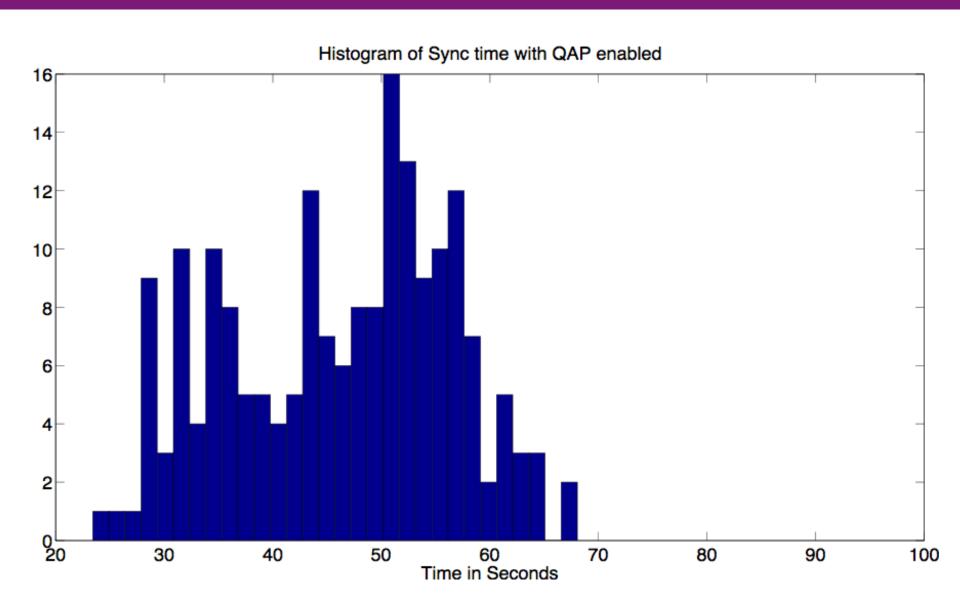
- We only know the layout at run time
  - Inverse network bandwidth D
- Inter-partitions communication
  - Communication requirement C
  - The more overlap, the higher is C
- Solve a quadratic assignment problem

$$T(\pi) = \sum_{kl} C_{kl} D_{\pi(k)\pi(l)} = \sum_{kl} C_{kl} \sum_{uv} \pi_{ku} \pi_{lv} D_{uv} = \operatorname{tr} C \pi D \pi^{\top}$$

# Sync time without QAP



# Sync time with QAP



### Summary

- Model as consensus problem
- Synchronous algorithms
  - Curse of the last reducer
- Asynchronous algorithm
  - Asynchronous parallel updates
  - Network topology optimization
  - Overlapping partitions

#### **Future Directions**

#### **Future Directions**

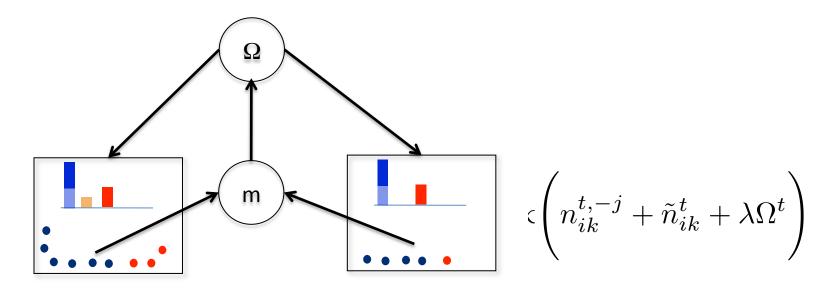
- Theoretical bounds and guarantees
- Non-parametric models
  - Learning structure from data
- Working under communication constraints
- A new release of Yahoo! LDA
- More applications
  - Citation analysis
    - Graph factorization + LDA

# Questions?

## Sampling $\Omega$

- Introduce auxiliary variable  $m_{\rm kt}$ 
  - How many times the global distribution was visited
  - $P(m_k^t|n_{1k}^t,\cdots,n_{ik}^t,\cdots)$ ~ AnotniaK

$$P(\Omega^t | \mathbf{m}^t, \tilde{\mathbf{m}}^t) \sim \text{Dir}(\tilde{\mathbf{m}}^t + \mathbf{m}^t + \alpha/K)$$



# Distributed Sampling Cycle

