

### Graphical Models for the Internet

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credits to Amr Ahmed, Yahoo Research, CA

### Outline

#### 1. Systems

- Hardware (computer architecture / networks / data centers)
- Storage and processing (file systems, MapReduce, Dryad, S4)
- Communication and synchronization (star, ring, hashtable, distributed star, tree)

#### 2. Applications on the internet

- User modeling (clustering, abuse, profiling)
- Content analysis (webpages, links, news)
- Search / sponsored search

#### 3. Probabilistic modeling

- Basic probability theory
- Naive Bayes
- Density estimation (exponential families)

#### 4. Directed graphical models

- Directed graph semantics (independence, factorization)
- Clustering and Markov models (basic model, EM, sampling)
- Dirichlet distribution

### Outline

#### 5. Scalable topic modeling

- Latent Dirichlet Allocation
- Sampling and parallel communication
- Applications to user profiling

#### 6. Applications of latent variable models

- Time dependent / context dependent models
- News articles / ideology estimation
- Recommendation systems

#### 7. Undirected graphical models

- Conditional independence and graphs (vertices, chains, trellis)
- Message passing (junction trees, variable elimination)
- Variational methods, sampling, particle filtering, message passing

#### 8. Applications of undirected graphical models (time permitting)

- Conditional random fields
- Information extraction
- Unlabeled data

# Part 1 - Systems



### Hardware

# Computers

#### • CPU

- 8-16 cores (Intel/AMD servers)
- 2-3 GHz (close to 1 IPC per core peak) 10-100 GFlops/socket
- 8-16 MB Cache (essentially accessible at clock speed)
- Vectorized multimedia instructions (128bit wide, e.g. add, multiply, logical)
- Deep pipeline architectures (branching is expensive)

#### RAM

- 8-128 GB depending on use
- 2-3 memory banks (each 32bit wide atomic writes!)
- DDR3 (10GB/s per chip, random access >10x slower)

#### Harddisk

- 2-3 TB/disk
- 100 MB/s sequential read from SATA2
- 5ms latency (no change over 10 years), i.e. random access is slow

#### Solid State Drives

- 500 MB/s sequential read
- Random writes are really expensive (read-erase-write cycle for a block)
- Latency is 0.5ms or lower (controller & flash cell dependent)
- Anything you can do in bulk & sequence is at least 10x faster









# Computers

#### Network interface

- Gigabit network (10-100MB/s throughput)
- Copying RAM across network takes 1-10 minutes
- Copying 1TB across network takes 1 day
- Dozens of computers in a rack (on same switch)

#### Power consumption

- 200-400W per unit (don't buy the fastest CPUs)
- Energy density big issue (cooling!)
- GPUs take much more power (150W each)

#### Systems designed to fail

- Commodity hardware
- Too many machines to ensure 100% uptime
- Design software to deal with it (monitoring, scheduler, data storage)
- Systems designed to grow







### Server Centers

- 10,000+ servers
- Aggregated into rack (same switch)
- Reduced bandwidth between racks
- Some failure modes
  - OS crash
  - Disk dies (one of many)
  - Computer dies
  - Network switch dies (lose rack)
  - Packet loss / congestion / DNS
- Several server centers worldwide
  - Applications running distributed (e.g. ranking, mail, images)
  - Load balancing
  - Data transfer between centers





### Some indicative prices (on Amazon)

Standard On-Demand Instances	
Small (Default)	\$0.085 per hour
Large	\$0.34 per hour
Extra Large	\$0.68 per hour
Micro On-Demand Instances	
Micro	\$0.02 per hour
Hi-Memory On-Demand Instances	
Extra Large	\$0.50 per hour
Double Extra Large	\$1.00 per hour
Quadruple Extra Large	\$2.00 per hour
Hi-CPU On-Demand Instances	
Medium	\$0.17 per hour
Extra Large	\$0.68 per hour
Cluster Compute Instances	
Quadruple Extra Large	\$1.60 per hour

### storage

#### Amazon EBS Volumes

- \$0.10 per GB-month of provisioned storage
- \$0.10 per 1 million I/O requests

#### Amazon EBS Snapshots to Amazon S3

- \$0.15 per GB-month of data stored
- \$0.01 per 1,000 PUT requests (when saving a snapshot)
- \$0.01 per 10,000 GET requests (when loading a snapshot)

\$0.100 per GB

### data transfer

### Quadruple Extra Large \$2.10 per h All data transfer in Data Transfer OUT

Data Transfer IN

server costs

Cluster GPU Instances

Data Transfer OUT	
First 1 GB / month	\$0.000 per GB
Up to 10 TB / month	\$0.150 per GB
Next 40 TB / month	\$0.110 per GB
Next 100 TB / month	\$0.090 per GB
Greater than 150 TB / month	\$0.080 per GB

# Processing in the cloud

### Data storage

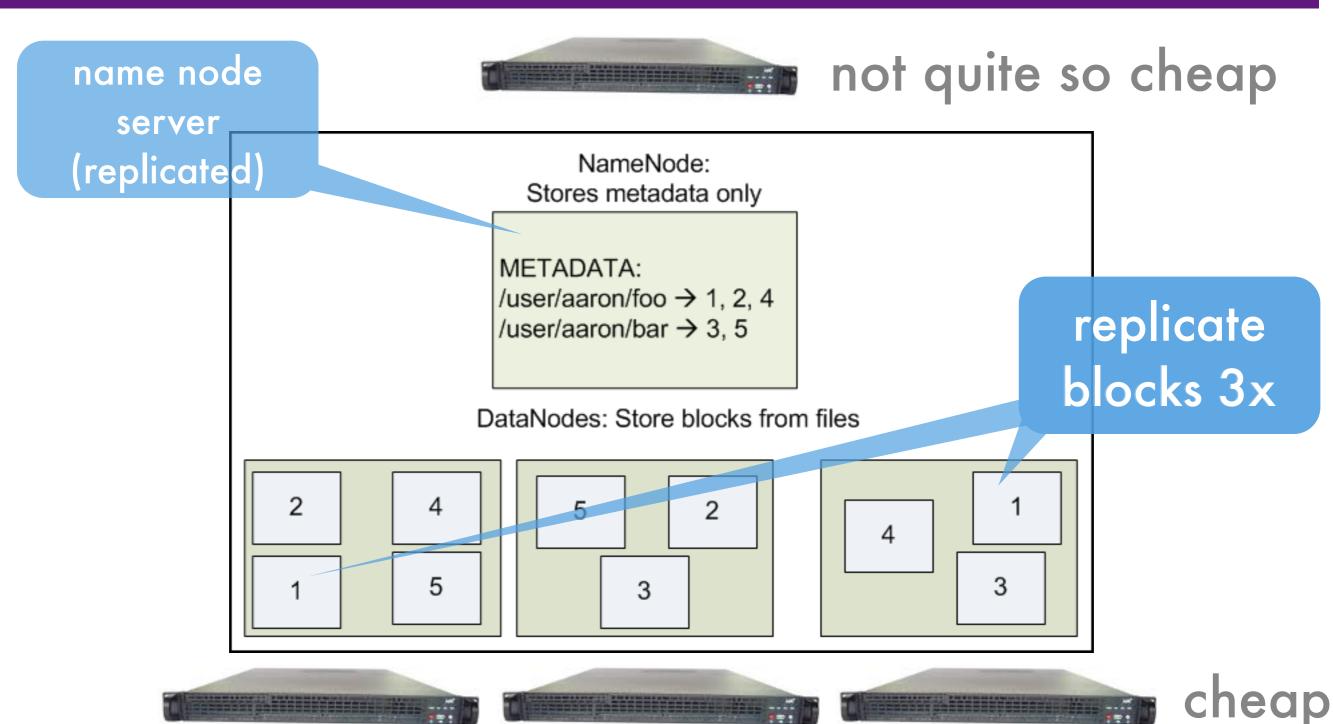
- Billions of webpages
- Billions of queries in query / click log
- Millions of data ranked by editors
- Storing data is cheap
  - less than 100 Billion interesting webpages
  - assume 10kB per webpage 1PB total
  - Amazon S3 price (1 month) \$10k (at \$0.01/GB)
- Processing data is expensive
  - 10 Billion webpages
  - 10ms per page (only simple algorithms work)
  - 10 days on 1000 computers (\$24k-\$240k at \$0.1-\$1/h)
- Crawling the data is very expensive
  - Assume 10 Gb/s link takes >100 days to gather APCN2 cable has 2.5 Tb/s bandwidth
  - Amazon EC2 price \$100k (at \$0.1/GB), with overhead \$1M)







### File Systems (GoogleFS/HDFS)



Ghemawat, Gobioff, Leung, 2003

servers

# File Systems Details

- Chunkservers
  - store 64MB (or larger) blocks
  - write to one, replicate transparently (3x)
- Name node
  - keeps directory of chunks
  - replicated
- Write
  - get chunkserver ID from name node
  - write block(s) to chunkserver (expensive to write parts of chunk)
  - largely write once scenario (read many times)
  - distributed write for higher bandwidth (each server 1 chunk)
- Read
  - from any of the replicated chunks
  - higher replication for hotspots (many reads of a chunk)
- Elasticity
  - Add additional chunkservers name node migrates chunks to new machine
  - Node failure (name node keeps track of chunk server status) requests replication

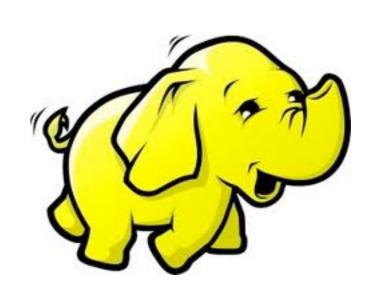
# Comparison

#### HDFS/GoogleFS

- Fast block writes / reads
- Fault tolerant
- Flexible size adjustment
- Terrible for random writes / bad for random writes
- Not really a filesystem
- NFS & co.
  - No distributed file management

#### Lustre

- Proper filesystem
- High bandwidth
- Explicitly exploits fast interconnects
- Cabinet servers replicated with RAID5
- Fails if cabinet dies
- Difficult to add more storage





# MapReduce

#### Scenario

- Lots of data (much more than what a single computer can process)
- Stored in a distributed fashion
- Move computation to data
- Map

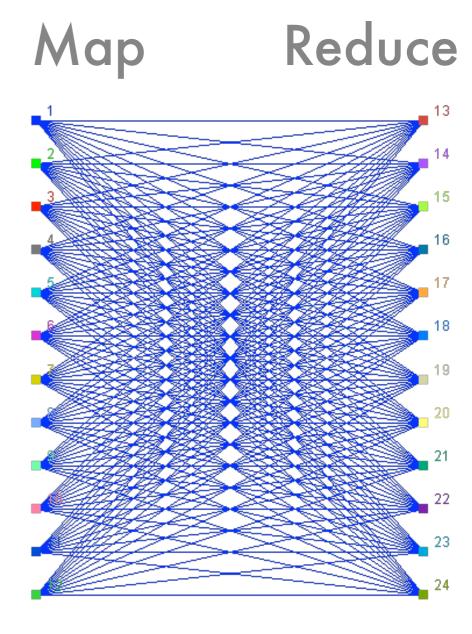
Apply function f to data as distributed over the mappers

This runs (if possible) where data sits

Reduce

Combine data given keys generated in the Map phase

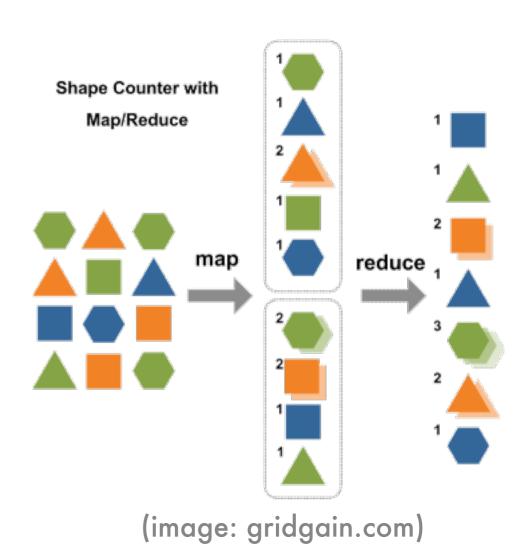
- Fault tolerance
  - If mapper dies, re-process the data
  - If reducer dies, re-send the data (cached) from mappers (requires considerable storage)



Dean, Ghemawat, 2004

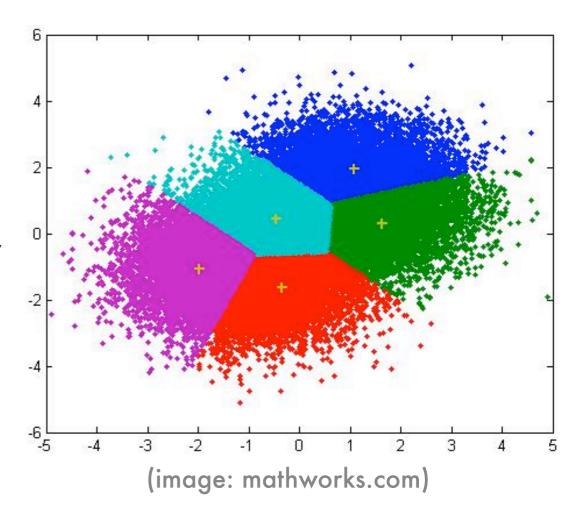
# Item count in MapReduce

- Task: object counting
- Map
  - Each mapper gets (unsorted) chunk of data
  - Preferably local on chunkservers
  - Perform local item counts
  - Emit (item, count) data
- Reduce
  - Aggregate all counts for a given item (all end up at same reducer)
  - Emit aggregate counts



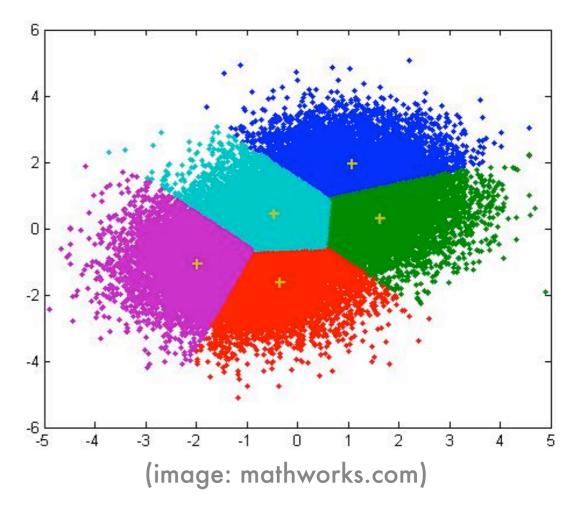
# k-means in MapReduce

- Initialize random cluster centers
- Map
  - Assign each data point to a cluster based on current model
  - Aggregate data per cluster
  - Send cluster aggregates to reducer (e.g. mean, variance, size)
- Reduce
  - Aggregate all data per cluster
  - Update cluster centers
  - Send new cluster centers to new reducers
- Repeat until converged



# k-means in MapReduce

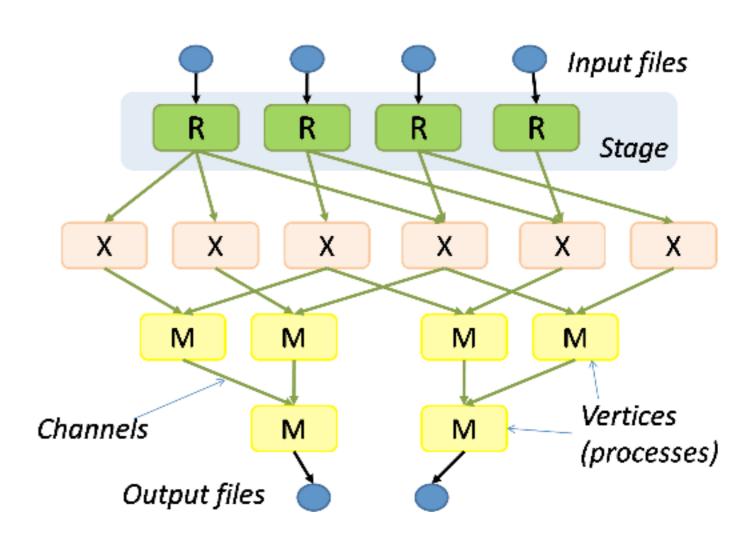
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needs to re-read data from disk for each MapReduce iteration

# Dryad

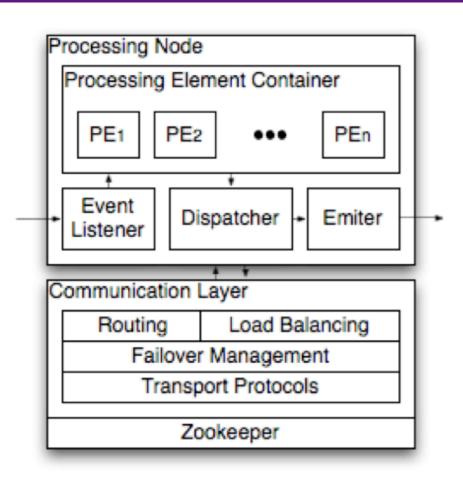
- Data flow graph (DAG rather than bipartite graph)
- Interface variety
  - Memory FIFO
  - Disk
  - Network
- Modular composition of computation graph



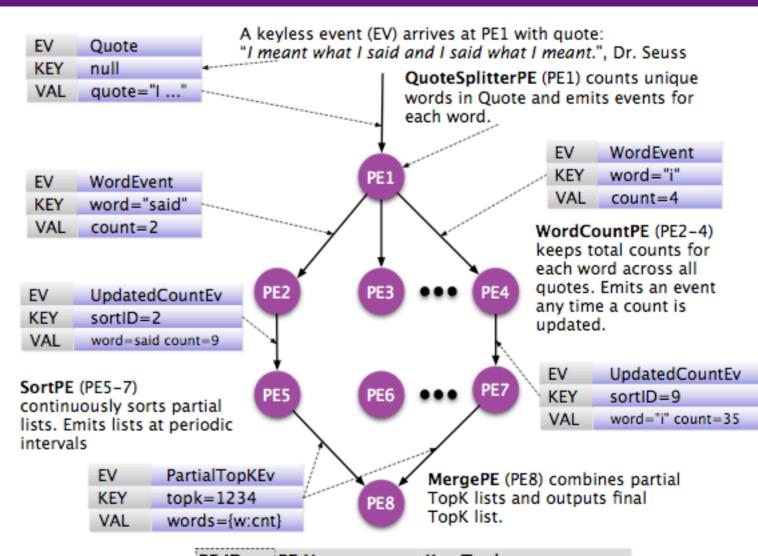
(image: Microsoft Research)

Isard, Budiu, Yu, Birrell & Fetterly, 2007

# S4 - Online processing



dispatch with distributed hash

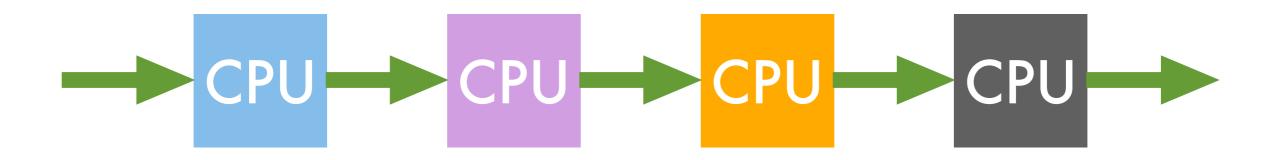


PE ID	PE Name	Key Tuple
PE1	QuoteSplitterPE	null
PE2	WordCountPE	word="said"
PE4	WordCountPE	word="i"
PE5	SortPE	sortID=2
PE7	SortPE	sortID=9
PE8	MergePE	topK=1234

Neumeyer, Robbins, Nair, Kesari, 2011

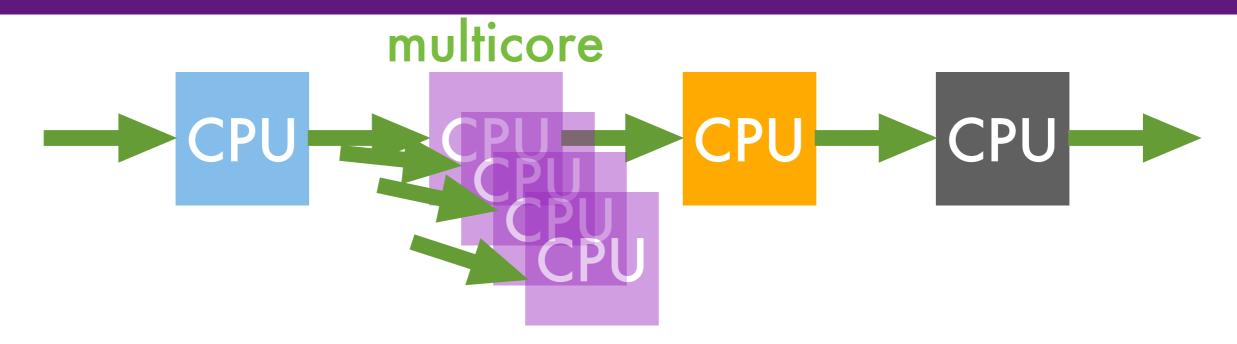
# Dataflow Paradigms

# Pipeline



- Process data sequentially
- Parallelizes up to number of tasks (disk read, feature extraction, logging, output)
- Reasonable for a single machine
- Parallelization per filter possible (see e.g. Intel Threading Building Blocks)

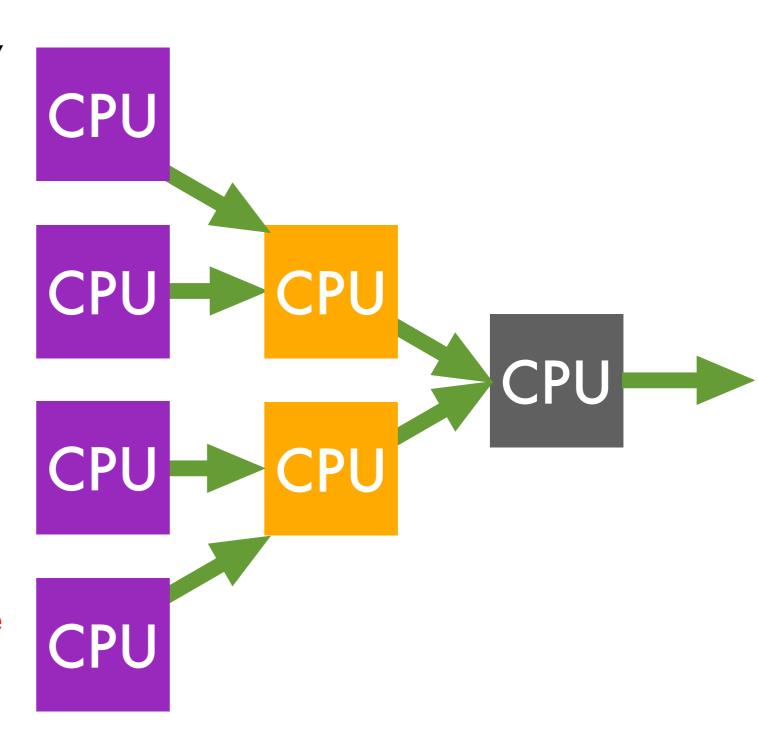
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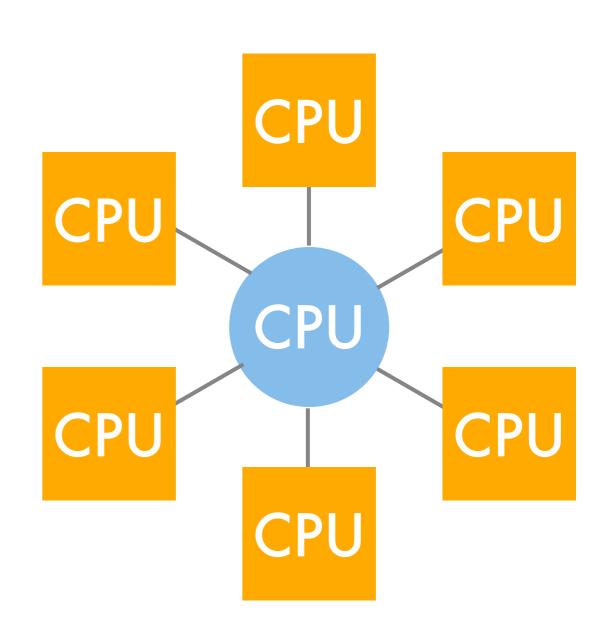
### Tree

- Aggregate data hierarchically
- Parallelizes at O(log n) cost (tree traversal)
- Communication at the interface is important (e.g. network latency)
- Good dataflow processing
- Poor efficiency for batched processing O(1/log n)
- Poor fault tolerance
- Does not map well onto server centers (need to ensure that we have lower leaves on rack)



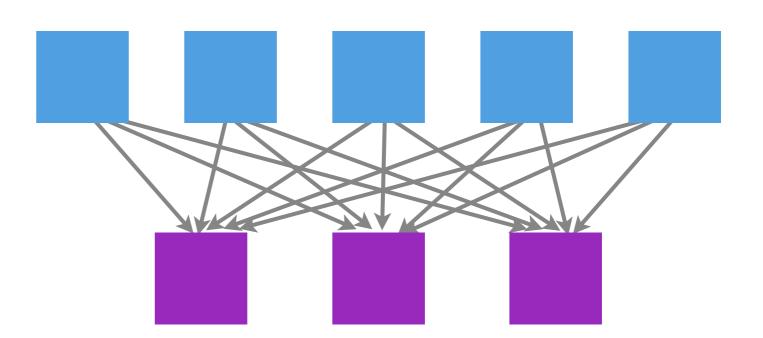
### Star

- Aggregate data centrally
- Does not parallelize at all if communication cost is high
- Perfect parallelization if CPU bound
- Latency is O(1) unless the network is congested.
- Network requirements are O(n)
- Central node becomes hotspot
- Synchronization is very easy
- Trivial to add more resources (just add leaves)
- Difficult to parallelize center



### Distributed (key, value) storage

- Caching problem
  - Store many (key,value) pairs
  - Linear scalability in clients and servers
  - Automatic key distribution mechanism
- memcached
  - (key, value) servers
  - client access library distributes access patterns
  - randomized O(n) bandwidth
  - aggregate O(n) bandwidth
  - load balancing via hashing
  - no versioned writes

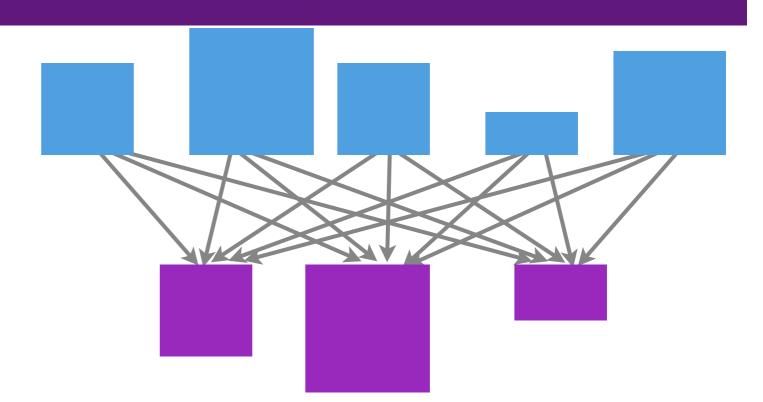


$$m(\text{key}, M) = \underset{m \in M}{\operatorname{argmin}} h(\text{key}, m)$$

P2P uses similar routing tables

### Proportional hashing/caching

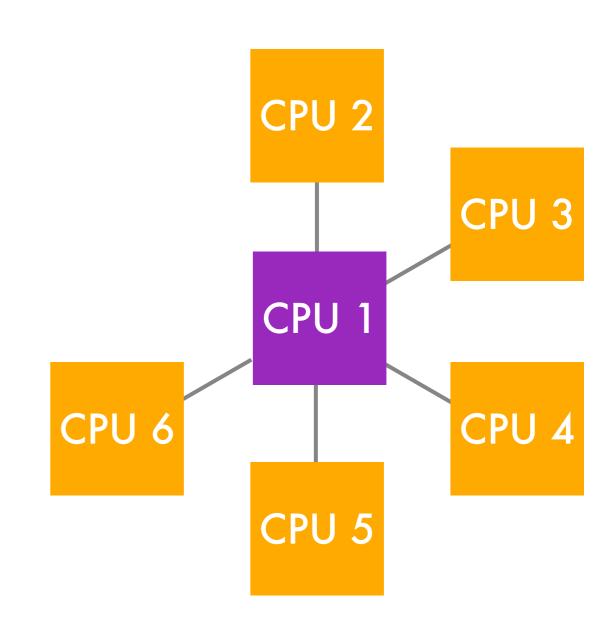
- Machines with different capacity
- Hotspots too hot for a single machine to handle
- Retain nearest neighbor hashing properties



- Sparse cover of keyspace proportional to machine capacities
- Repeatedly hash a key until it hits a key-range, i.e. h<sup>n</sup>(key)
- Keep busy table and advance to next hash if machine already used

### Distributed Star

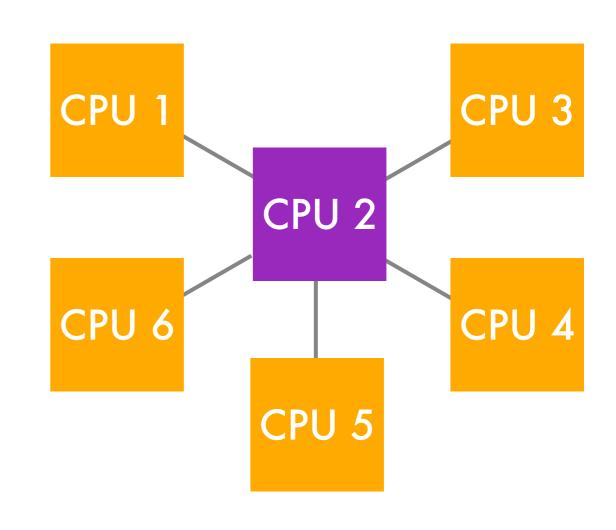
- Aggregate data centrally
- Use different center for each key, as selected by distributed hashing
- Linear bandwidth for synchronization
- Perfect scalability O(n) bandwidth required
- Each CPU performs local computation and stores small fraction of global data
- Works best if all nodes on the same switch / rack



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### Distributed Star

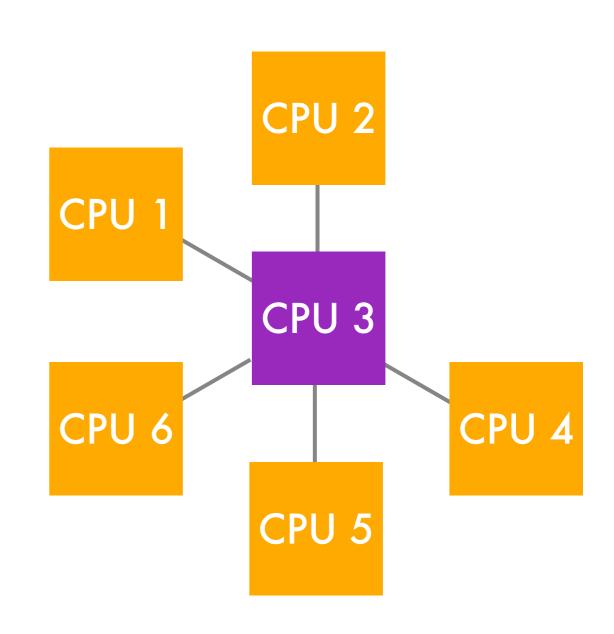
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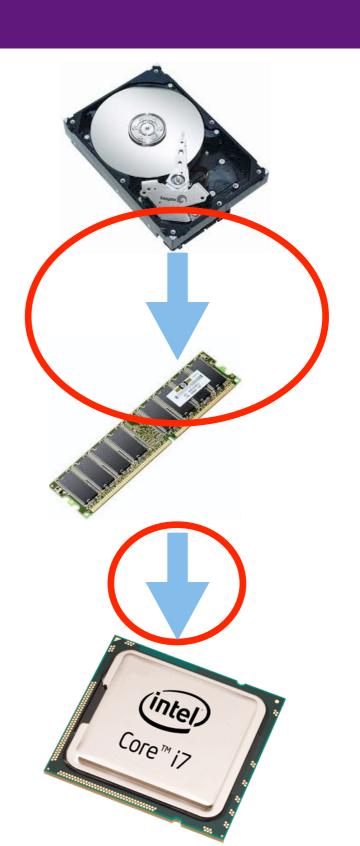
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# Ringbuffer

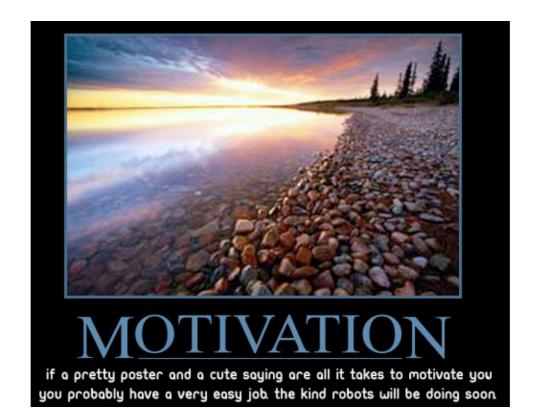
- Problem
  - Disk, RAM and CPU operate at different speeds (>10x difference)
  - Want to do maximum data processing (e.g. optimization)
- Idea
  - Load data from disk into ringbuffer
  - Process data continuously on buffer
  - Chain ringbuffers
- Yields consistently maximum throughput for each resource



### Summary

- Hardware
   Servers, networks, amounts of data
- Processing paradigms
   MapReduce, Dryad, S4
- Communication templates
   Stars, pipelines, distributed hash table, caching

### Part 2 - Motivation



### Data on the Internet

- Webpages (content, graph)
- Clicks (ad, page, social)
- Users (OpenID, FB Connect)
- e-mails (Hotmail, Y!Mail, Gmail)
- Photos, Movies (Flickr, YouTube, Vimeo ...)
- Cookies / tracking info (see Ghostery)
- Installed apps (Android market etc.)
- Location (Latitude, Loopt, Foursquared)
- User generated content (Wikipedia & co)
- Ads (display, text, DoubleClick, Yahoo)
- Comments (Disqus, Facebook)
- Reviews (Yelp, Y!Local)
- Third party features (e.g. Experian)
- Social connections (LinkedIn, Facebook)
- Purchase decisions (Netflix, Amazon)
- Instant Messages (YIM, Skype, Gtalk)
- Search terms (Google, Bing)
- Timestamp (everything)
- News articles (BBC, NYTimes, Y!News)
- Blog posts (Tumblr, Wordpress)
- Microblogs (Twitter, Jaiku, Meme)

#### Finite resources

- Editors are expensive
- Editors don't know users
- Barrier to i18n
- Abuse (intrusions are novel)
- Implicit feedback
- Data analysis (find interesting stuff rather than find x)
- Integrating many systems
- Modular design for data integration
- Integrate with given prediction tasks

Invest in modeling and naming rather than data generation

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now users unlimited amounts of data

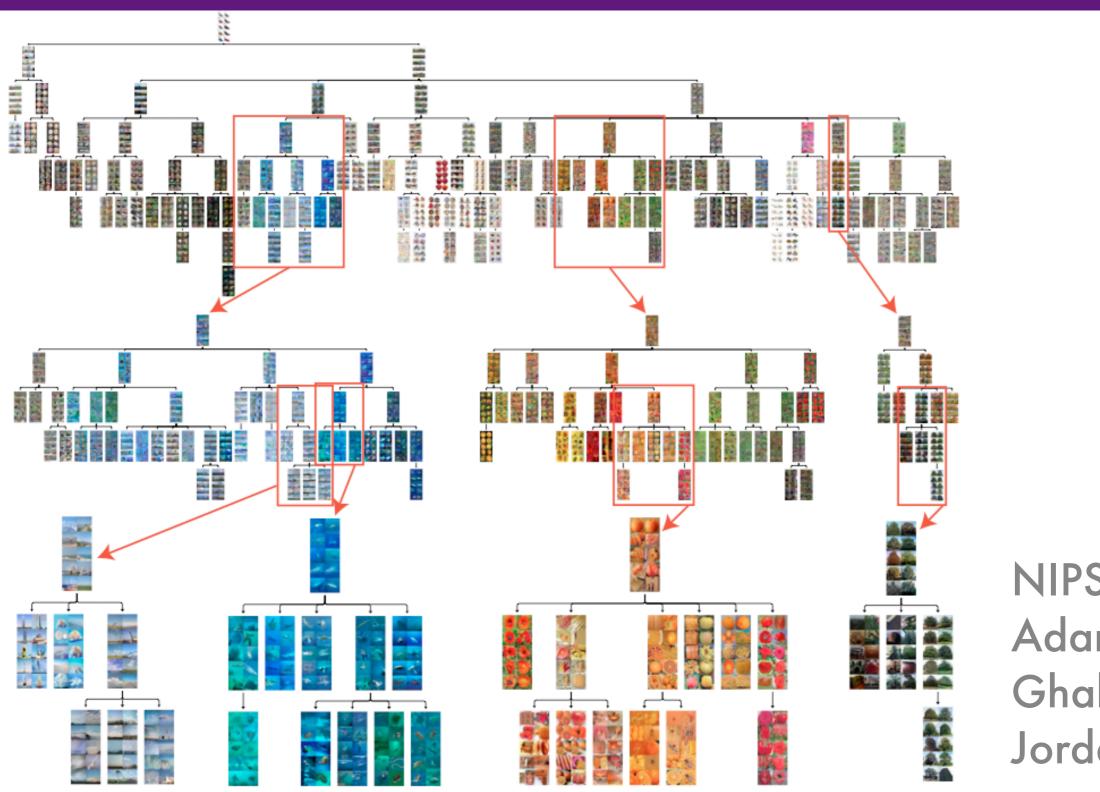
ns are novel)

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# Unsupervised Modeling

### Hierarchical Clustering



NIPS 2010 Adams, Ghahramani, Jordan

### Topics in text

The William Randolph Hearst Foundation will give \$1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be \$200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive \$400,000 each. The Juilliard School, where music and the performing arts are taught, will get \$250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual \$100,000 donation, too.

Latent Dirichlet Allocation; Blei, Ng, Jordan, JMLR 2003

### Word segmentation

first, shedreamed of little aliceherself, and once again the tiny handswere clasped upon her knee, and the brighte agereyes were looking up into her she could hear the very tone so fher voice, and see that queer little toss of her head to keep back the wandering hair that would always get into her eyes and still as she listened, or seemed to listen, the whole place around her became alive the strange creatures of her little sister's dream. The long rass rustled at her feet as the whiter abbit hurried by the fright ened mouses plashed. His way through the neighbouring pools he could hear the rattle of the teacups as the march hare and his friends shared their never ending meal, and the shrill voice of the queen...



first, she dream ed of little alice herself ,and once again the tiny hand s were clasped upon her knee ,and the bright eager eyes were looking up into hers -- shecould hearthe very tone s of her voice , and see that queer little toss of herhead to keep back the wandering hair that would always get into hereyes -- and still as she listened , or seemed to listen , thewhole place a round her became alive the strange creatures of her little sister 'sdream. thelong grass rustled ather feet as thewhitera bbit hurried by -- the frightened mouse splashed his way through the neighbour ing pool -- shecould hearthe rattle ofthe tea cups as the marchhare and his friends shared their never -endingme a I ,and the ...

Mochihashi, Yamada, Ueda, ACL 2009

## Language model

nevertheless,
he was admired
by many of his immediate subordinates
for his long work hours
and dedication to building northwest
into what he called a " mega carrier
"

although
preliminary findings
were reported
more than a year ago ,
the latest results
appear
in today 's
new england journal of medicine ,
a forum
likely to bring new attention to the problem

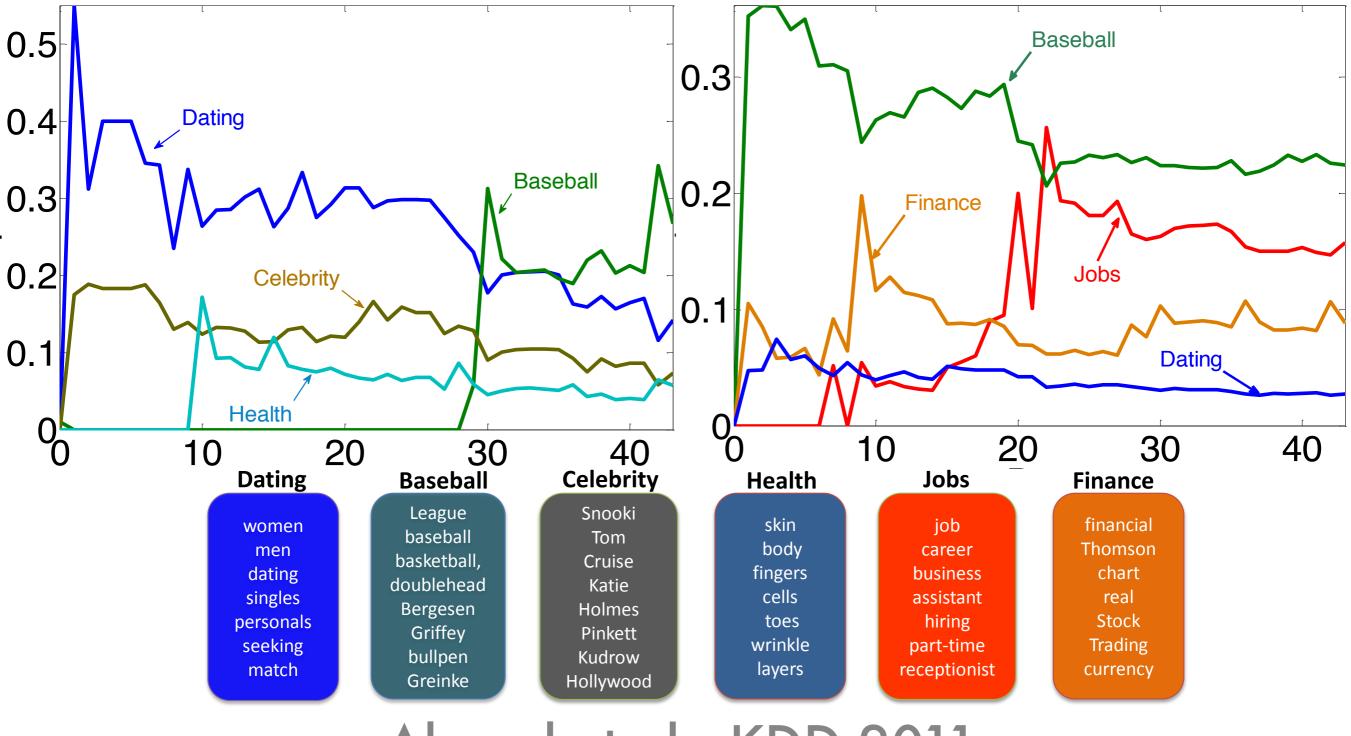
registered a trade deficit of \$ 101 million in october

- , reflecting the country 's economic sluggishness
- , according to government figures released wednesday

automatically synthesized from Penn Treebank

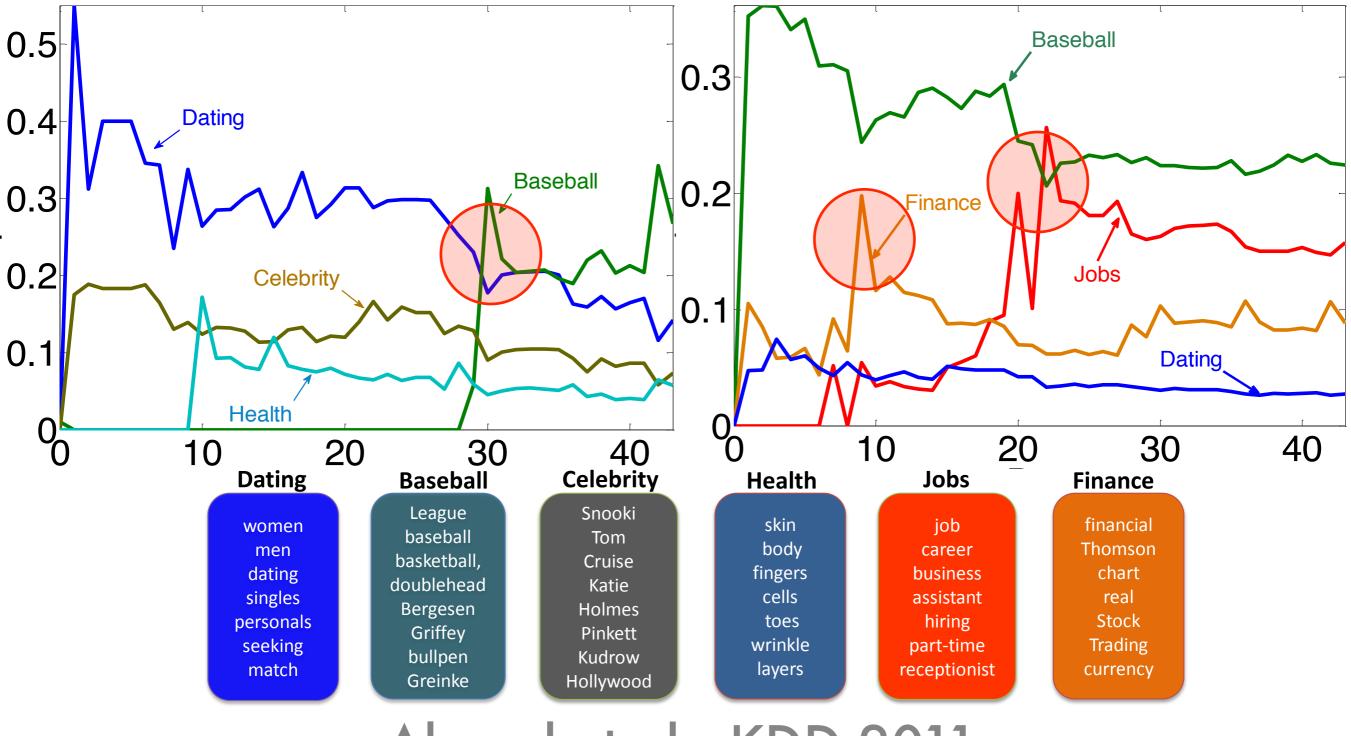
Mochihashi, Yamada, Ueda ACL 2009

### User model over time



Ahmed et al., KDD 2011

### User model over time

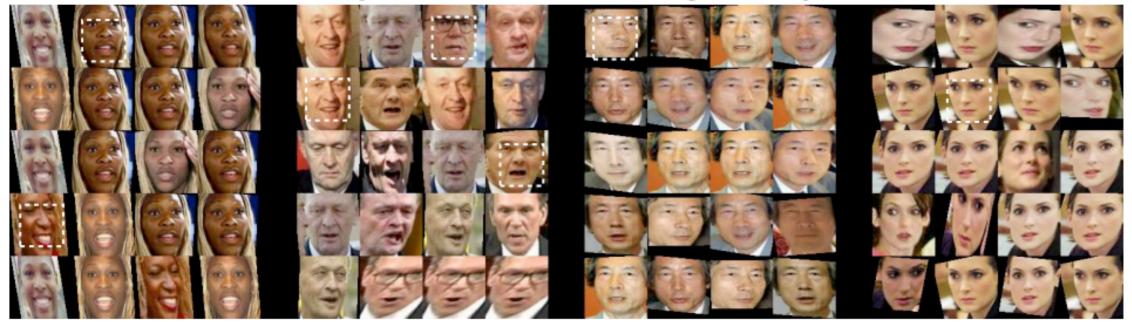


Ahmed et al., KDD 2011

### Face recognition from captions



(a) Random samples from four clusters obtained using LDA on caption text [6].



(b) The corresponding clusters obtained by People-LDA.

Jain, Learned-Miller, McCallum, ICCV 2007

### Storylines from news

TOPICS

# STORYLINES

### Sports

games won team final season league held

### **Politics**

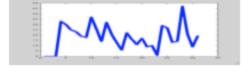
government minister authorities opposition officials leaders group

### Unrest

police attack run man group arrested move Ahmed et al, AISTATS 2011

### **UEFA-soccer**

champions Juventus
goal AC Milan
leg Real Madrid
coach Milan
striker Lazio
midfield Ronaldo
penalty Lyon



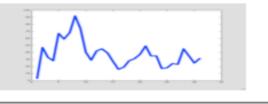
### Tax bills

tax Bush
billion Senate
cut US
plan Congress
budget Fleischer
economy White House
lawmakers Republican



### **India-Pakistan tension**

nuclear Pakistan
border India
dialogue Kashmir
diplomatic New Delhi
militant Islamabad
insurgency Musharraf
wissile Vajpayee



# Ideology detection

### US role

Israeli View

arafat state leader roadmap election month iraq yasir senior involvement clinton terrorism

bush US president american sharon administration prime pressure policy washington

Roadmap process

powell minister colin visit internal policy statement express pro previous package work transfer european

**Palestinian** View

palestinian

palestinian israeli

peace year political process state end right government need conflict way security

process force terrorism unit interim discussion union

provide confidence element succee point build positive recognize present timetable

iihad time

roadmap phase security ceasefire state plan international step authority

end settlement implementation obligation stop expansion commitment fulfill unit illegal present previous

assassination meet forward

israeli Peace political occupation process end security conflict way governmen t people time year

force

negotiation

### Arab Involvement

peace strategic plo hizballah islamic neighbor territorial radical iran relation think obviou countri mandate greater conventional intifada affect

syria syrian negotiate lebanon deal conference concession asad agreement regional october initiative relationship

track negotiation official leadership position withdrawal time victory present second stand circumstance represent sense talk strategy issue participant parti negotiator

Ahmed et al, 2010; Bitterlemons collection

### Hypertext topic extraction

### Topic 1

0.067 Artificial neural network neural 0.047 network 0.004 0.039 networks 0.027 learning Neural network 0.003 artificial 0.017 0.015 data 0.014 models function 0.014

### Topic 2

Speech recognition recognition 0.058 speech 0.033 0.004 0.015 language 0.012 pattern Pattern recognition handwriting 0.011 0.004 evaluation 0.010 robots 0.010 0.009 systems

### Topic 3

0.051 vancouver 0.043 denver 0.041 city retrieved 0.024 0.011 colorado 0.009 area 0.009 population 0.008 canada

### Denver, Colorado



### Topic 4

0.047 brain Cognitive science 0.003 0.026 cognitive 0.016 science 0.011 press Neuroscience 0.002 0.010 neurons 0.010 mind Neuroscience systems 0.010 0.010 human

Gruber, Rosen-Zvi, Weiss; UAI 2008

## Supervised Modeling

### Ontologies

dmoz open directory project

In partnership with Aol Search.

about dmoz | dmoz blog | suggest URL | help | link | editor login

(Search) advanced

Arts <u>Business</u> <u>Computers</u>

Movies, Television, Music... Jobs, Real Estate, Investing... Internet, Software, Hardware...

Games Health Home

Video Games, RPGs, Gambling... Fitness, Medicine, Alternative... Family, Consumers, Cooking...

Kids and Teens News Recreation

Arts, School Time, Teen Life... Media, Newspapers, Weather... Travel, Food, Outdoors, Humor...

Reference Regional Science

Maps, Education, Libraries... US, Canada, UK, Europe... Biology, Psychology, Physics...

Shopping Society Sports

Clothing, Food, Gifts... People, Religion, Issues... Baseball, Soccer, Basketball...

<u>World</u>

Català, Dansk, Deutsch, Español, Français, Italiano, 日本語, Nederlands, Polski, Русский, Svenska...

Become an Editor Help build the largest human-edited directory of the web



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- continuous maintenance
- no guarantee of coverage
- difficult categories

### Ontologies

dmoz open directory project

In partnership with Aol Search.

about dmoz | dmoz blog | suggest URL | help | link | editor login

Search advanced

Arts <u>Business</u> <u>Computers</u>

Movies, Television, Music... Jobs, Real Estate, Investing... Internet, Software, Hardware...

Games Health Home

Video Games, RPGs, Gambling... Fitness, Medicine, Alternative... Family, Consumers, Cooking...

Kids and Teens News Recreation

Arts, School Time, Teen Life... Media, Newspapers, Weather... Travel, Food, Outdoors, Humor...

Reference Regional Science

Maps, Education, Libraries... US, Canada, UK, Europe... Biology, Psychology, Physics...

Shopping Society Sports

Clothing, Food, Gifts... People, Religion, Issues... Baseball, Soccer, Basketball...

<u>World</u>

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continuous maintenance

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4,855,150 sites - 90,367 editors - over 1,005,887 categories

### Face Classification/Recognition



### Sampels



Frontal, Pose Age: 4



Frontal, Pos Age: 11



Frontal, Pose, Expression Age: 16



Frontal, Pose, Express Age: 21



Frontal, Pose, Ex



Age: 46



Frontal, Pose, Expressi Age: 65



Frontal, Pose, Expression
Age: 82

- 100-1000people
- 10k faces
- curated (not realistic)
- expensive to generate

## Topic Detection & Tracking

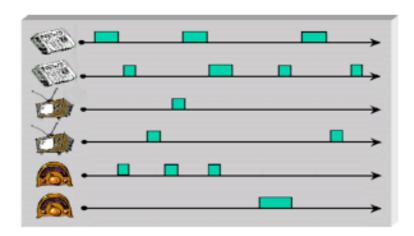
Information Technology Laboratory

Information Access Division (IAD)



- <u>Multimodal Information</u>
   Group Home
- Benchmark Tests
- Tools
- Test Beds
- Publications
- Links
- Contacts

### **Topic Detection and Tracking Evaluation**



Topic Detection and Tracking research was pursued under the DARPA Translingual Information Detection, Extraction, and Summarization (TIDES) program:

Topic Detection and Tracking is an integral part of the DARPA Translingual Information Detection, Extraction, and Summarization (TIDES) program. The goal of the TIDES program is to enable English-speaking users to access, correlate, and interpret multilingual sources of real-time information and to share the essence of this information with collaborators.

As a TIDES evaluation community, TDT provides a forum to discuss applications and techniques for detecting and tracking events that occur in real-time and the infrastructure to support common evaluations of component technologies. The TIDES project currently has one other evaluation community, The Text REtrieval Conference (TREC), and planning has begun for three new evaluations in the areas of Text Summarization, Question Answering and Quick Machine Translation.

- editorially curated training data
- expensive to generate
- subjective in selection of threads
- language specific

# Advertising Targeting

**Browse Ad Solutions** 

**Media Spotlight** 

AUDIENCE

**Affluents** 

**Boomer Men** 

**Boomer Women** 

Men 18-34

Men 18-49

Millennials

Online Dads

Online Moms

Women 18-34

Women 18-49

Your categories

Below you can edit the interests and inferred demographics that Google has associated with your cookie:

Category	
Arts & Entertainment - TV & Video - Online Video	Remove
Computers & Electronics	Remove
Computers & Electronics - Hardware Chips & Processors	Remove
Computers & Electronics - Software - Operating Systems - Mac OS	Remove
Games - Computer & Video Games - Shooter Games	Remove
Games - Online Games - Massive Multiplayer	Remove
News - Politics	Remove
News - Sports News	Remove
Shopping - Coupons & Discount Offers	Remove
Sports - Team Sports - American Football	Remove
Add categories Coogle does not associate consitive interest estegories with your add	proformano

Add categories

Google does not associate sensitive interest categories with your ads preferences.

- Needs training data in every language
- Is it really relevant for better ads?
- Does it cover relevant areas?

# Advertising Targeting



Below you can edit the interests and inferred demographics that Google has associated with your cookie:

	gpg	,
Category		
Arts & Entertainment - TV & Video - Online Vi	ideo	Remove
Computers & Electronics		Remove
Computers & Electronics - Hardware Chi	ips & Processors	Remove
Computers & Electronics - Software - Operating	ng Systems - Mac OS	Remove
Games - Computer & Video Games - Shooter	Games	Remove
Games - Online Games - Massive Multiplayer		
News - Politics		
News - Sports News		
Shopping - Coupons & Discount Offers		7
Sports - Team Sports - American Football		
(Add annuarian) Congle does not apposint		

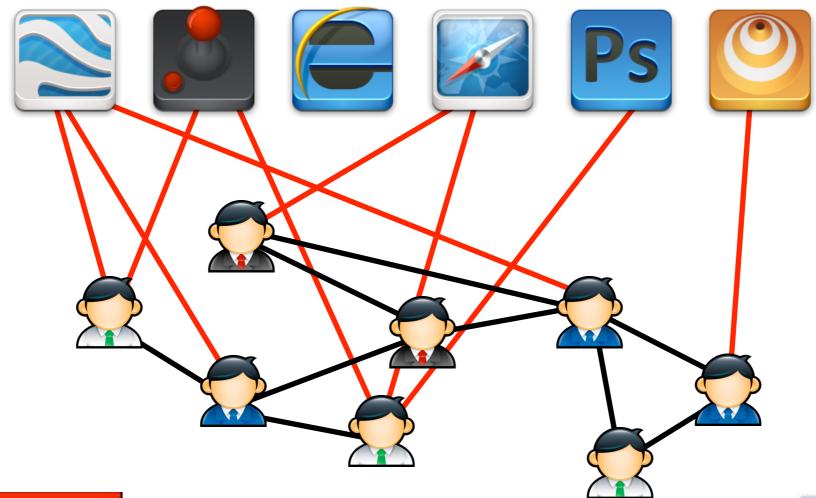
Needs training data in every language

Add categories

Google does not associate

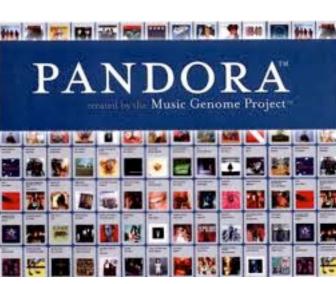
- Is it really relevant for better ads?
- Does it cover relevant areas?

### Collaborative Filtering









# Challenges

- Scale
  - Millions to billions of instances (documents, clicks, users, messages, ads)
  - Rich structure of data (ontology, categories, tags)
  - Model description typically larger than memory of single workstation
- Modeling
  - Usually clustering or topic models do not solve the problem
  - Temporal structure of data
  - Side information for variables
  - Solve problem. Don't simply apply a model!
- Inference
  - 10k-100k clusters for hierarchical model
  - 1M-100M words
  - Communication is an issue for large state space

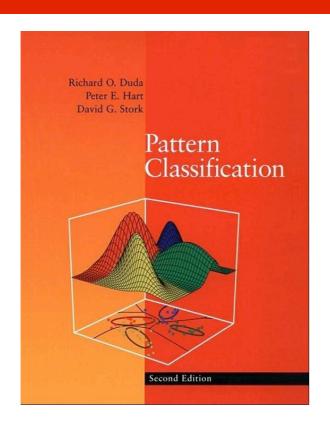
### Summary

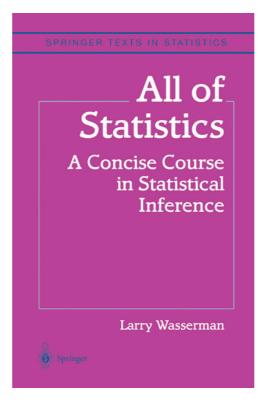
- Essentially infinite amount of data
- Labeling is (in many cases) prohibitively expensive
- Editorial data not scalable for i18n
- Even for supervised problems unlabeled data abounds. Use it.
- User-understandable structure for representation purposes
- Solutions are often customized to problem
   We can only cover building blocks in tutorial.

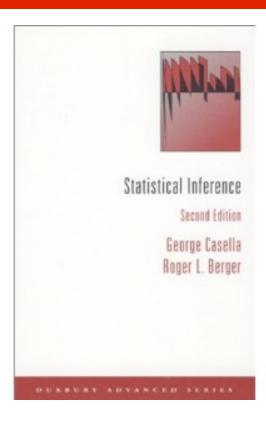
### Part 3 - Basic Tools



### Statistics 101







## Probability

- Space of events X
  - server working; slow response; server broken
  - income of the user (e.g. \$95,000)
  - query text for search (e.g. "statistics tutorial")
- Probability axioms (Kolmogorov)

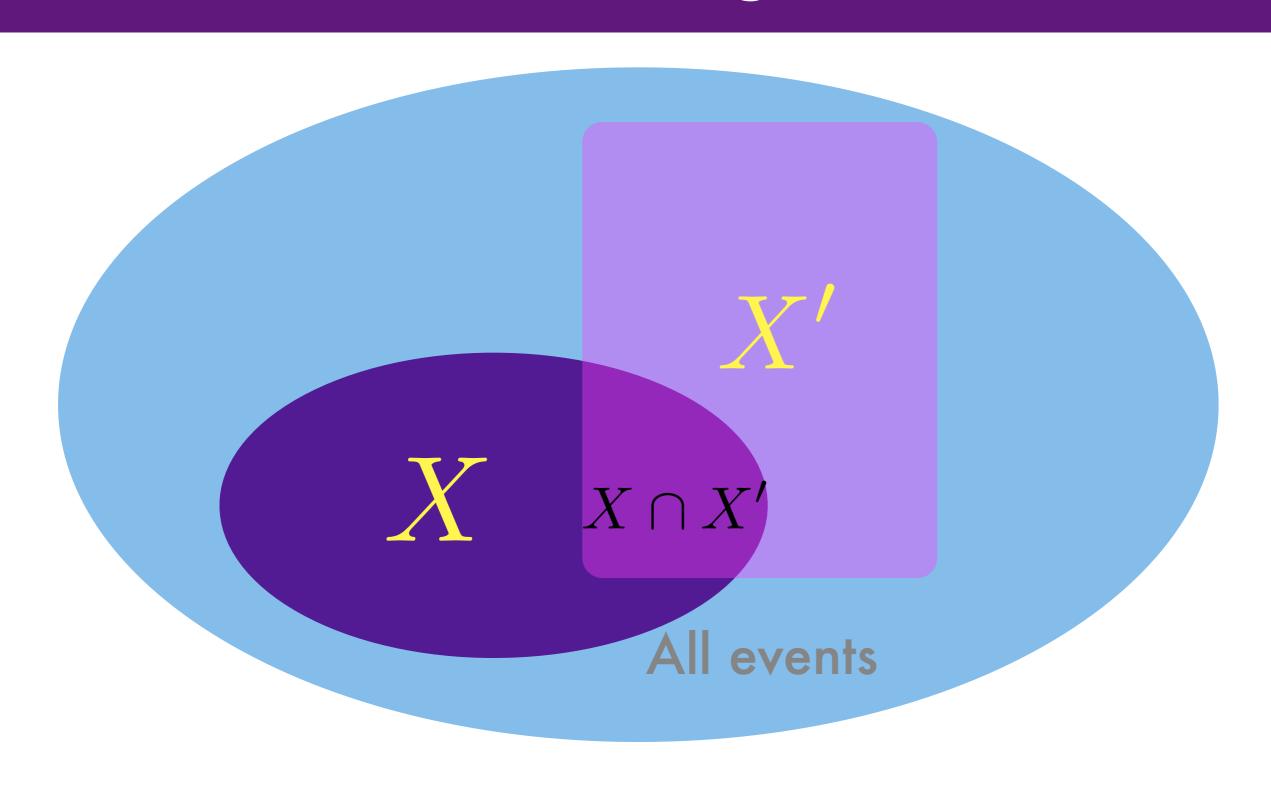
$$\Pr(X) \in [0, 1], \Pr(\mathcal{X}) = 1$$
  
 $\Pr(\bigcup_i X_i) = \sum_i \Pr(X_i) \text{ if } X_i \cap X_j = \emptyset$ 

- Example queries
  - P(server working) = 0.999
  - P(90,000 < income < 100,000) = 0.1

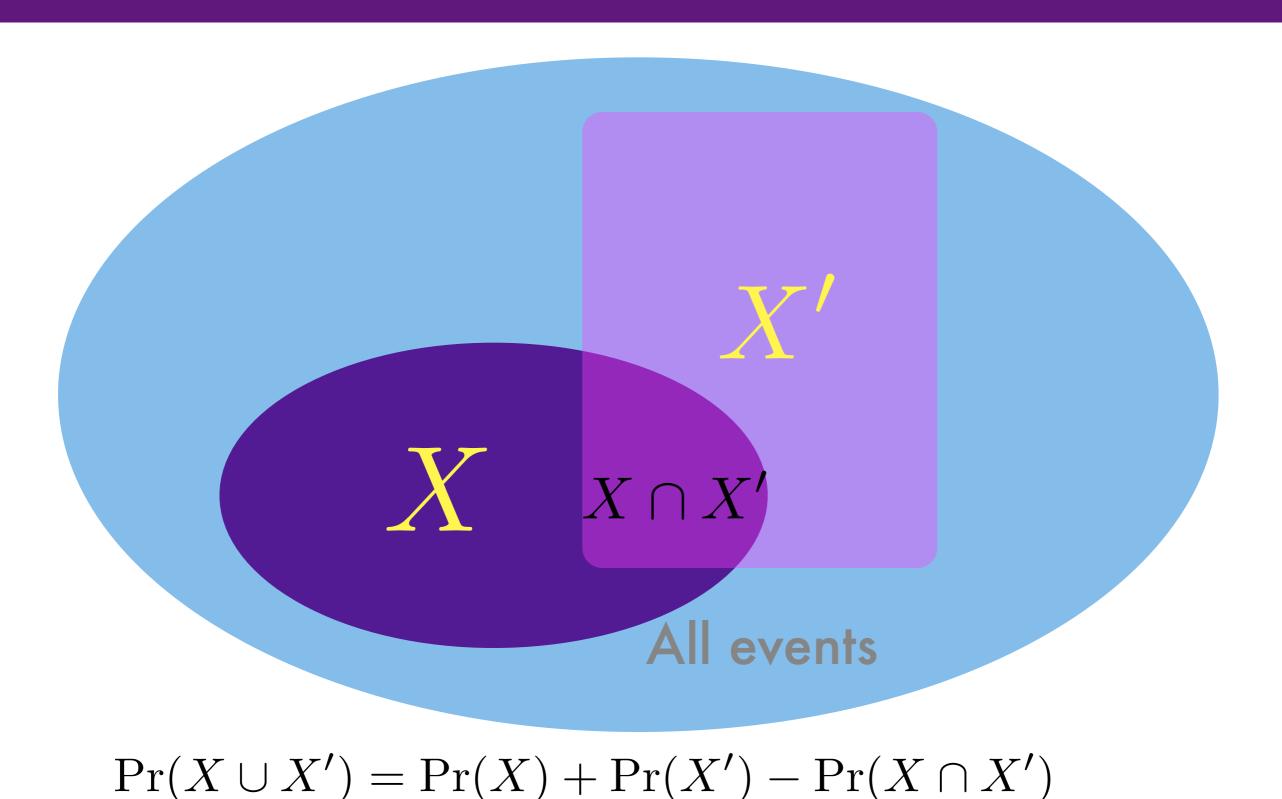
### Venn Diagram

All events

### Venn Diagram



### Venn Diagram



### (In)dependence

- Independence  $Pr(x, y) = Pr(x) \cdot Pr(y)$ 
  - Login behavior of two users (approximately)
  - Disk crash in different colos (approximately)

### (In)dependence

- Independence  $Pr(x, y) = Pr(x) \cdot Pr(y)$ 
  - Login behavior of two users (approximately)
  - Disk crash in different colos (approximately)
- Dependent events
  - Emails  $\Pr(x,y) \neq \Pr(x) \cdot \Pr(y)$
  - Queries
  - News stream / Buzz / Tweets
  - IM communication
  - Russian Roulette

### (In)dependence

- Independence  $Pr(x, y) = Pr(x) \cdot Pr(y)$ 
  - Login behavior of two users (approximately)
  - Disk crash in different colos (approximately)
- Dependent events
  - Emails

$$\Pr(x,y) \neq \Pr(x) \land \Pr(y)$$

- Queries
- News stream / Buzz / Tweets
- IM communication
- Russian Roulette

Everywhere!

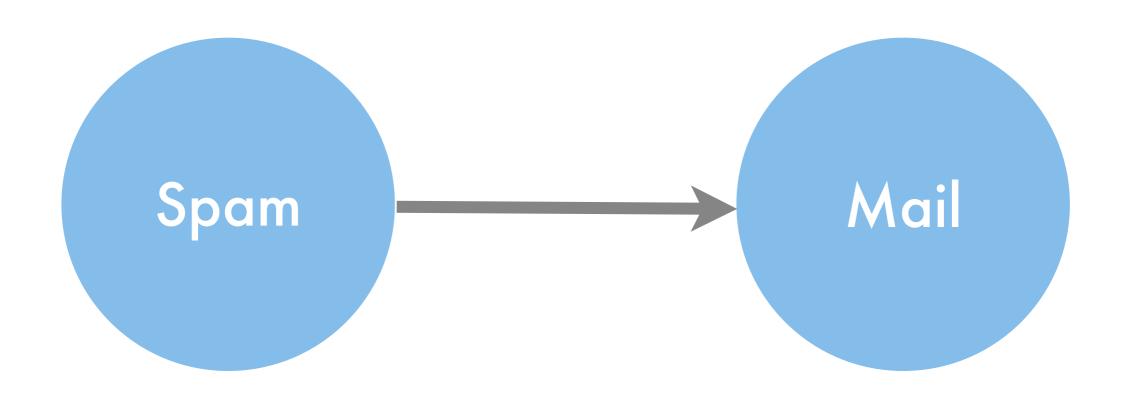
## Independence

0.25	0.25
0.25	0.25

# Dependence

0.45	0.05
0.05	0.45

### A Graphical Model



p(spam, mail) = p(spam) p(mail|spam)

### Bayes Rule

Joint Probability

$$Pr(X,Y) = Pr(X|Y) Pr(Y) = Pr(Y|X) Pr(X)$$

Bayes Rule

$$Pr(X|Y) = \frac{Pr(Y|X) \cdot Pr(X)}{Pr(Y)}$$

- Hypothesis testing
- Reverse conditioning

## AIDS test (Bayes rule)

- Data
  - Approximately 0.1% are infected
  - Test detects all infections
  - Test reports positive for 1% healthy people
- Probability of having AIDS if test is positive

## AIDS test (Bayes rule)

- Data
  - Approximately 0.1% are infected
  - Test detects all infections
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- Probability of having AIDS if test is positive

$$\Pr(a = 1|t) = \frac{\Pr(t|a = 1) \cdot \Pr(a = 1)}{\Pr(t)}$$

$$= \frac{\Pr(t|a = 1) \cdot \Pr(a = 1)}{\Pr(t|a = 1) \cdot \Pr(a = 1) + \Pr(t|a = 0) \cdot \Pr(a = 0)}$$

$$= \frac{1 \cdot 0.001}{1 \cdot 0.001 + 0.01 \cdot 0.999} = 0.091$$

# Improving the diagnosis

# Improving the diagnosis

- Use a follow-up test
  - Test 2 reports positive for 90% infections
  - Test 2 reports positive for 5% healthy people

$$\frac{0.01 \cdot 0.05 \cdot 0.999}{1 \cdot 0.9 \cdot 0.001 + 0.01 \cdot 0.05 \cdot 0.999} = 0.357$$

# Improving the diagnosis

- Use a follow-up test
  - Test 2 reports positive for 90% infections
  - Test 2 reports positive for 5% healthy people

$$\frac{0.01 \cdot 0.05 \cdot 0.999}{1 \cdot 0.9 \cdot 0.001 + 0.01 \cdot 0.05 \cdot 0.999} = 0.357$$

Why can't we use Test 1 twice?
 Outcomes are not independent but tests 1 and 2 are conditionally independent

# Improving the diagnosis

- Use a follow-up test
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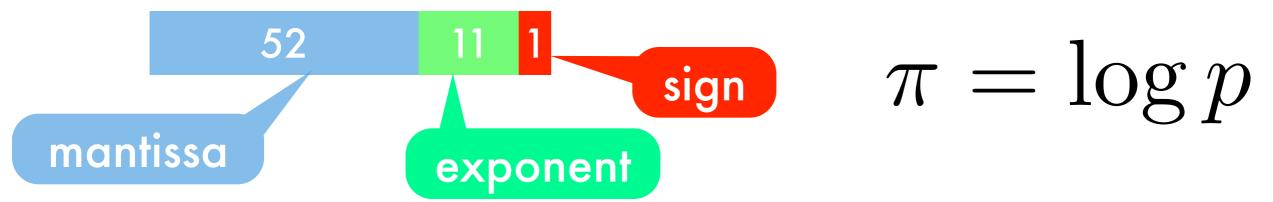
$$\frac{0.01 \cdot 0.05 \cdot 0.999}{1 \cdot 0.9 \cdot 0.001 + 0.01 \cdot 0.05 \cdot 0.999} = 0.357$$

Why can't we use Test 1 twice?
 Outcomes are not independent but tests 1 and 2 are conditionally independent

$$p(t_1, t_2|a) = p(t_1|a) \cdot p(t_2|a)$$

# Logarithms are good

Floating point numbers



- Probabilities can be very small. In particular products of many probabilities. Underflow!
- Store data in mantissa, not exponent

$$\prod_{i} p_{i} \to \sum_{i} \pi_{i} \qquad \sum_{i} p_{i} \to \max \pi + \log \sum_{i} \exp \left[\pi_{i} - \max \pi\right]$$

Known bug e.g. in Mahout Dirichlet clustering

# Application: Naive Bayes



Key assumption
 Words occur independently of each other
 given the label of the document

$$p(w_1, \dots, w_n | \text{spam}) = \prod_{i=1}^n p(w_i | \text{spam})$$

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 Words occur independently of each other
 given the label of the document

$$p(w_1, \dots, w_n | \text{spam}) = \prod p(w_i | \text{spam})$$

ullet Spam classification via Bayes Rule

$$p(\operatorname{spam}|w_1,\ldots,w_n) \propto p(\operatorname{spam}) \prod_{i=1}^n p(w_i|\operatorname{spam})$$

Key assumption
 Words occur independently of each other
 given the label of the document

$$p(w_1, \dots, w_n | \text{spam}) = \prod p(w_i | \text{spam})$$

• Spam classification via Bayes n

$$p(\operatorname{spam}|w_1,\ldots,w_n) \propto p(\operatorname{spam}) \prod_{i=1}^n p(w_i|\operatorname{spam})$$

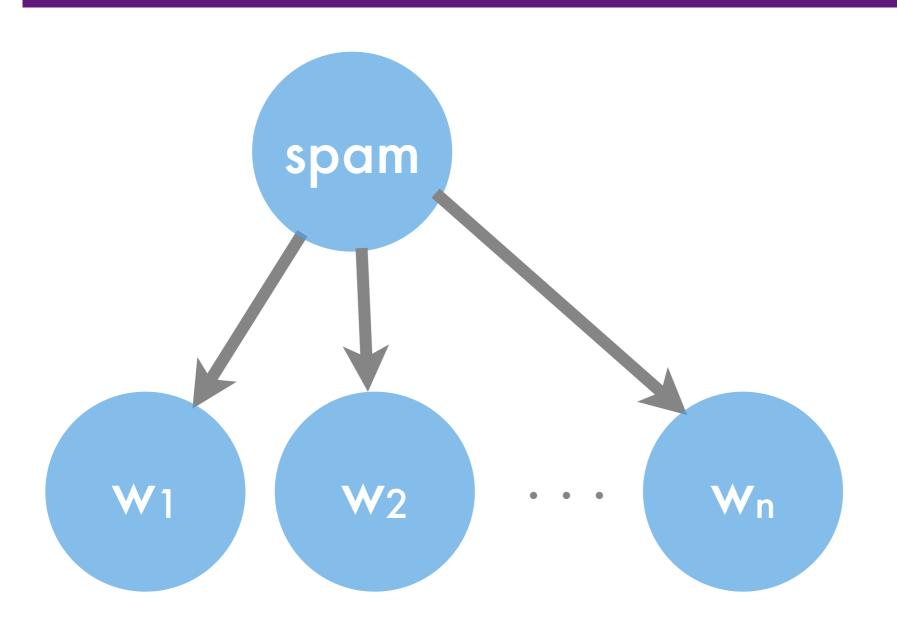
• Parameter estimation Compute spam probability and word distributions for spam and ham

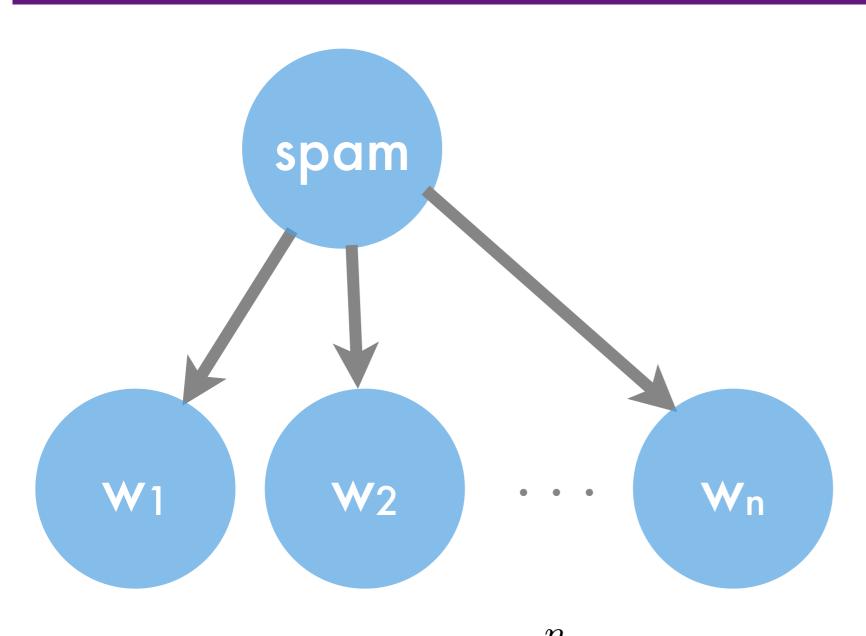
#### Equally likely phrases

- Get rich quick. Buy WWW stock.
- Buy Viagra. Make your WWW experience last longer.
- You deserve a PhD from WWW University.
   We recognize your expertise.

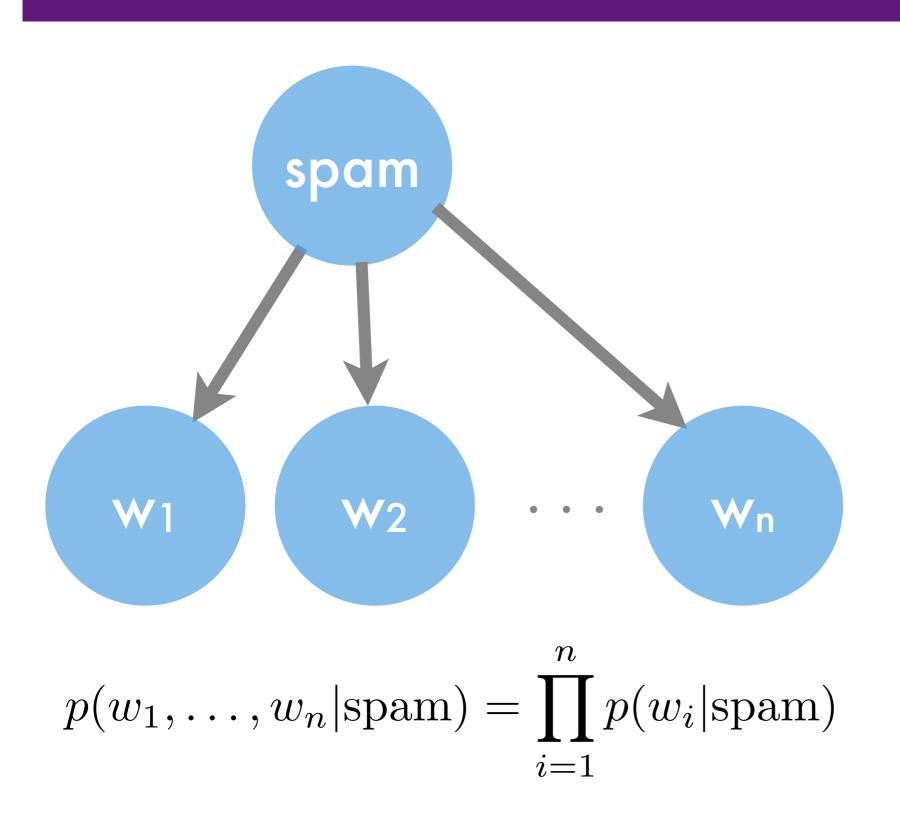
#### Equally likely phrases

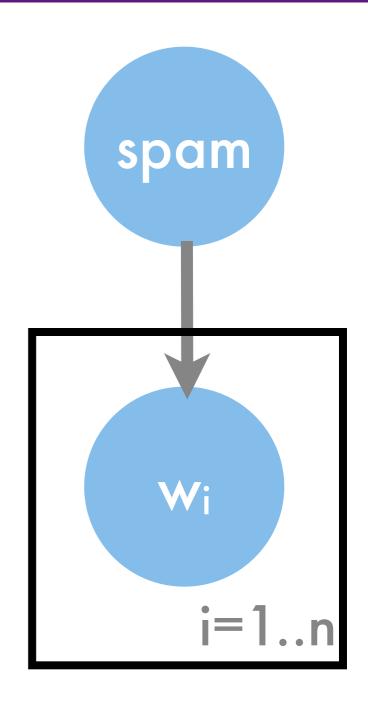
- Get rich quick. Buy WWW stock.
- Buy Viagra. Make your WWW experience last longer.
- You deserve a PhD from WWW University.
   We recognize your expertise.
- Make your rich WWW PhD experience last longer.

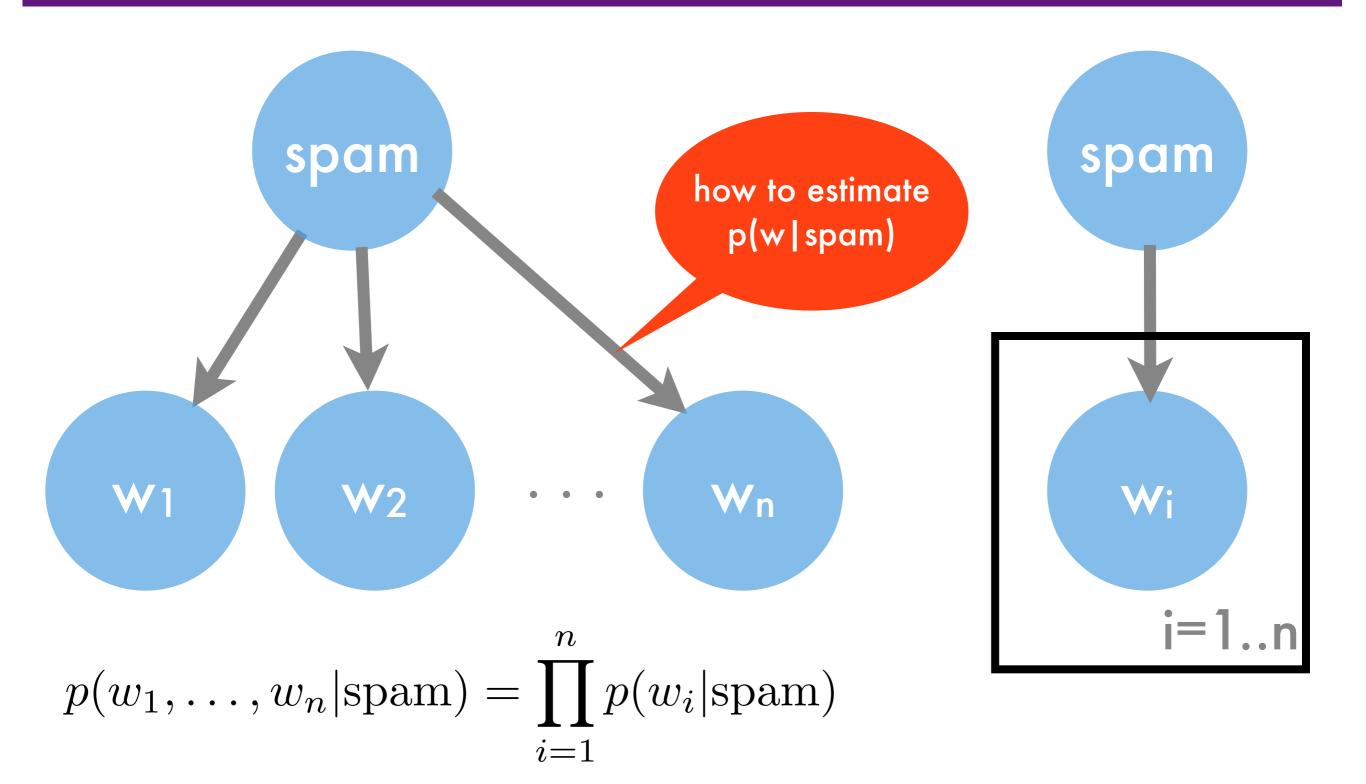




$$p(w_1, \dots, w_n | \text{spam}) = \prod_{i=1}^n p(w_i | \text{spam})$$







# Simple Algorithm

- For each document (x,y) do
  - Aggregate label counts given y
  - For each feature x<sub>i</sub> in x do
    - Aggregate statistic for (x<sub>i</sub>, y) for each y
- For y estimate distribution p(y)
- For each (x<sub>i</sub>,y) pair do
   Estimate distribution p(x<sub>i</sub>|y), e.g. Parzen Windows,
   Exponential family (Gauss, Laplace, Poisson, ...), Mixture
- Given new instance compute

$$p(y|x) \propto p(y) \prod_{i} p(x_{i}|y)$$

# Simple Algorithm

- For each document (x,y) do
  - Aggregate label counts given y pass over all data
  - For each feature x<sub>i</sub> in x do
    - Aggregate statistic for (x<sub>i</sub>, y) for each y
- For y estimate distribution p(y)
- For each (x<sub>i</sub>,y) pair do
   Estimate distribution p(x<sub>i</sub>|y), e.g. Parzen Windows,
   Exponential family (Gauss, Laplace, Poisson, ...), Mixture
- Given new instance compute

$$p(y|x) \propto p(y) \prod_{i} p(x_{i}|y)$$

## MapReduce Algorithm

- Map(document (x,y))
  - For each mapper for each feature  $x_i$  in x do
    - Aggregate statistic for (x<sub>i</sub>, y) for each y
  - Send aggregate statistics to reducer
- Reduce(x<sub>i</sub>, y)
  - Aggregate over all messages from mappers
  - Estimate distribution  $p(x_i|y)$ , e.g. Parzen Windows, Exponential family (Gauss, Laplace, Poisson, ...), Mixture
  - Send coordinate-wise model to global storage
- Given new instance compute

$$p(y|x) \propto p(y) \prod_{i} p(x_{i}|y)$$

## MapReduce Algorithm

- Map(document (x,y))
  - For each mapper for each feature x<sub>i</sub> in x do ocal per
    - Aggregate statistic for (xi, y) for each y chunkserver
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- only aggregates
- Aggregate over all messages from mappers needed
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   Exponential family (Gauss, Laplace, Poisson, ...), Mixture
- Send coordinate-wise model to global storage
- Given new instance compute

$$p(y|x) \propto p(y) \prod_{i} p(x_{i}|y)$$

## Naive NaiveBayes Classifier

- Two classes (spam/ham)
- Binary features (e.g. presence of \$\$\$, viagra)
- Simplistic Algorithm
  - Count occurrences of feature for spam/ham
  - Count number of spam/ham mails

     spam probability

feature probability

$$p(x_i = \text{TRUE}|y) = \frac{n(i,y)}{n(y)} \text{ and } p(y) = \frac{n(y)}{n}$$

$$p(y|x) \propto \frac{n(y)}{n} \prod_{i:x_i = \text{TRUE}} \frac{n(i,y)}{n(y)} \prod_{i:x_i = \text{FALSE}} \frac{n(y) - n(i,y)}{n(y)}$$

## Naive NaiveBayes Classifier

what if n(i,y)=n(y)?

what if n(i,y)=0?

$$p(y|x) \propto \frac{n(y)}{n} \prod_{i:x_i = \text{TRUE}} \frac{n(i,y)}{n(y)} \prod_{i:x_i = \text{FALSE}} \frac{n(y) - n(i,y)}{n(y)}$$

# Naive NaiveBayes Classifier



what if n(i,y)=0?

$$p(y|x) \propto \frac{n(y)}{n} \prod_{i:x_i = \text{TRUE}} \frac{n(i,y)}{n(y)} \prod_{i:x_i = \text{FALSE}} \frac{n(y) - n(i,y)}{n(y)}$$

### Estimating Probabilities



#### Binomial Distribution

- Two outcomes (head, tail); (0,1)
- Data likelihood

$$p(X;\pi) = \pi^{n_1} (1-\pi)^{n_0}$$





- Incorporate constraint via  $p(x;\theta) = \frac{e^{x\theta}}{1 + e^{\theta}}$
- Taking derivatives yields

$$\theta = \log \frac{n_1}{n_0 + n_1} \iff p(x = 1) = \frac{n_1}{n_0 + n_1}$$



#### ... in detail ...

$$p(X;\theta) = \prod_{i=1}^{n} p(x_i;\theta) = \prod_{i=1}^{n} \frac{e^{\theta x_i}}{1 + e^{\theta}}$$

$$\Longrightarrow \log p(X;\theta) = \theta \sum_{i=1}^{n} x_i - n \log \left[1 + e^{\theta}\right]$$

$$\Longrightarrow \partial_{\theta} \log p(X;\theta) = \sum_{i=1}^{n} x_i - n \frac{e^{\theta}}{1 + e^{\theta}}$$

$$\Longleftrightarrow \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{e^{\theta}}{1 + e^{\theta}} = p(x = 1)$$

#### ... in detail ...

$$p(X;\theta) = \prod_{i=1}^{n} p(x_i;\theta) = \prod_{i=1}^{n} \frac{e^{\theta x_i}}{1 + e^{\theta}}$$

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$$\Longleftrightarrow \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{e^{\theta}}{1 + e^{\theta}} = p(x = 1)$$

empirical probability of x=1

#### Discrete Distribution

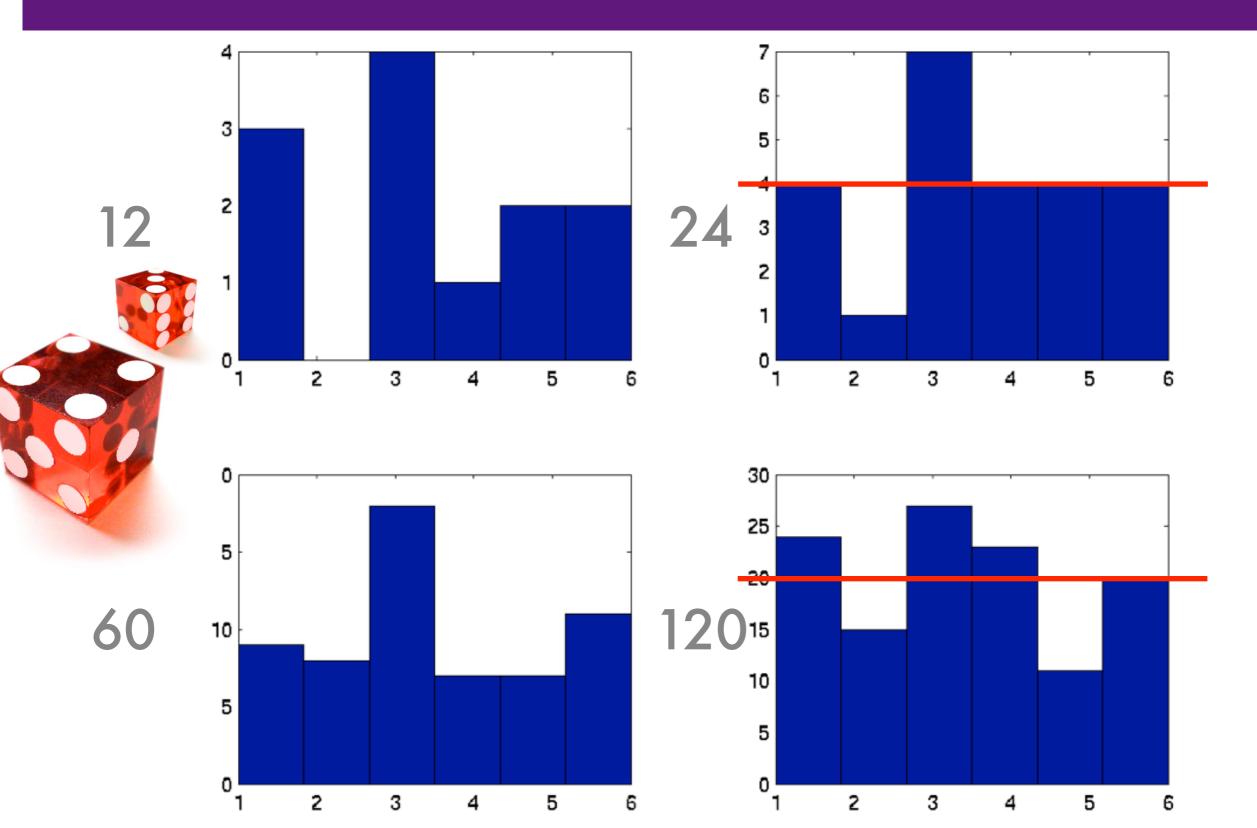
- n outcomes (e.g. USA, Canada, India, UK, NZ)
- Data likelihood

$$p(X;\pi) = \prod \pi_i^{n_i}$$

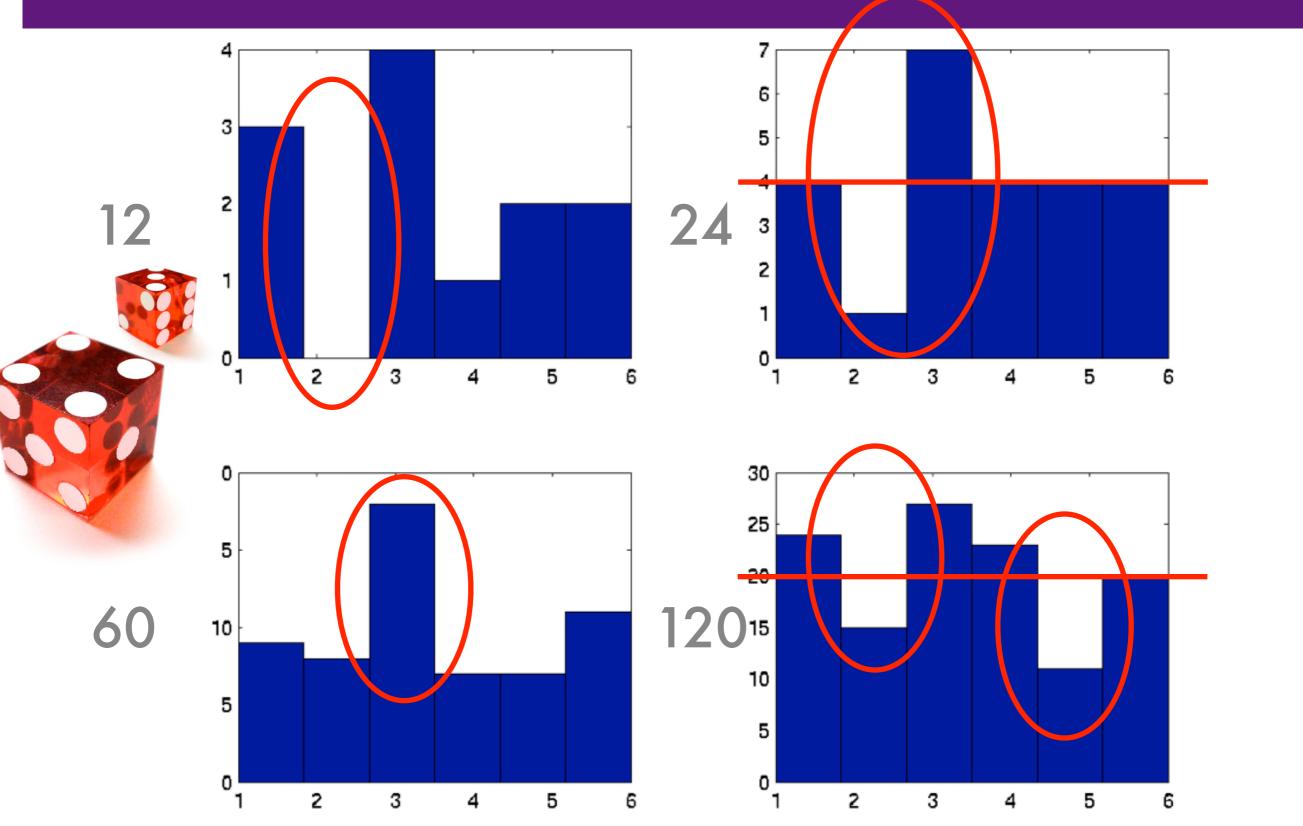
- Maximum Likelihood Estimation
  - Constrained optimization problem ... or ...
  - Incorporate constraint via  $p(x;\theta) = \frac{\exp \theta_x}{\sum_{x'} \exp \theta_{x'}}$
  - Taking derivatives yields

$$\theta_i = \log \frac{n_i}{\sum_j n_j} \iff p(x = i) = \frac{n_i}{\sum_j n_j}$$

# Tossing a Dice



# Tossing a Dice





#### Density function

$$p(x; \theta) = \exp\left(\langle \phi(x), \theta \rangle - g(\theta)\right)$$
where  $g(\theta) = \log \sum_{x'} \exp\left(\langle \phi(x'), \theta \rangle\right)$ 

#### Density function

$$p(x; \theta) = \exp\left(\langle \phi(x), \theta \rangle - g(\theta)\right)$$
where  $g(\theta) = \log \sum_{x'} \exp\left(\langle \phi(x'), \theta \rangle\right)$ 

Log partition function generates cumulants

$$\partial_{\theta} g(\theta) = \mathbf{E} \left[ \phi(x) \right]$$
  
 $\partial_{\theta}^{2} g(\theta) = \operatorname{Var} \left[ \phi(x) \right]$ 

Density function

$$p(x; \theta) = \exp\left(\langle \phi(x), \theta \rangle - g(\theta)\right)$$
where  $g(\theta) = \log \sum_{x'} \exp\left(\langle \phi(x'), \theta \rangle\right)$ 

Log partition function generates cumulants

$$\partial_{\theta} g(\theta) = \mathbf{E} \left[ \phi(x) \right]$$
  
 $\partial_{\theta}^{2} g(\theta) = \operatorname{Var} \left[ \phi(x) \right]$ 

• g is convex (second derivative is p.s.d.)

#### Examples

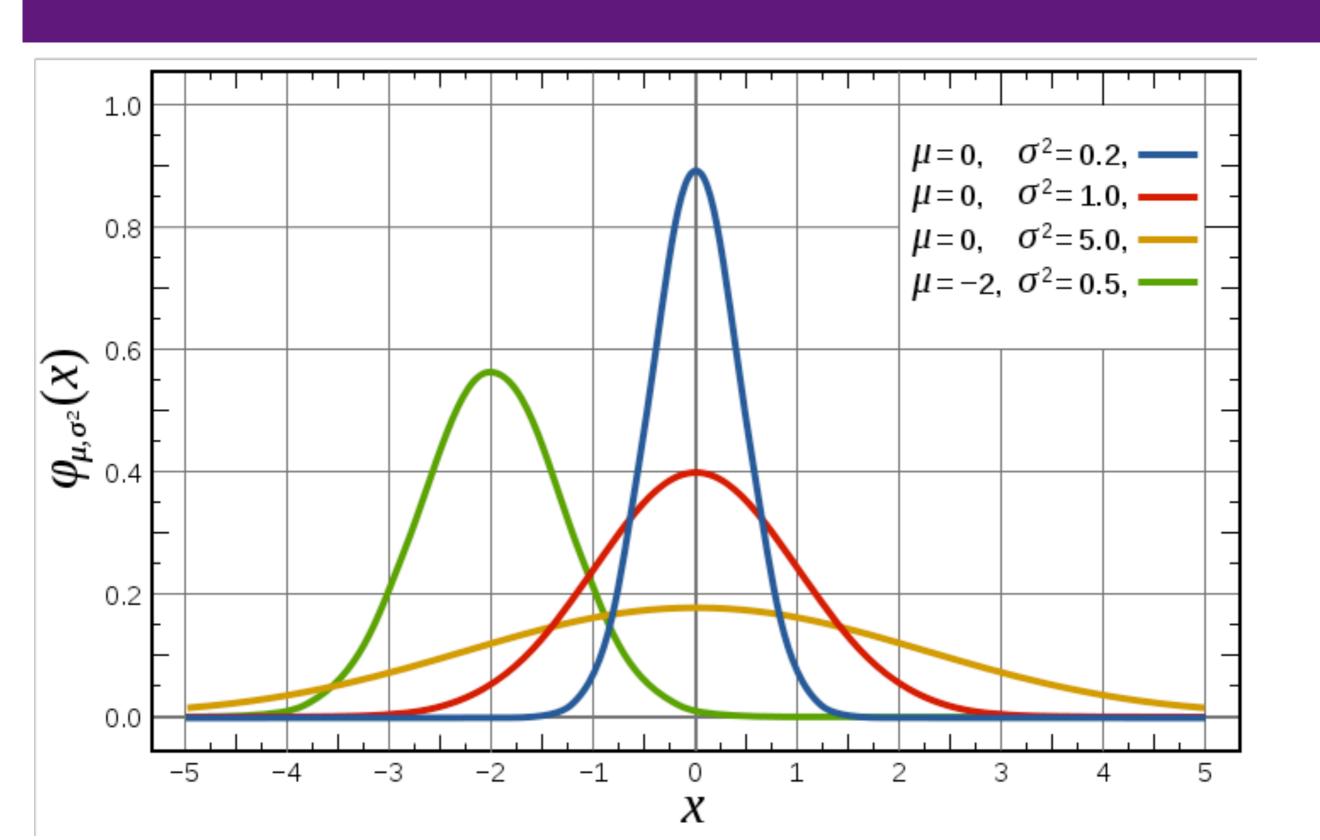
- Binomial Distribution
- Discrete Distribution
   (e<sub>x</sub> is unit vector for x)
- Gaussian
- Poisson (counting measure 1/x!)  $\phi(x) = x$
- Dirichlet, Beta, Gamma, Wishart, ...

$$\phi(x) = x$$

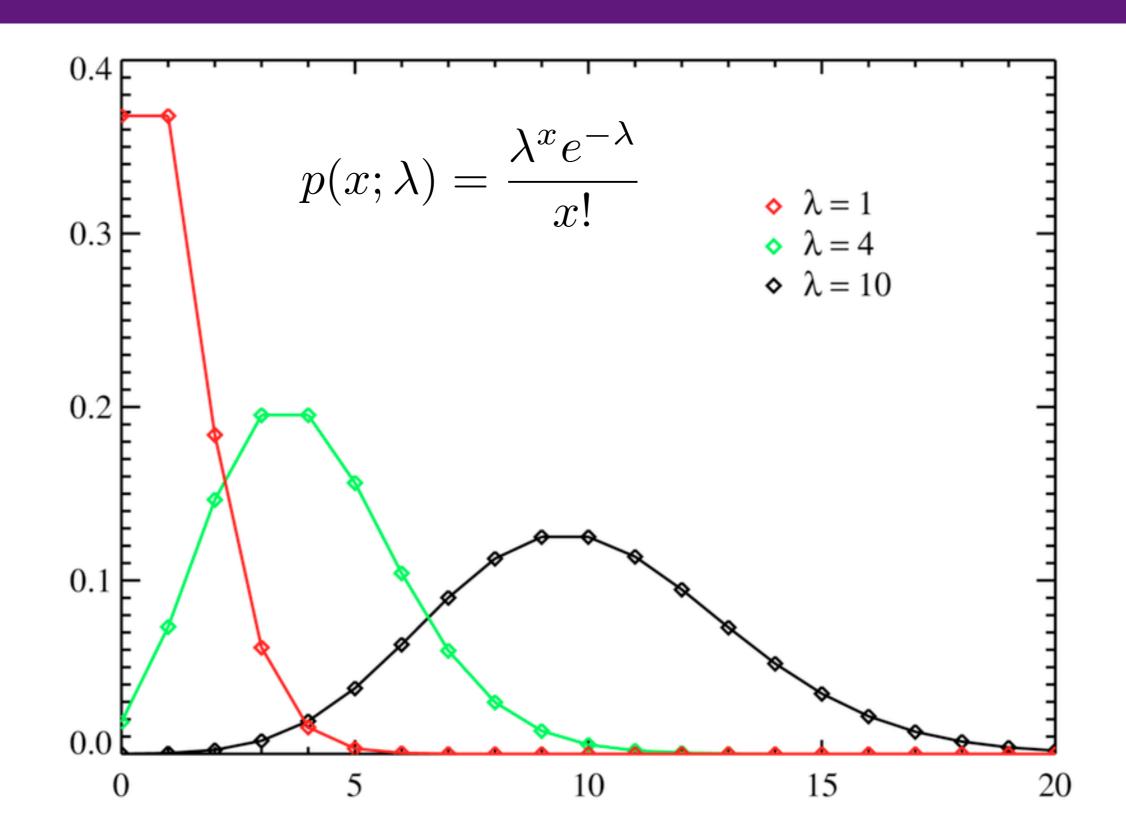
$$\phi(x) = e_x$$

$$\phi(x) = \left(x, \frac{1}{2}xx^{\top}\right)$$

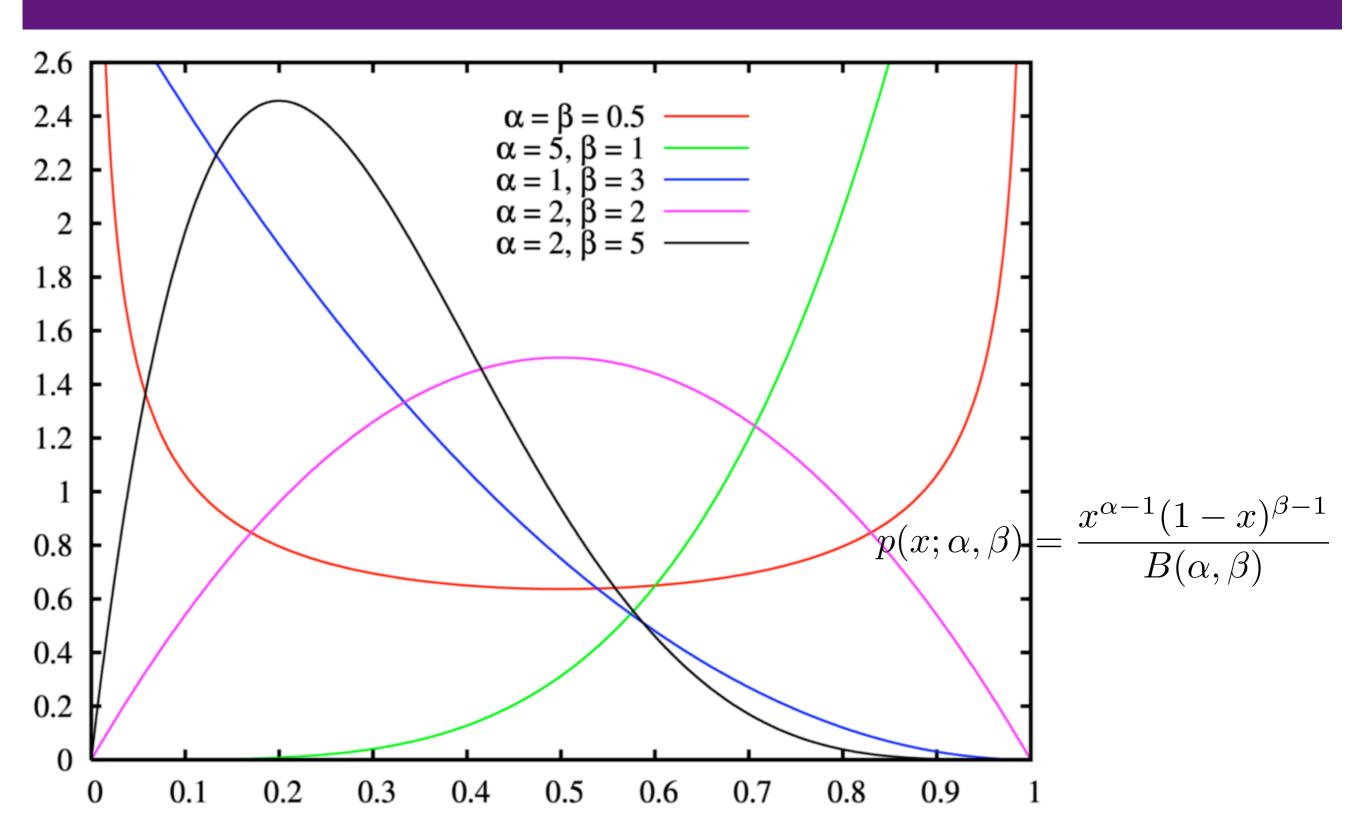
#### Normal Distribution



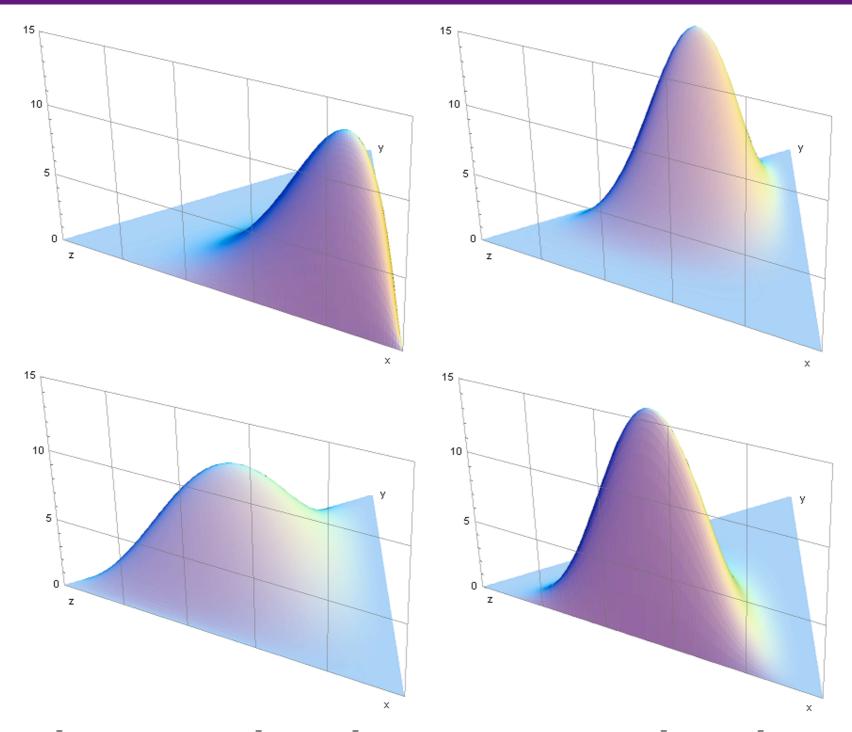
## Poisson Distribution



### Beta Distribution



### Dirichlet Distribution



... this is a distribution over distributions ...

# Maximum Likelihood

#### Maximum Likelihood

Negative log-likelihood

$$-\log p(X;\theta) = \sum_{i=1}^{n} g(\theta) - \langle \phi(x_i), \theta \rangle$$

#### Maximum Likelihood

Negative log-likelihood

$$-\log p(X;\theta) = \sum_{i=1}^{n} g(\theta) - \langle \phi(x_i), \theta \rangle$$

• Taking derivatives 
$$-\partial_{\theta} \log p(X;\theta) = m \left[ \mathbf{E}[\phi(x)] - \frac{1}{m} \sum_{i=1}^{n} \phi(x_i) \right]$$

We pick the parameter such that the distribution matches the empirical average.

empirical average

# Conjugate Priors

- Unless we have lots of data estimates are weak
- Usually we have an idea of what to expect

$$p(\theta|X) \propto p(X|\theta) \cdot p(\theta)$$

we might even have 'seen' such data before

Solution: add 'fake' observations

$$p(\theta) \propto p(X_{\text{fake}}|\theta) \text{ hence } p(\theta|X) \propto p(X|\theta)p(X_{\text{fake}}|\theta) = p(X \cup X_{\text{fake}}|\theta)$$

Inference (generalized Laplace smoothing)

$$\frac{1}{n}\sum_{i=1}^n\phi(x_i)\longrightarrow\frac{1}{n+m}\sum_{i=1}^n\phi(x_i)+\frac{m}{n+m}\mu_0$$
 fake count

# Example: Gaussian Estimation

- Sufficient statistics:  $x, x^2$
- Mean and variance given by

$$\mu = \mathbf{E}_x[x] \text{ and } \sigma^2 = \mathbf{E}_x[x^2] - \mathbf{E}_x^2[x]$$

Maximum Likelihood Estimate

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i \text{ and } \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \hat{\mu}^2$$

Maximum a Posteriori Estimate

smoother

$$\hat{\mu} = \frac{1}{n+n_0} \sum_{i=1}^{n} x_i \text{ and } \sigma^2 = \frac{1}{n+n_0} \sum_{i=1}^{n} x_i^2 + \frac{n_0}{n+n_0} \mathbf{1} - \hat{\mu}^2$$

# Collapsing

Conjugate priors

$$p(\theta) \propto p(X_{\rm fake}|\theta)$$

Hence we know how to compute normalization

• Prediction  $p(x|X) = \int p(x|\theta)p(\theta|X)d\theta$ 

(Beta, binomial)
(Dirichlet, multinomial)
(Gamma, Poisson)
(Wishart, Gauss)

$$\propto \int p(x|\theta)p(X|\theta)p(X_{\rm fake}|\theta)d\theta$$
 
$$= \int p(\{x\} \cup X \cup X_{\rm fake}|\theta)d\theta$$
 look up closed form expansions

http://en.wikipedia.org/wiki/Exponential\_family

## Conjugate Prior in action

 $m_i = m \cdot [\mu_0]_i$ 

$$p(x=i) = \frac{n_i}{n} \longrightarrow p(x=i) = \frac{n_i + m_i}{n+m}$$

Outcome	1	2	3	4	5	6
Counts	3	6	2	1	4	4
MLE	0.15	0.30	0.10	0.05	0.20	0.20
MAP $(m_0 = 6)$	0.15	0.27	0.12	0.08	0.19	0.19
MAP $(m_0 = 100)$	0.16	0.19	0.16	0.15	0.17	0.17

# Conjugate Prior in action

 $m_i = m \cdot [\mu_0]_i$ 

#### Discrete Distribution

$$p(x=i) = \frac{n_i}{n} \longrightarrow p(x=i) = \frac{n_i + m_i}{n+m}$$
   
 • Tossing a dice

Outcome	1		3	4	5	6
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# Conjugate Prior in action

 $m_i = m \cdot [\mu_0]_i$ 

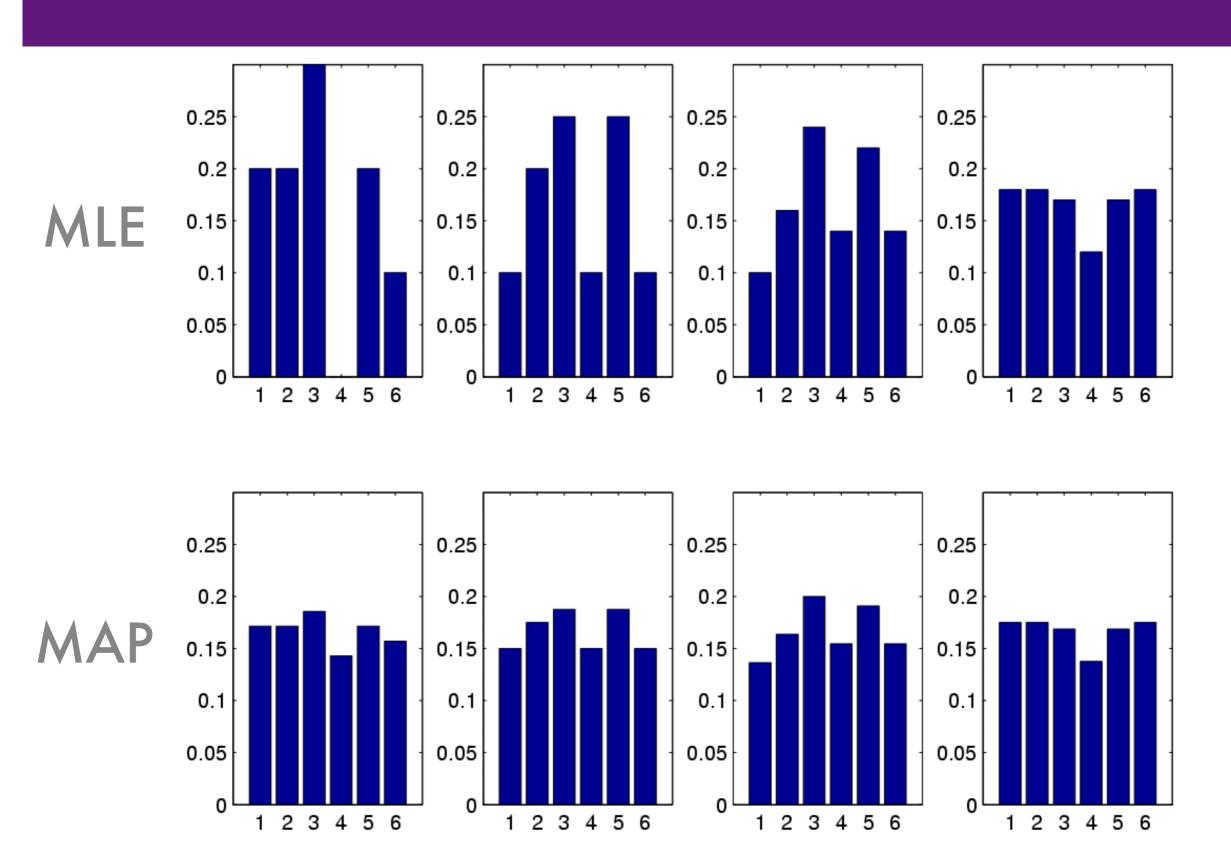
#### Discrete Distribution

$$p(x=i) = \frac{n_i}{n} \longrightarrow p(x=i) = \frac{n_i + m_i}{n+m}$$
   
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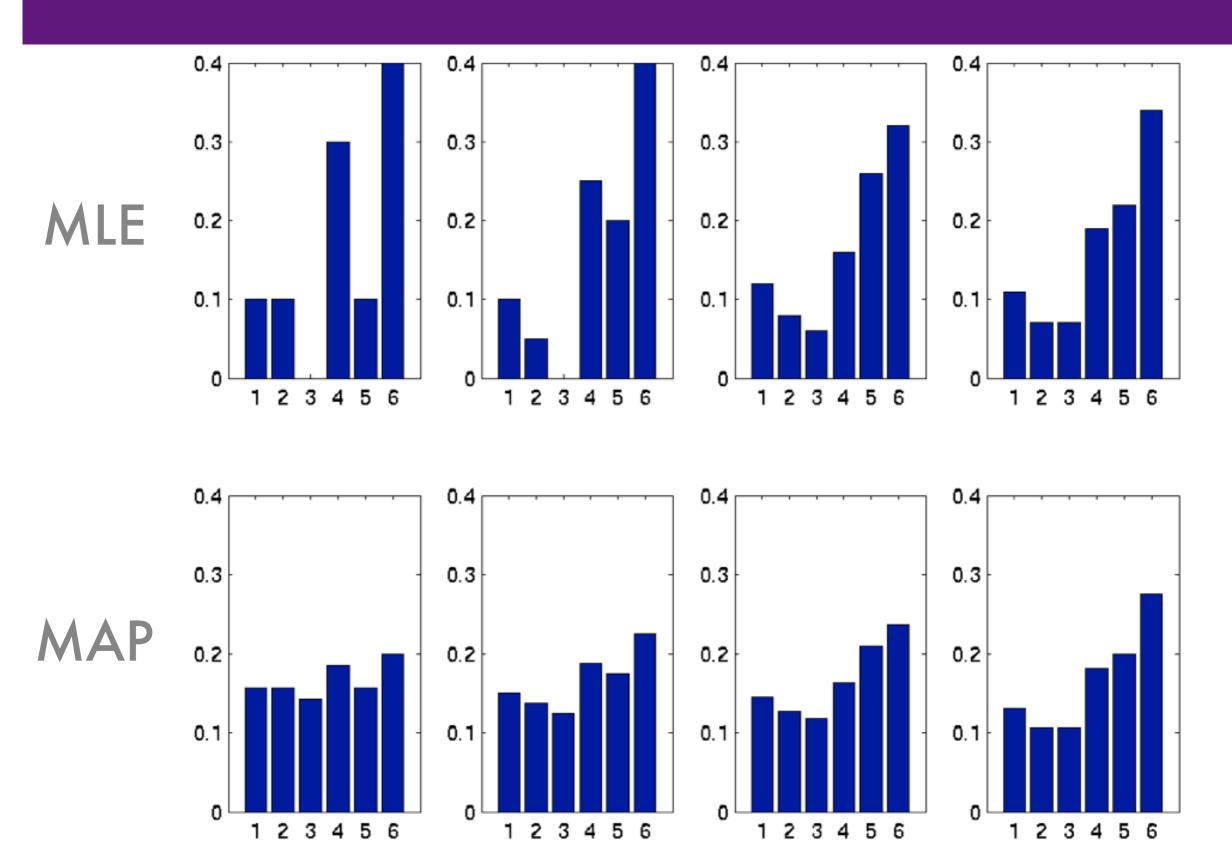
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MAP $(m_0 = 100)$	0.16	0.19	0.16	0.15	0.17	0.17

 Rule of thumb need 10 data points (or prior) per parameter

## Honest dice



## Tainted dice



# Priors (part deux)

Parameter smoothing

$$p(\theta) \propto \exp(-\lambda \|\theta\|_1) \text{ or } p(\theta) \propto \exp(-\lambda \|\theta\|_2^2)$$

Posterior

$$p(\theta|x) \propto \prod_{i=1}^{m} p(x_i|\theta)p(\theta)$$
$$\propto \exp\left(\sum_{i=1}^{m} \langle \phi(x_i), \theta \rangle - mg(\theta) - \frac{1}{2\sigma^2} \|\theta\|_2^2\right)$$

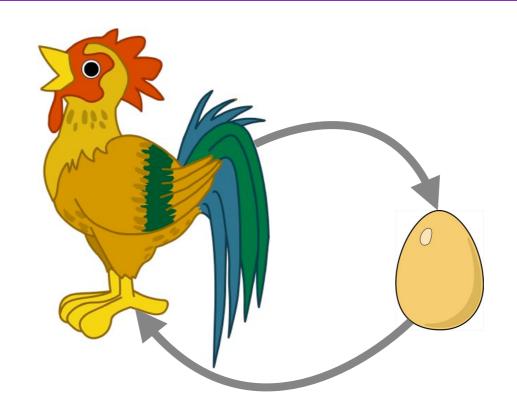
Convex optimization problem (MAP estimation)

$$\underset{\theta}{\text{minimize }} g(\theta) - \left\langle \frac{1}{m} \sum_{i=1}^{m} \phi(x_i), \theta \right\rangle + \frac{1}{2m\sigma^2} \|\theta\|_2^2$$

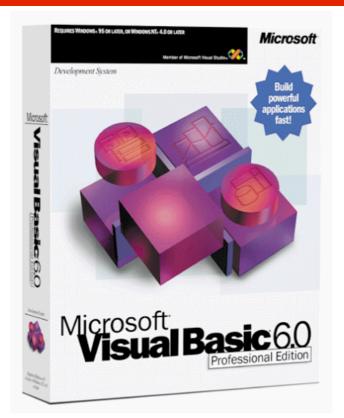
# Summary

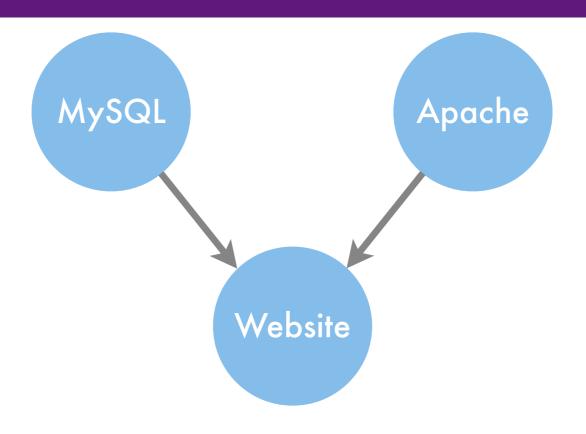
- Basic statistics tools
- Estimating probabilities (mainly scalar)
- Exponential family introduction

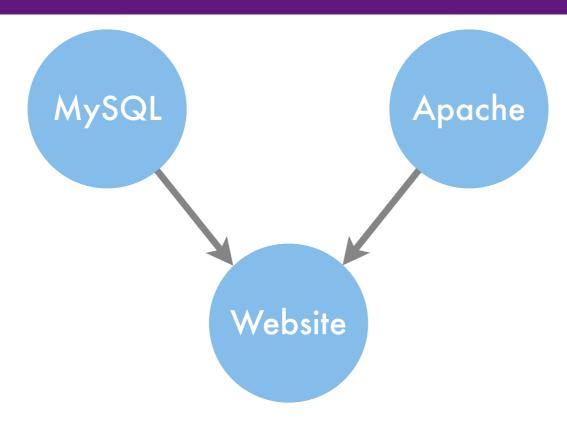
# Part 4: Directed Graphical



## Basics

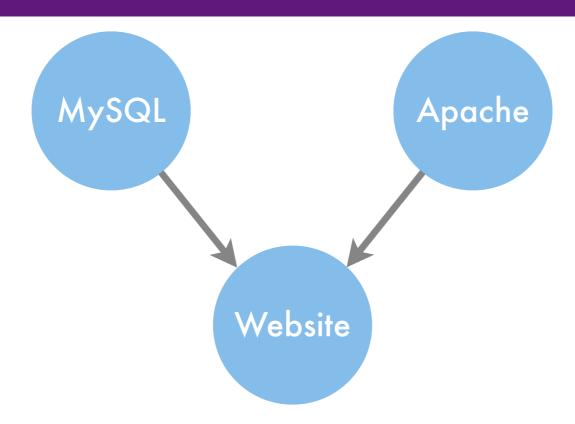






Joint distribution (assume a and m are independent)

$$p(m, a, w) = p(w|m, a)p(m)p(a)$$



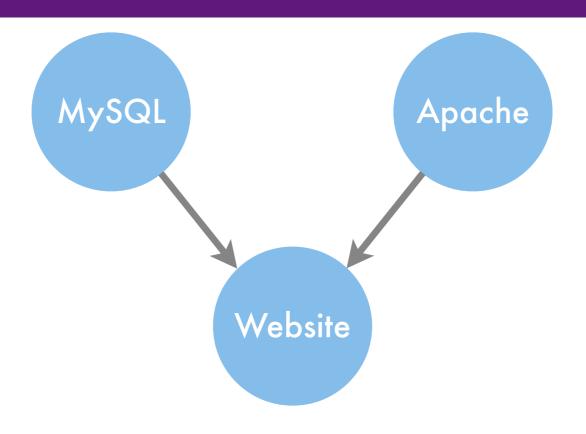
Joint distribution (assume a and m are independent)

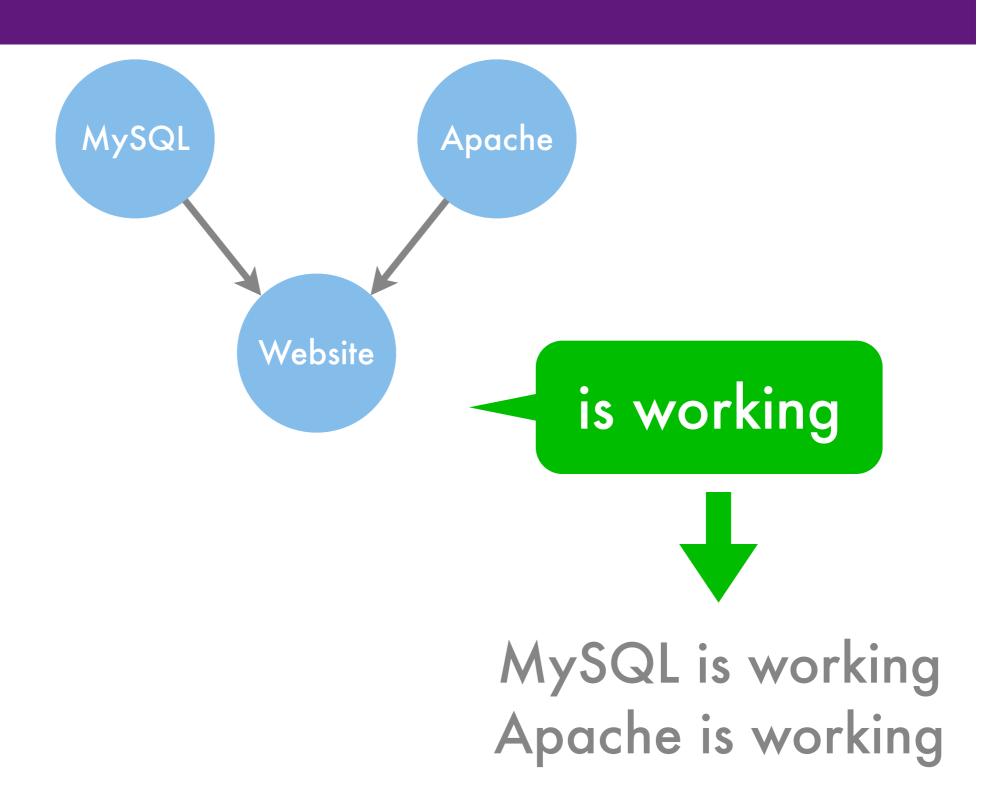
$$p(m, a, w) = p(w|m, a)p(m)p(a)$$

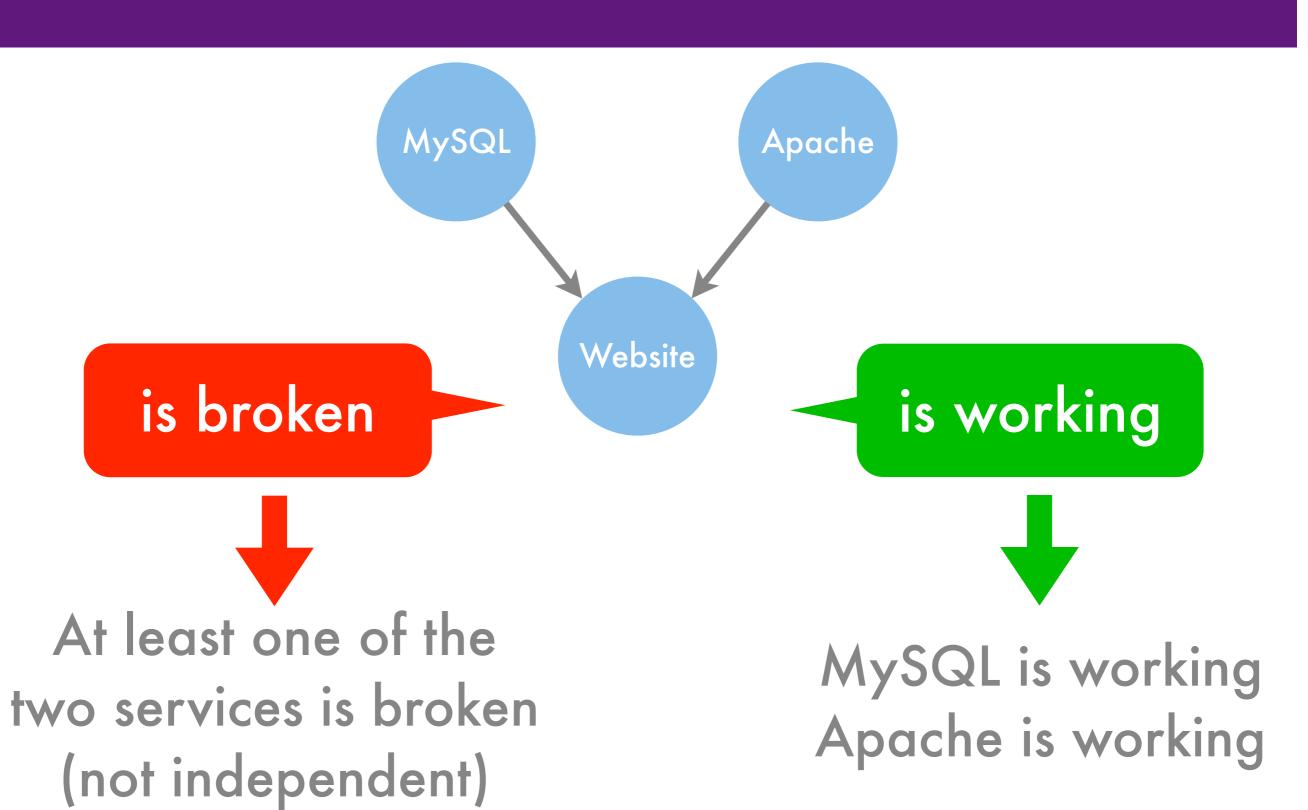
Explaining away

$$p(m, a|w) = \frac{p(w|m, a)p(m)p(a)}{\sum_{m', a'} p(w|m', a')p(m')p(a')}$$

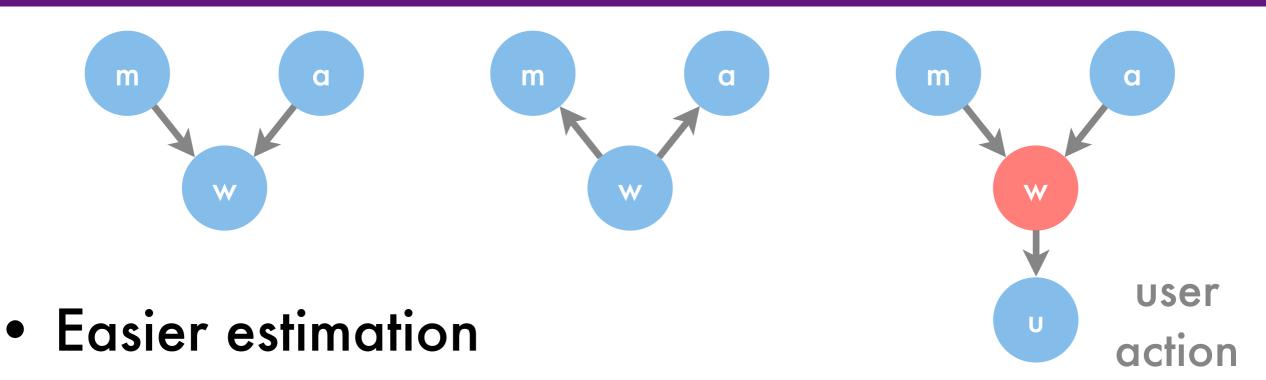
a and m are dependent conditioned on w







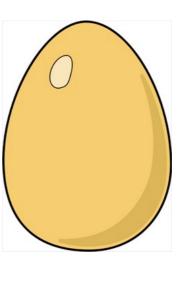
# Directed graphical model



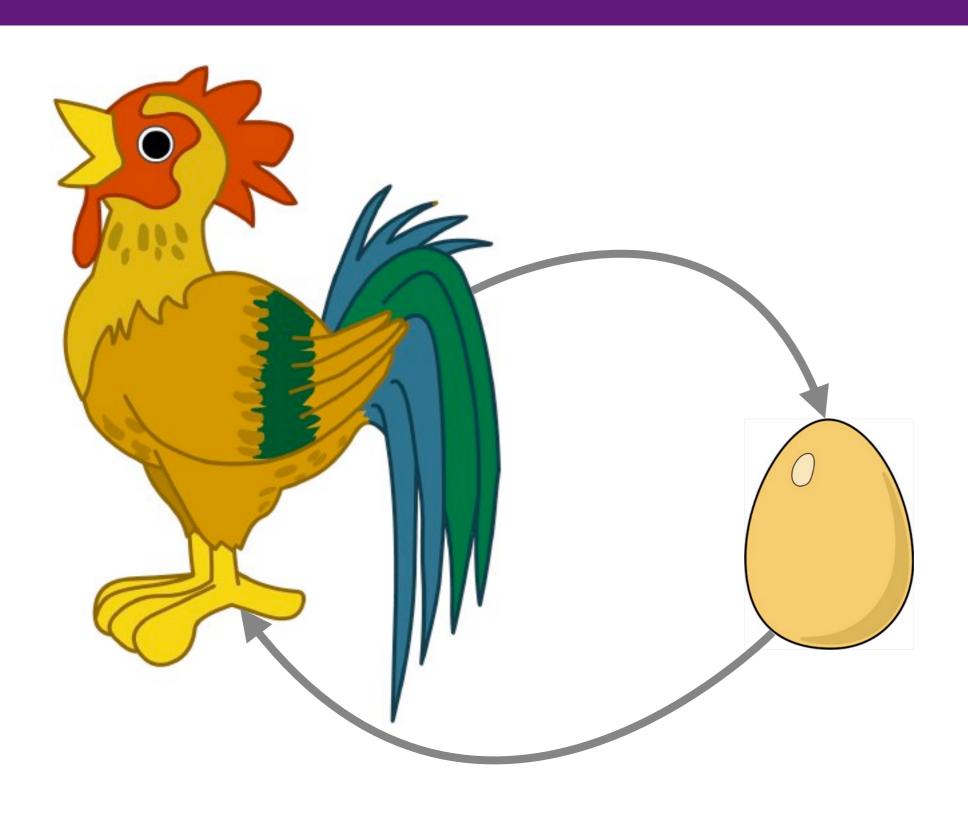
- 15 parameters for full joint distribution
- 1+1+3+1 for factorizing distribution
- Causal relations
- Inference for unobserved variables

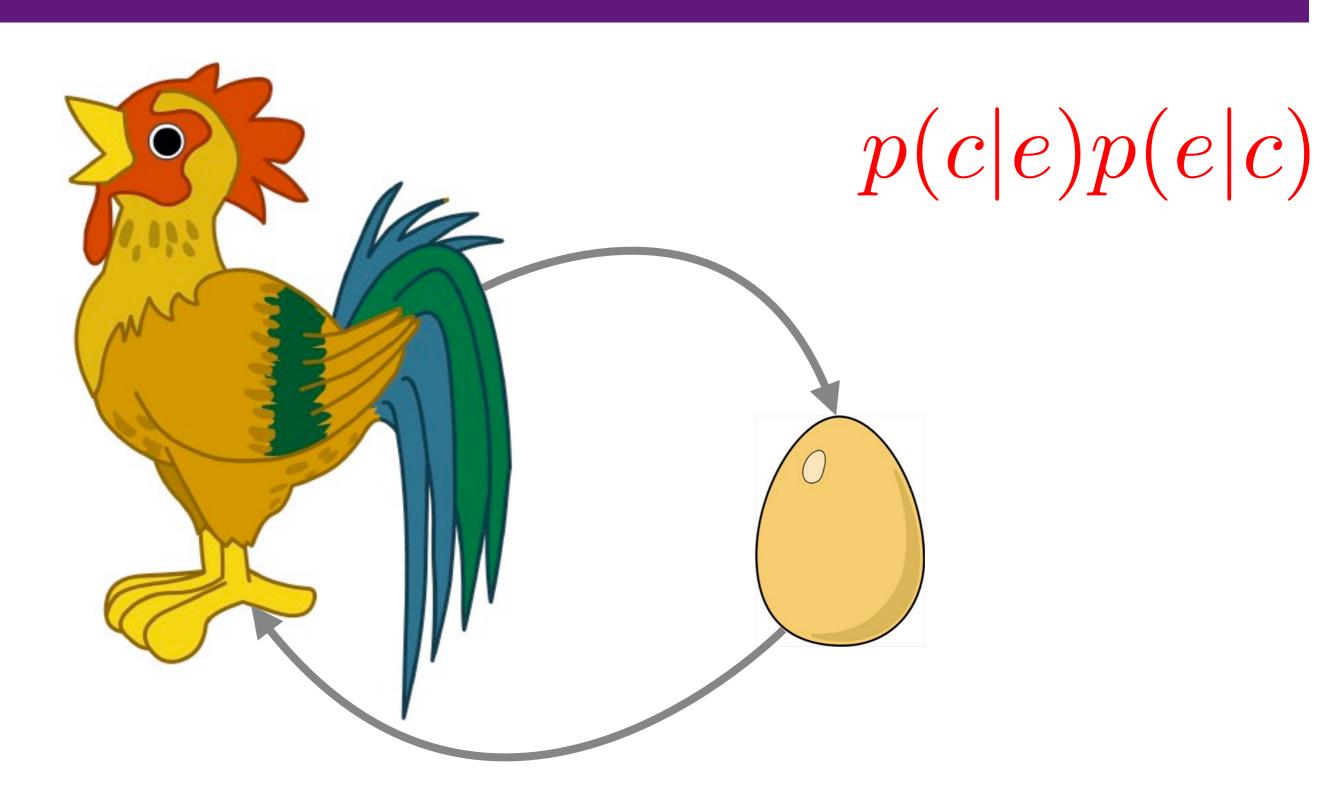


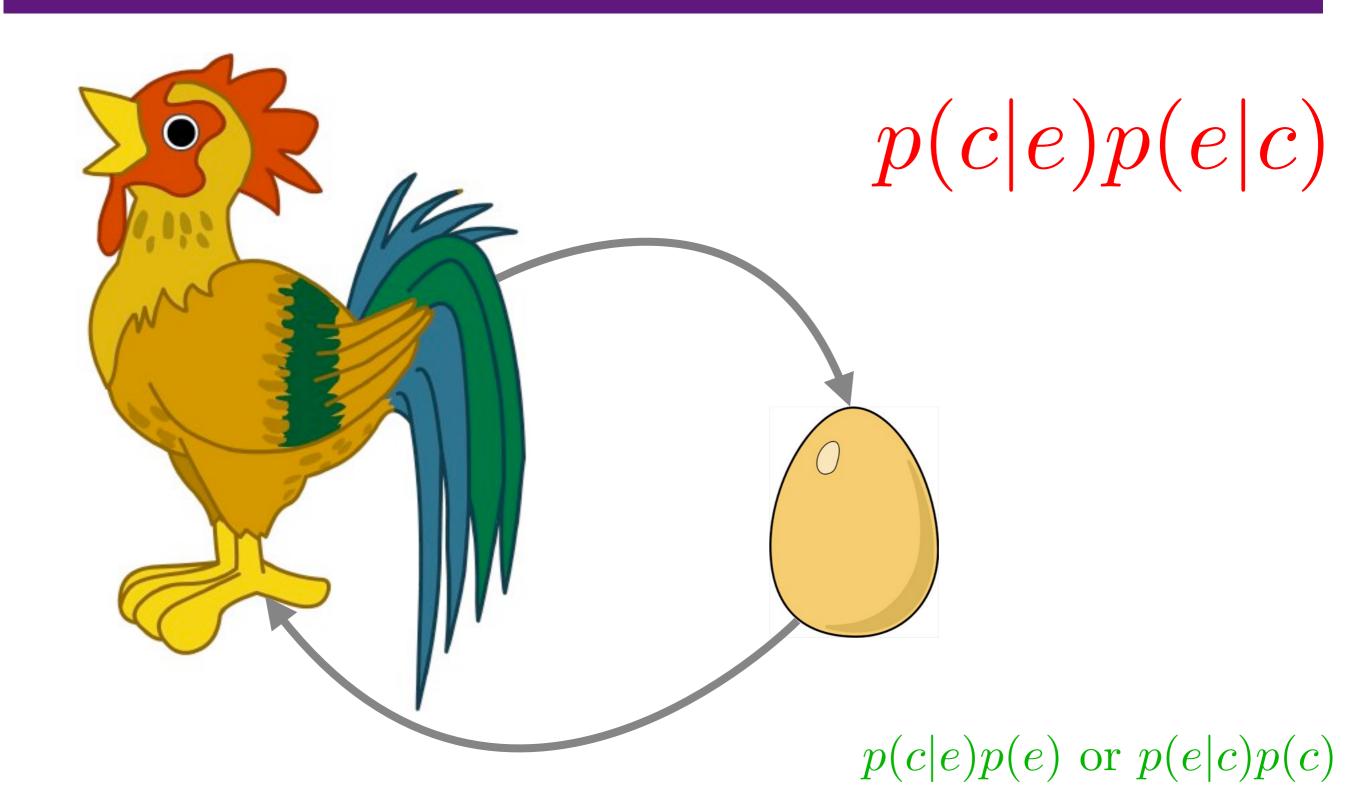


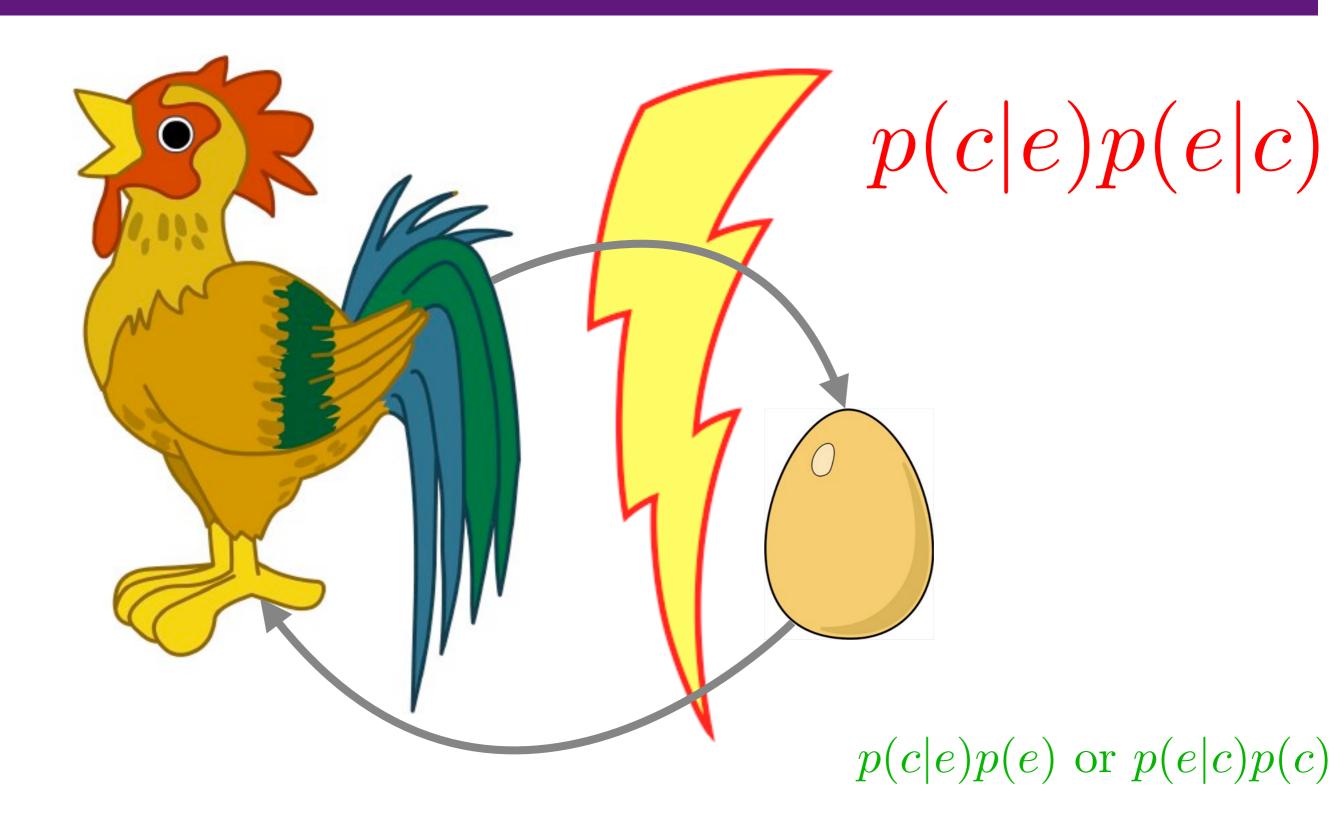












# Directed Graphical Model

• Joint probability distribution

$$p(x) = \prod_{i} p(x_i | x_{\text{parents(i)}})$$



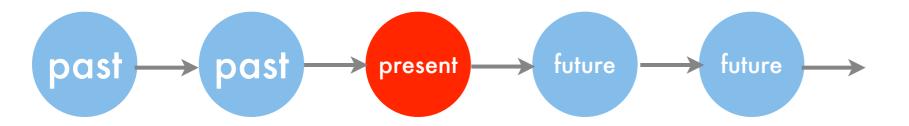


$$\log p(x|\theta) = \sum \log p(x_i|x_{\text{parents(i)}}, \theta)$$

• If x is partially observed things get interesting maximization, EM, variational, sampling ...

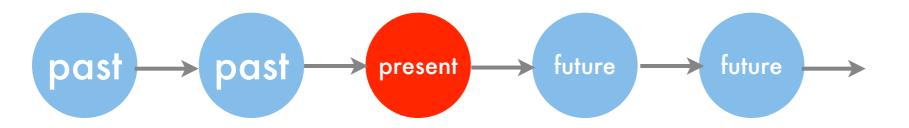
## Chains

#### Markov Chain

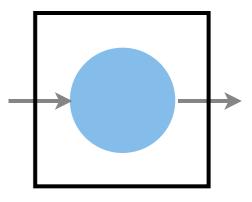


## Chains

#### Markov Chain

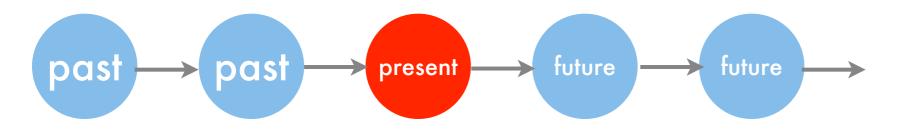


#### Plate

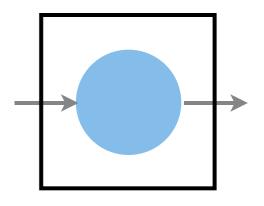


#### Chains

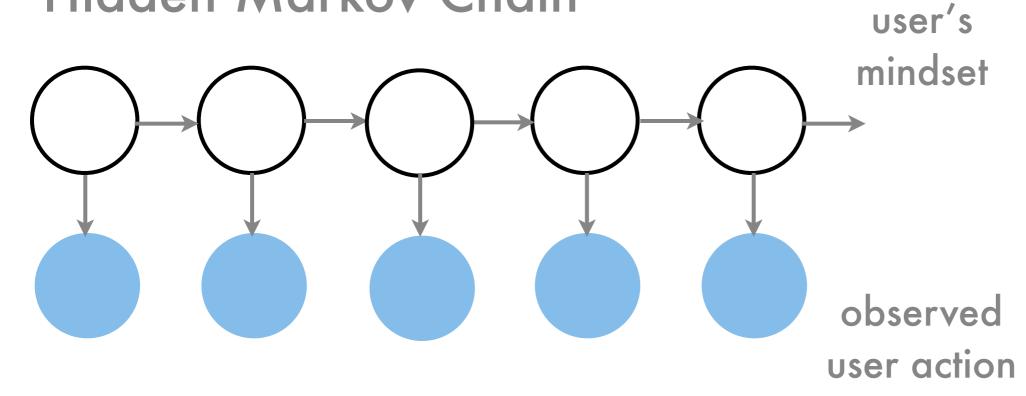
#### Markov Chain



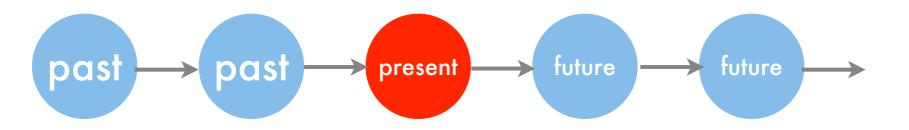
#### Plate



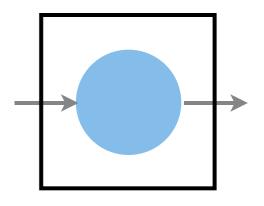
#### Hidden Markov Chain



#### Markov Chain

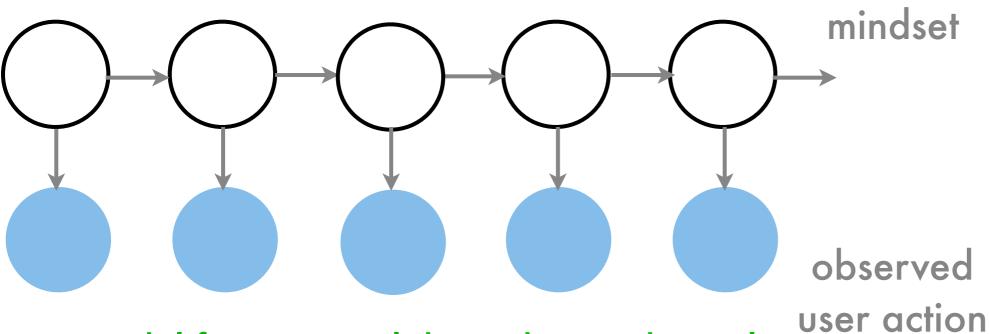


### Plate



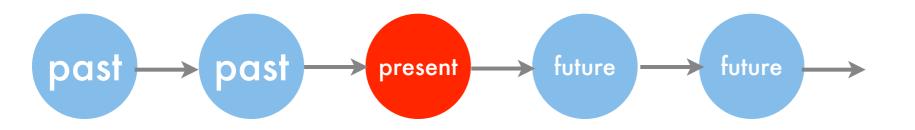
user's

### Hidden Markov Chain

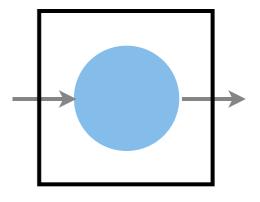


user model for traversal through search results

#### Markov Chain

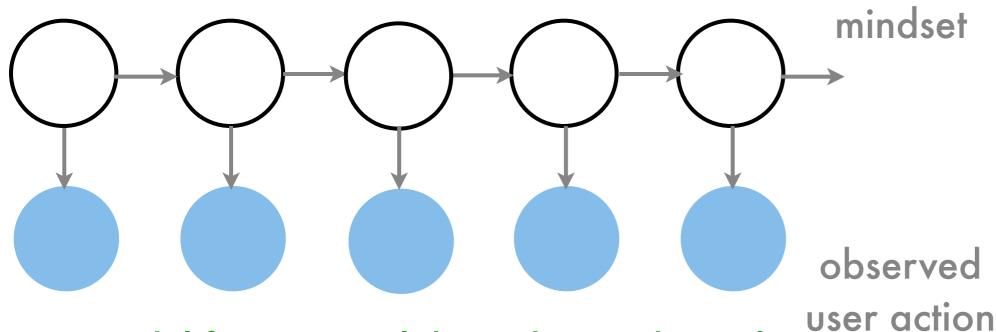


### Plate



user's

### Hidden Markov Chain



user model for traversal through search results

### Markov Chain

$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta)$$

### Hidden Markov Chain

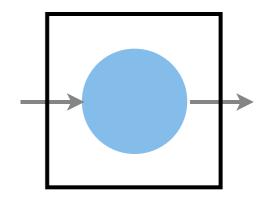
$$p(x, y; \theta) = p(x_0; \theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i; \theta) \prod_{i=1}^{n} p(y_i|x_i)$$

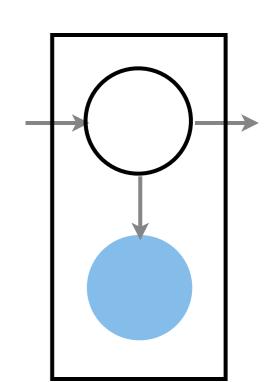
observed user action

user's

mindset

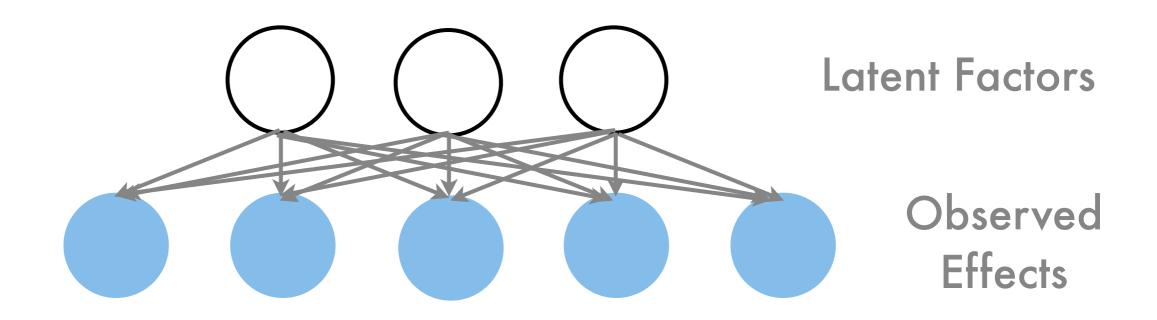
#### Plate



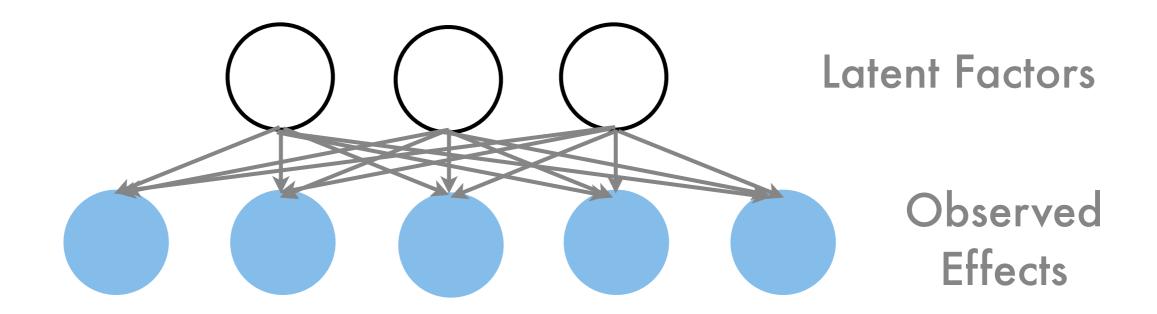


user model for traversal through search results

# Factor Graphs

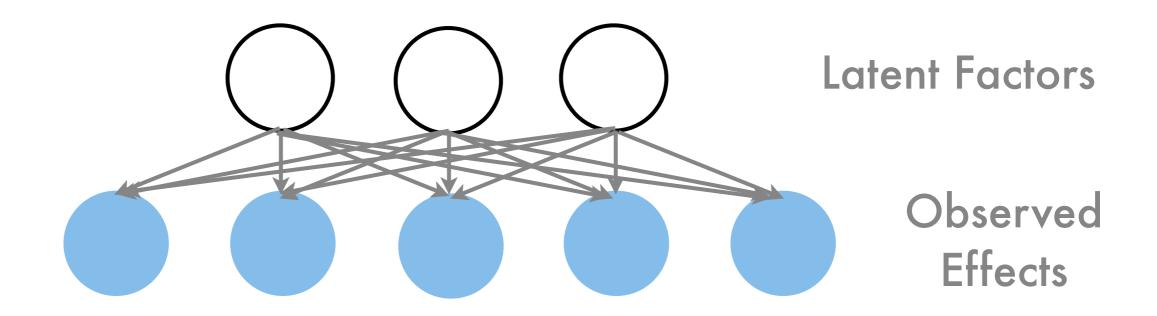


# Factor Graphs

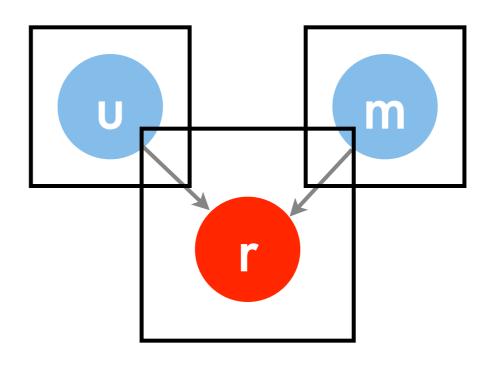


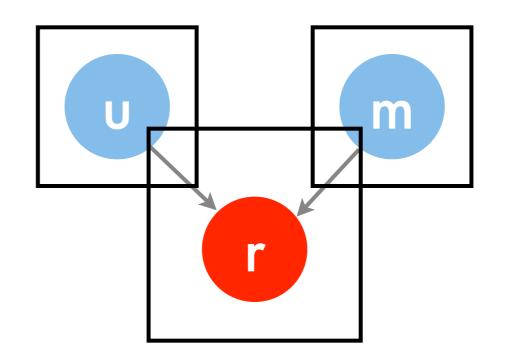
Observed effects
 Click behavior, queries, watched news, emails

# Factor Graphs

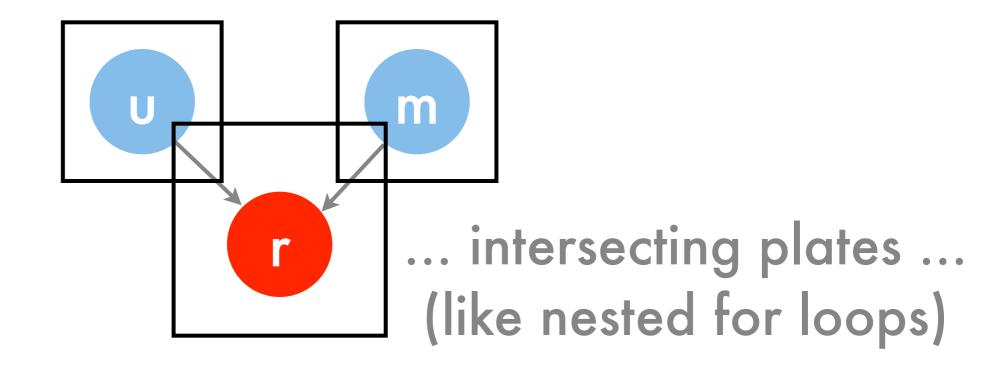


- Observed effects
   Click behavior, queries, watched news, emails
- Latent factors
   User profile, news content, hot keywords, social connectivity graph, events



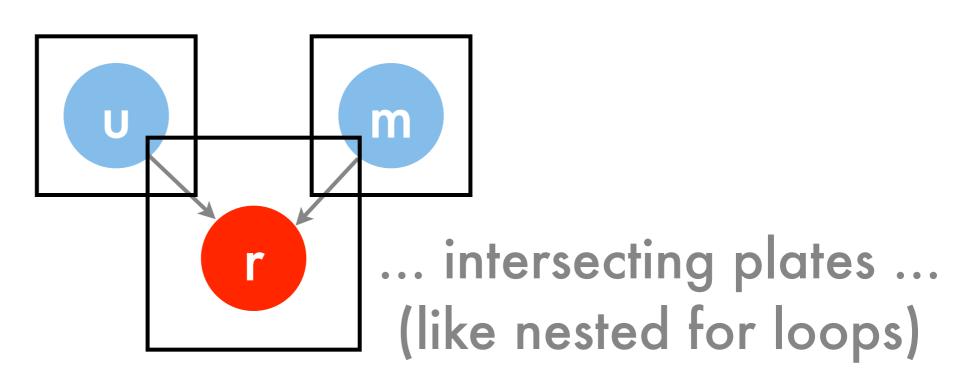


- Users u
- Movies m
- Ratings r (but only for a subset of users)



- Users u
- Movies m
- Ratings r (but only for a subset of users)

news,
SearchMonkey
answers
social
ranking
OMG
personals



- Users u
- Movies m
- Ratings r (but only for a subset of users)

your job

my job

- How to design models
  - Common (engineering) sense
  - Computational tractability

your job

my job

- How to design models
  - Common (engineering) sense
  - Computational tractability
- Dependency analysis
  - Bayes ball (not in this lecture)

your job

my job

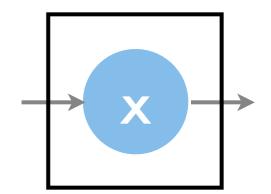
How to design models

- your job
- Common (engineering) sense
- Computational tractability
- Dependency analysis

- my job
- Bayes ball (not in this lecture)
- Inference
  - Easy for fully observed situations
  - Many algorithms if not fully observed
  - Dynamic programming / message passing

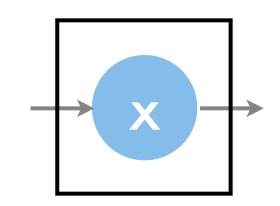
# Dynamic Programming 101

$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta) \qquad \mathbf{\chi}_0 \longrightarrow \mathbf{\chi}_1 \longrightarrow \mathbf{\chi}_2 \longrightarrow \mathbf{\chi}_3$$



$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta) \qquad \chi_0 \longrightarrow \chi_1 \longrightarrow \chi_2 \longrightarrow \chi_3$$

$$p(x_i) = \sum_{x_0, \dots x_{i-1}, x_{i+1} \dots x_n} \underbrace{p(x_0)}_{j=1} \prod_{j=1}^n p(x_j | x_{j-1})$$



$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta) \qquad \mathbf{x}_0 \longrightarrow \mathbf{x}_1 \longrightarrow \mathbf{x}_2 \longrightarrow \mathbf{x}_3$$

$$p(x_i) = \sum_{x_0, \dots x_{i-1}, x_{i+1} \dots x_n} \underbrace{p(x_0)}_{:=l_0(x_0)} \prod_{j=1}^n p(x_j|x_{j-1}) = \sum_{x_1, \dots x_{i-1}, x_{i+1} \dots x_n} \underbrace{\sum_{x_0} [l_0(x_0)p(x_1|x_0)]}_{:=l_1(x_1)} \prod_{j=2}^n p(x_j|x_{j-1}) \longrightarrow \mathbf{x}_1 \longrightarrow \mathbf{x}_2 \longrightarrow \mathbf{x}_3$$

$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta) \qquad \mathbf{x}_0 \longrightarrow \mathbf{x}_1 \longrightarrow \mathbf{x}_2 \longrightarrow \mathbf{x}_3$$

$$p(x_i) = \sum_{x_0, \dots x_{i-1}, x_{i+1} \dots x_n} \underbrace{p(x_0)}_{:=l_0(x_0)} \prod_{j=1}^n p(x_j|x_{j-1})$$

$$= \sum_{x_1, \dots x_{i-1}, x_{i+1} \dots x_n} \underbrace{\sum_{i=l_1(x_1)} [l_0(x_0)p(x_1|x_0)]}_{:=l_1(x_1)} \prod_{j=2}^n p(x_j|x_{j-1})$$

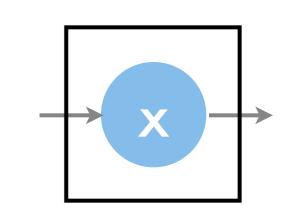
$$= \sum_{x_2, \dots x_{i-1}, x_{i+1} \dots x_n} \underbrace{\sum_{i=l_2(x_2)} [l_1(x_1)p(x_2|x_1)]}_{:=l_2(x_2)} \prod_{j=3}^n p(x_j|x_{j-1})$$

$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta) \qquad \qquad \chi_0 \longrightarrow \chi_1 \longrightarrow \chi_2 \longrightarrow \chi_3$$
$$p(x_i) = l_i(x_i) \qquad \sum_{i=1}^{n-1} p(x_{j+1}|x_j)$$

$$= l_i(x_i) \sum_{x_{i+1}...x_{n-1}} \prod_{j=i}^{n-2} p(x_{j+1}|x_j) \sum_{x_n} p(x_n|x_{n-1})$$

$$:= r_{n-1}(x_{n-1})$$

 $x_{i+1}...x_n j=i$ 



$$= l_i(x_i) \sum_{x_{i+1}...x_{n-2}} \prod_{j=i}^{n-3} p(x_{j+1}|x_j) \sum_{x_{n-1}} p(x_{n-1}|x_{n-2})r_{n-1}(x_{n-1})$$

$$:= r_{n-2}(x_{n-2})$$

$$p(x;\theta) = p(x_0;\theta) \prod_{i=1}^{n-1} p(x_{i+1}|x_i;\theta) \qquad \chi_0 \longrightarrow \chi_1 \longrightarrow \chi_2 \longrightarrow \chi_3$$

Forward recursion

$$l_0(x_0) := p(x_0)$$
 and  $l_i(x_i) := \sum_{x_{i-1}} l_{i-1}(x_{i-1})p(x_i|x_{i-1})$ 

Backward recursion

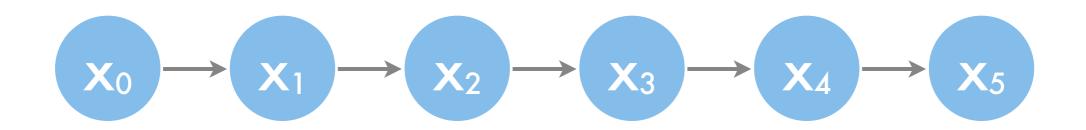
$$r_n(x_n) := 1 \text{ and } r_i(x_i) := \sum_{x_{i+1}} r_{i+1}(x_{i+1}) p(x_{i+1}|x_i)$$

Marginalization & conditioning

$$p(x_i) = l_i(x_i)r_i(x_i)$$

$$p(x_{-i}|x_i) = \frac{p(x)}{p(x_i)}$$

$$p(x_i, x_{i+1}) = l_i(x_i)p(x_{i+1}|x_i)r_i(x_{i+1})$$

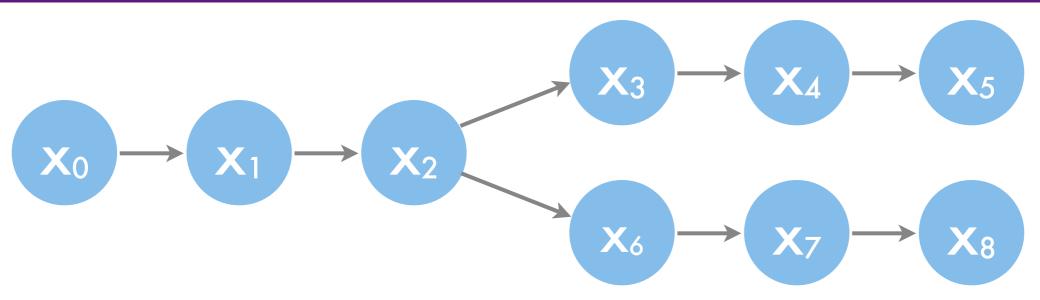


Send forward messages starting from left node

$$m_{i-1\to i}(x_i) = \sum_{x_{i-1}} m_{i-2\to i-1}(x_{i-1}) f(x_{i-1}, x_i)$$

Send backward messages starting from right node

$$m_{i+1 \to i}(x_i) = \sum_{x_{i+1}} m_{i+2 \to i+1}(x_{i+1}) f(x_i, x_{i+1})$$

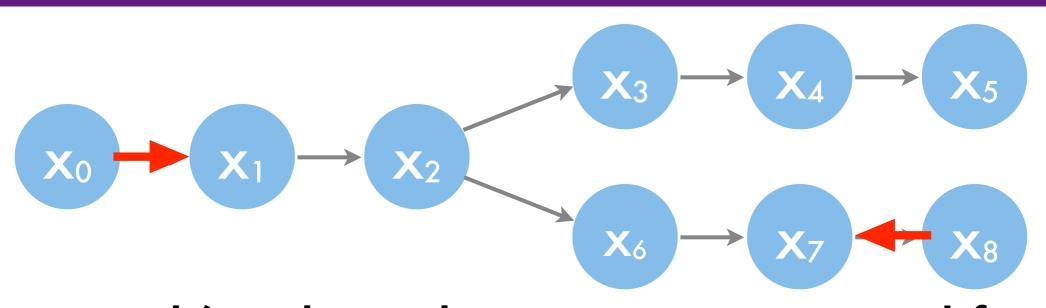


- Forward/Backward messages as normal for chain
- When we have more edges for a vertex use ...

$$m_{2\to 3}(x_3) = \sum_{x_2} m_{1\to 2}(x_2) m_{6\to 2}(x_2) f(x_2, x_3)$$

$$m_{2\to 6}(x_6) = \sum_{x_2} m_{1\to 2}(x_2) m_{3\to 2}(x_2) f(x_2, x_6)$$

$$m_{2\to 1}(x_1) = \sum_{x_2} m_{3\to 2}(x_2) m_{6\to 2}(x_2) f(x_1, x_2)$$

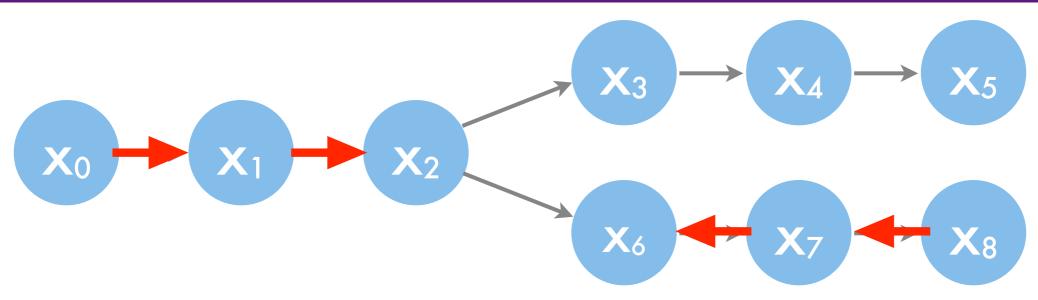


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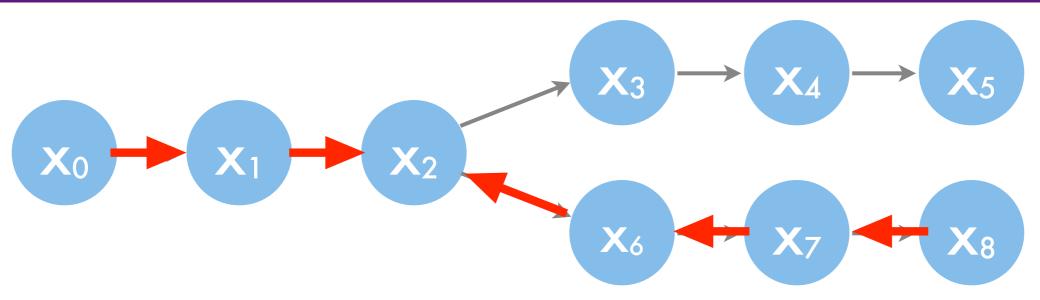


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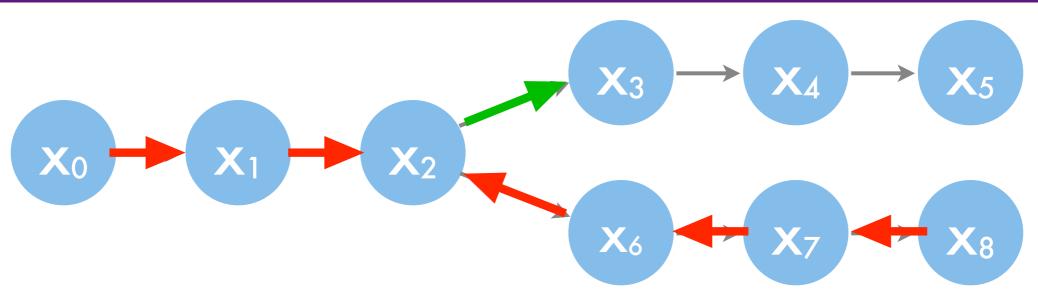


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- When we have more edges for a vertex use ...

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$$m_{2\to 1}(x_1) = \sum_{x_2} m_{3\to 2}(x_2) m_{6\to 2}(x_2) f(x_1, x_2)$$

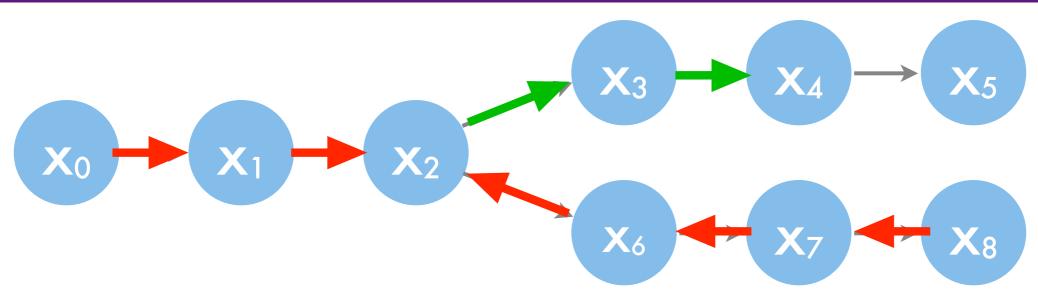


- Forward/Backward messages as normal for chain
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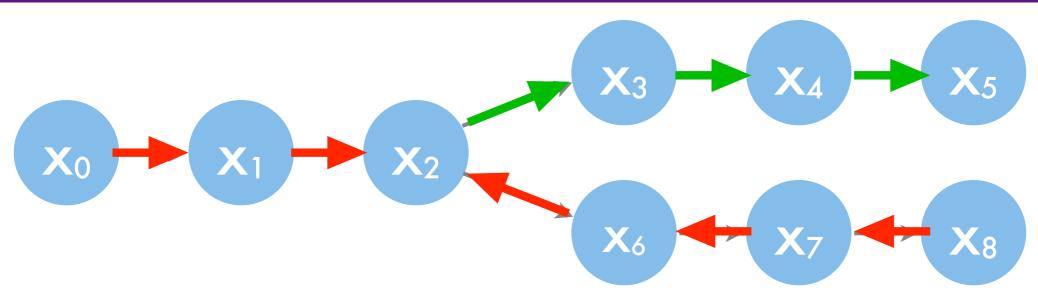


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$$m_{2\to 3}(x_3) = \sum_{x_2} m_{1\to 2}(x_2) m_{6\to 2}(x_2) f(x_2, x_3)$$

$$m_{2\to 6}(x_6) = \sum_{x_2} m_{1\to 2}(x_2) m_{3\to 2}(x_2) f(x_2, x_6)$$

$$m_{2\to 1}(x_1) = \sum_{x_2} m_{3\to 2}(x_2) m_{6\to 2}(x_2) f(x_1, x_2)$$

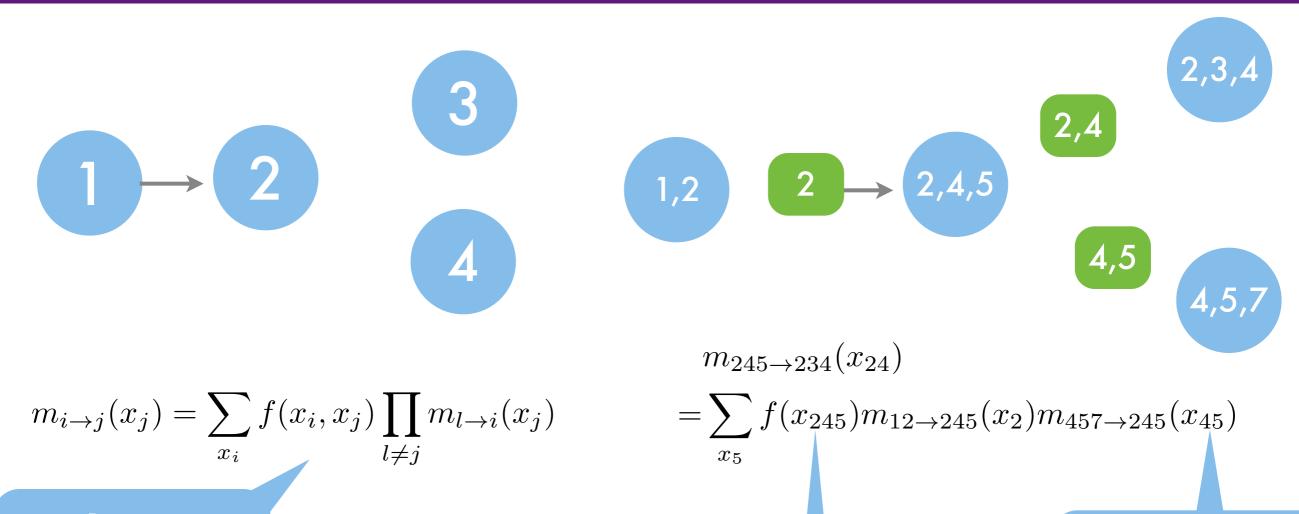


- Forward/Backward messages as normal for chain
- When we have more edges for a vertex use ...

$$m_{2\to 3}(x_3) = \sum_{x_2} m_{1\to 2}(x_2) m_{6\to 2}(x_2) f(x_2, x_3)$$

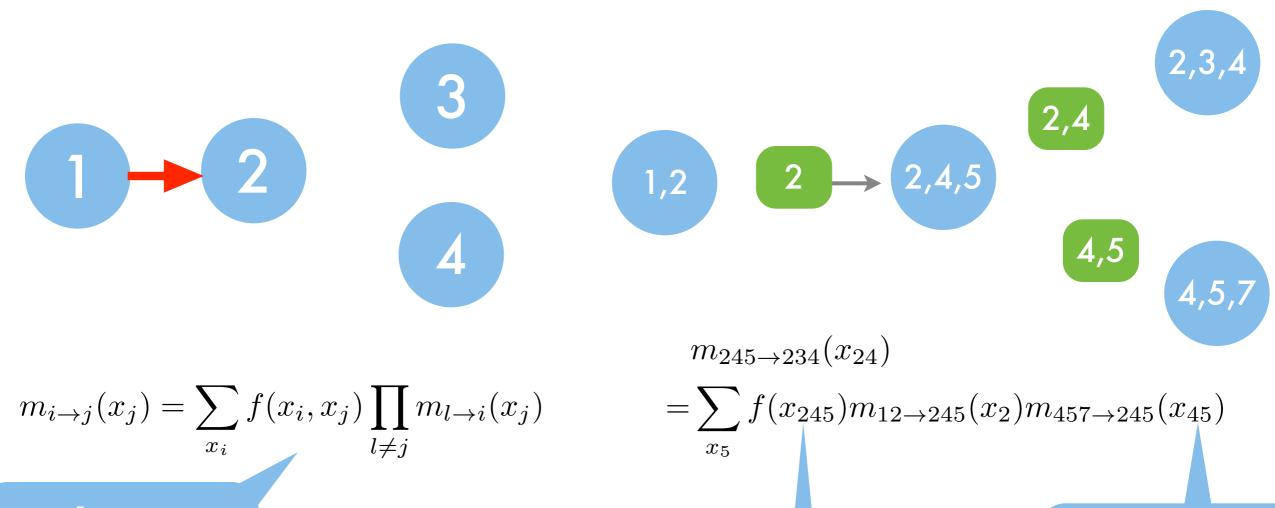
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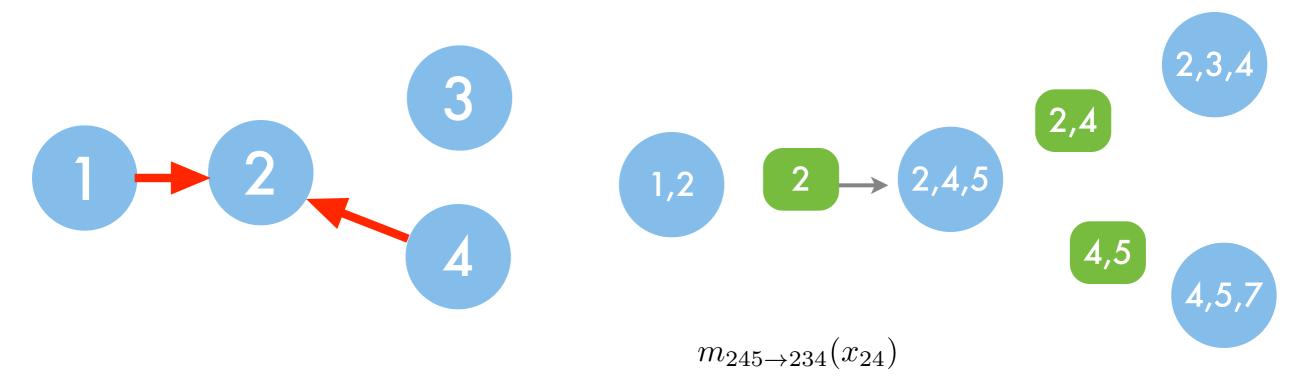
clique potential

clique potential



clique potential

clique potential

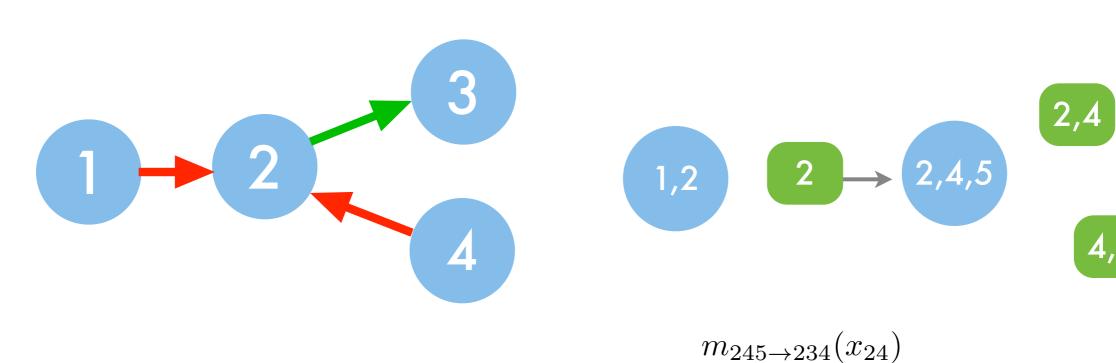


$$m_{i \to j}(x_j) = \sum_{x_i} f(x_i, x_j) \prod_{l \neq j} m_{l \to i}(x_j)$$

clique potential

clique potential

 $= \sum f(x_{245}) m_{12 \to 245}(x_2) m_{457 \to 245}(x_{45})$ 



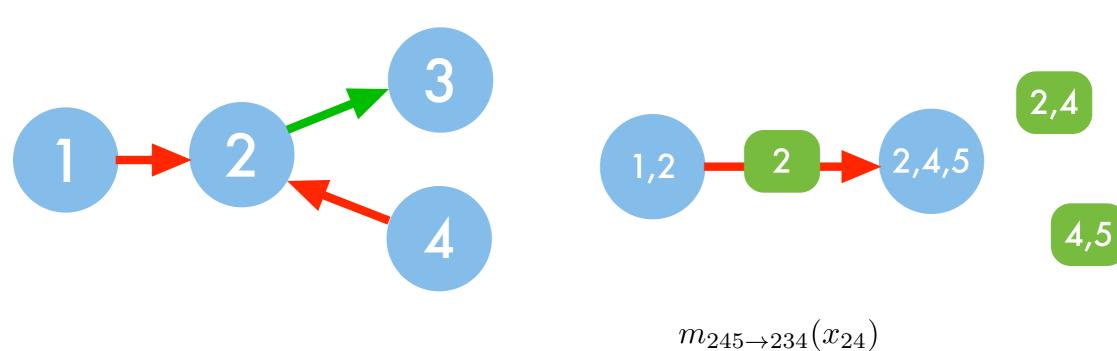
2,3,4

$$m_{i \to j}(x_j) = \sum_{x_i} f(x_i, x_j) \prod_{l \neq j} m_{l \to i}(x_j)$$

$$= \sum_{x_5} f(x_{245}) m_{12 \to 245}(x_2) m_{457 \to 245}(x_{45})$$

clique potential

clique potential



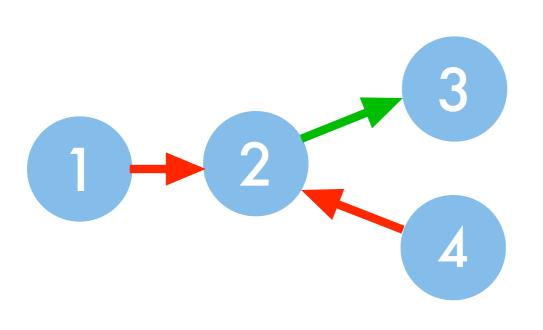
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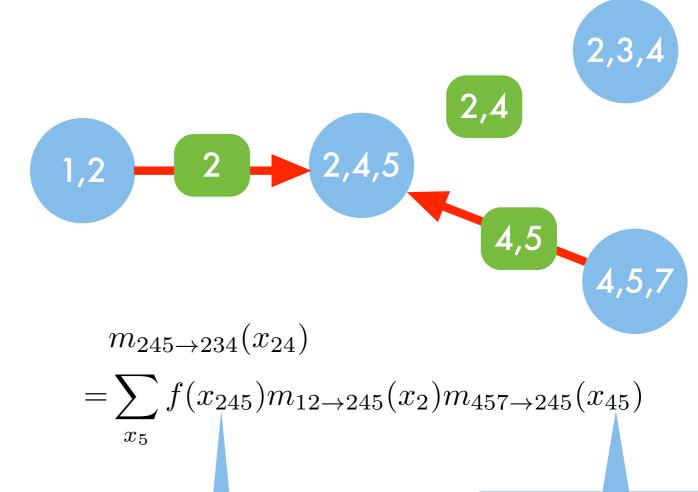
clique potential

clique potential

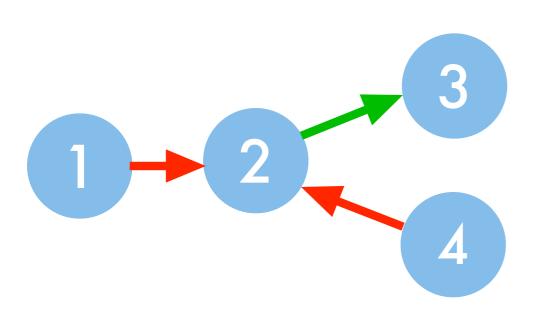


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clique potential

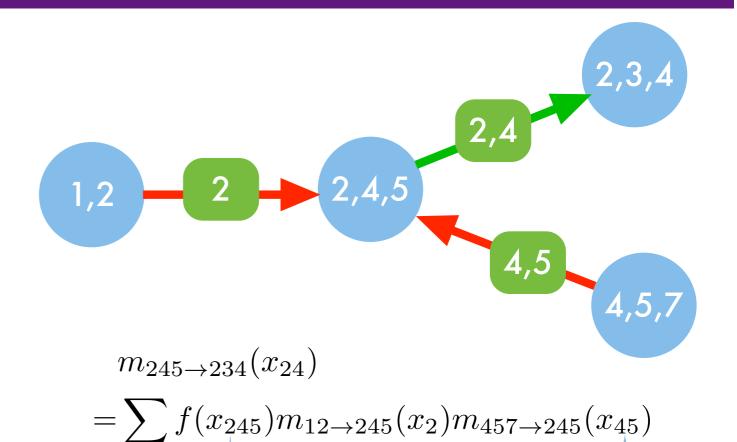


clique potential

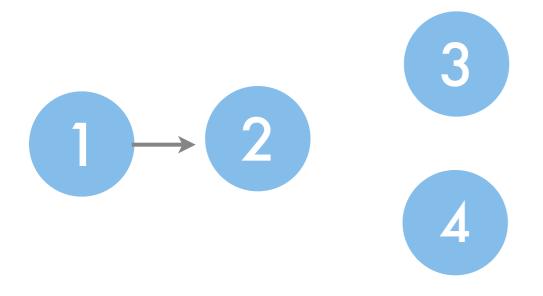


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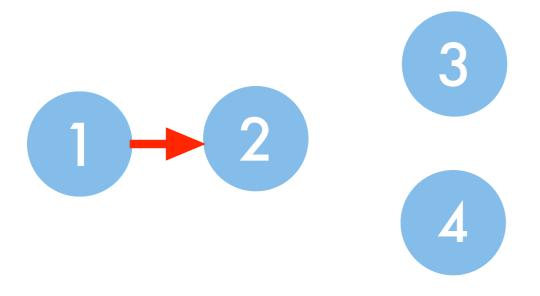
clique potential



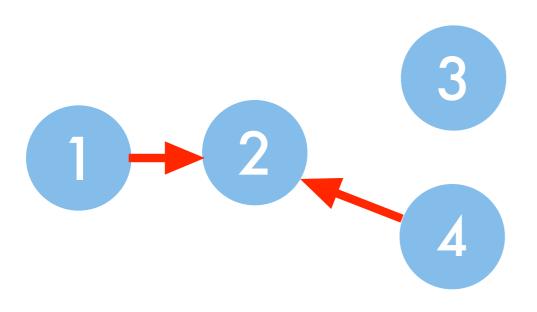
clique potential



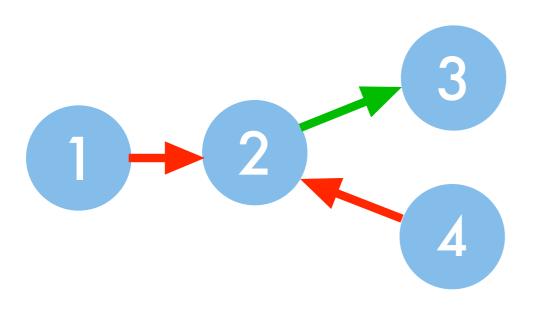
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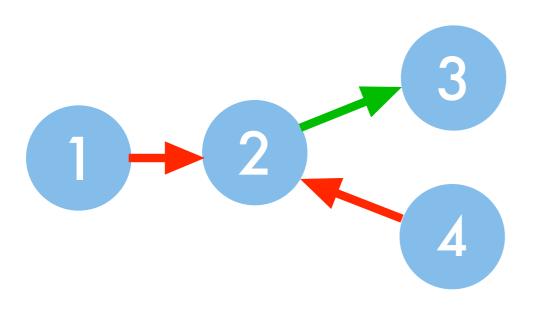
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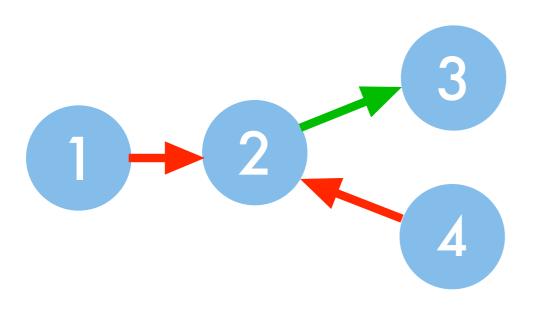
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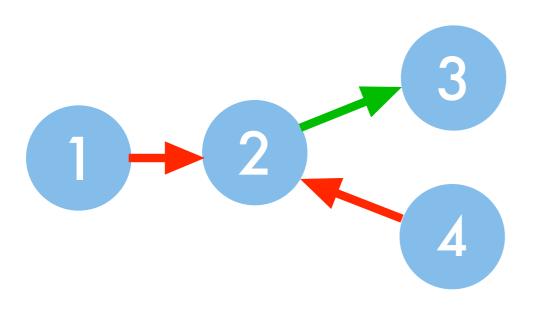
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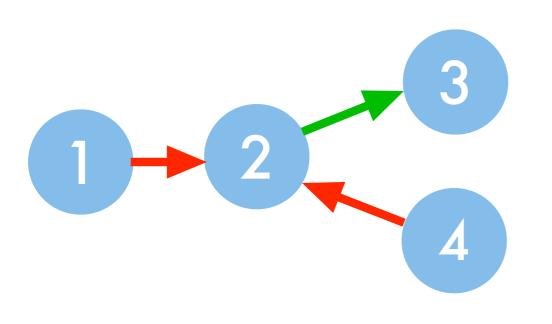
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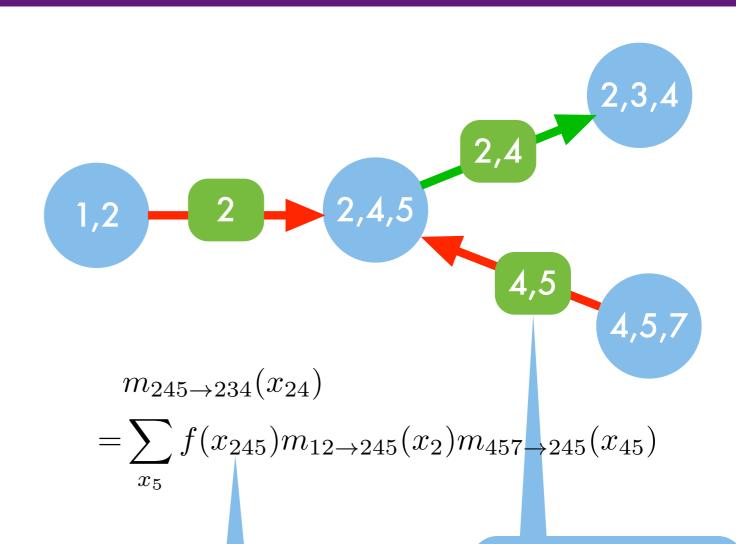


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clique potential



clique potential separator set

#### Generalized Distributive Law

- Key Idea
   Dynamic programming uses only sums and multiplications, hence replace them with equivalent operations from other semirings
- Semiring
  - 'addition' and 'summation' equivalent
  - Associative law: (a+b)+c = a+(b+c)
  - Distributive law: a(b+c) = ab + ac

#### Generalized Distributive Law

Integrating out probabilities (sum, product)

$$a \cdot (b+c) = a \cdot b + a \cdot c$$

Finding the maximum (max, +)

$$a + \max(b, c) = \max(a + b, a + c)$$

Set algebra (union, intersection)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- Boolean semiring (AND, OR)
- Probability semiring (log +, +)
- Tropical semiring (min, +)

# Chains ... again

$$\bar{s} = \max_{x} s(x_0) + \sum_{i=1}^{n-1} s(x_{i+1}|x_i)$$
  $x_0 \to x_1 \to x_2 \to x_3$ 

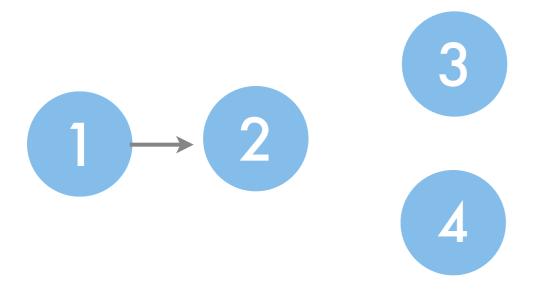
$$\bar{s} = \max_{x_{0...n}} \underbrace{s(x_0)}_{s(x_0)} + \sum_{j=1}^n s(x_j|x_{j-1})$$

$$= \max_{x_1...n} \max_{x_0} \left[ l_0(x_0) s(x_1|x_0) \right] + \sum_{j=2}^n s(x_j|x_{j-1})$$

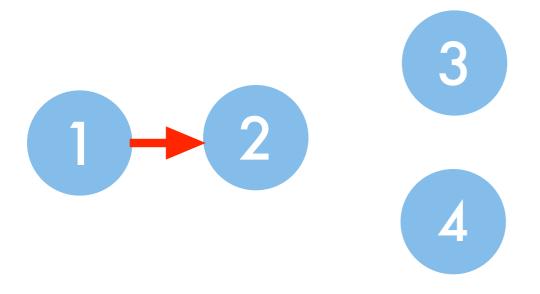
$$:= l_1(x_1)$$

$$= \max_{x_{2...n}} \max_{x_1} \left[ l_1(x_1) s(x_2|x_1) \right] + \sum_{j=3}^n s(x_j|x_{j-1})$$

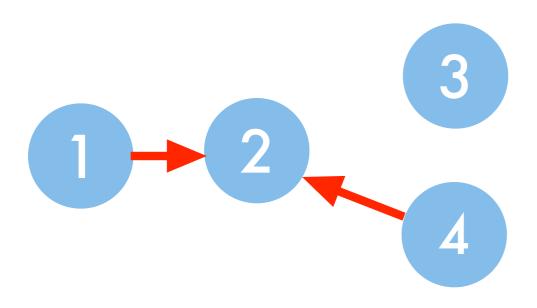
$$:= l_2(x_2)$$



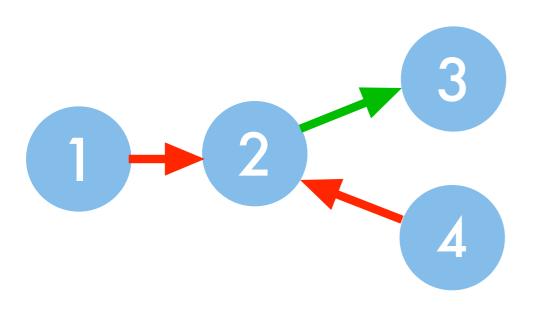
$$m_{i \to j}(x_j) = \max_{x_i} f(x_i, x_j) + \sum_{l \neq j} m_{l \to i}(x_j)$$



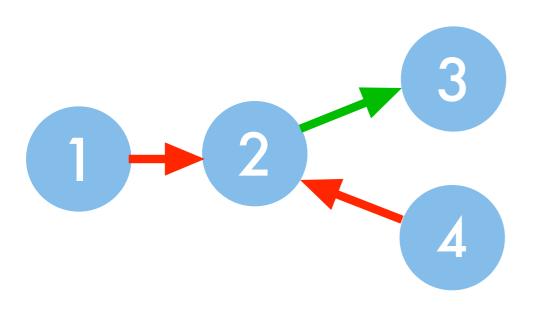
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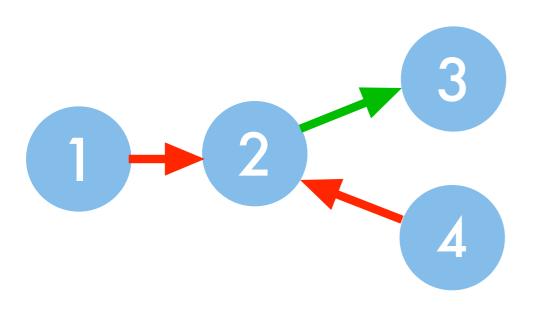
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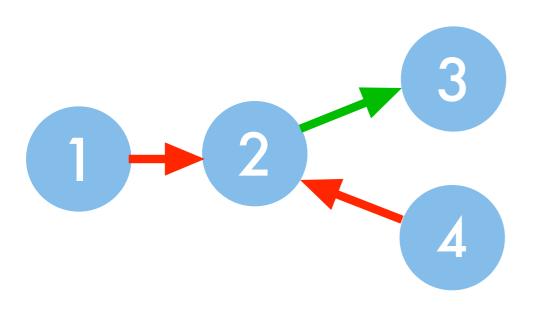
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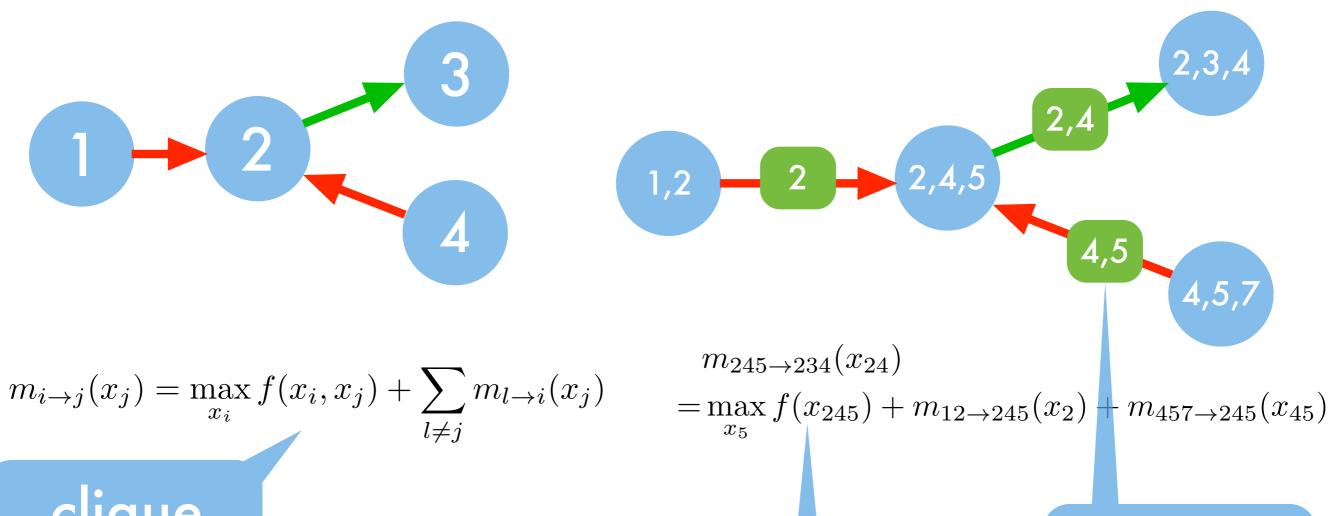
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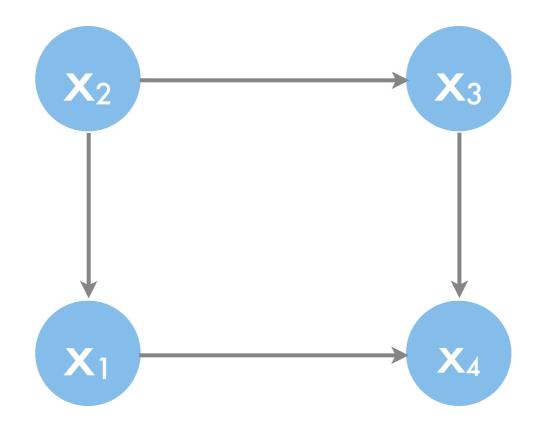
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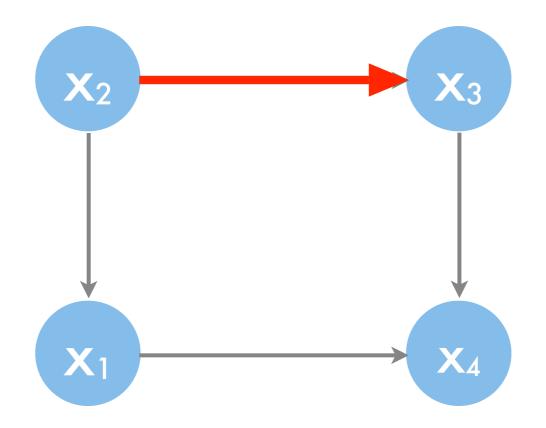
clique potential

clique potential separator set

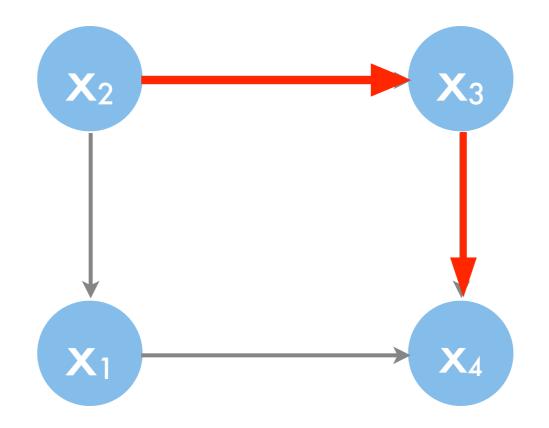
$$s(x_1, x_2) + s(x_2, x_3) + s(x_3, x_4) + s(x_4, x_1)$$



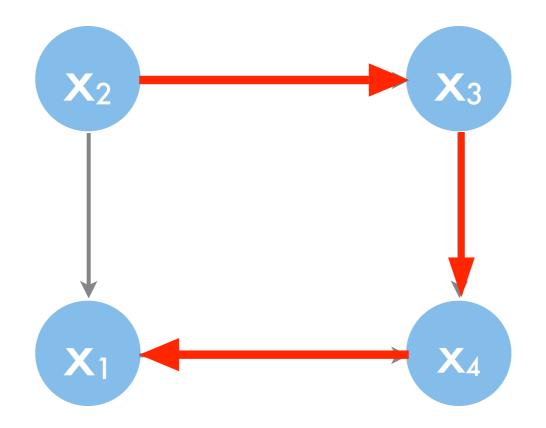
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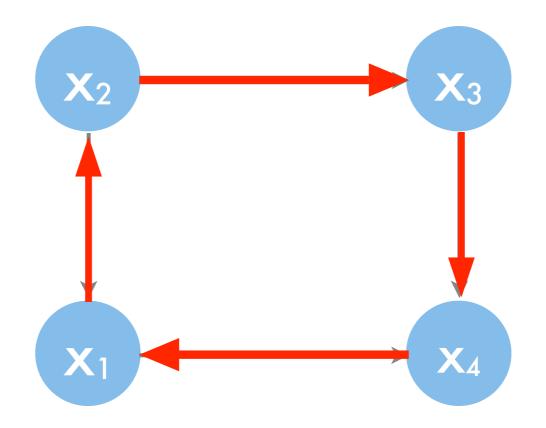
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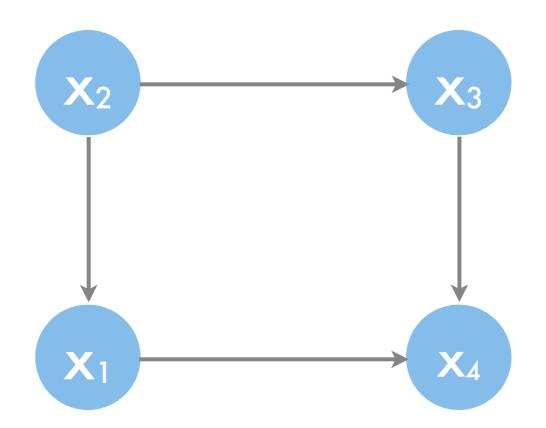
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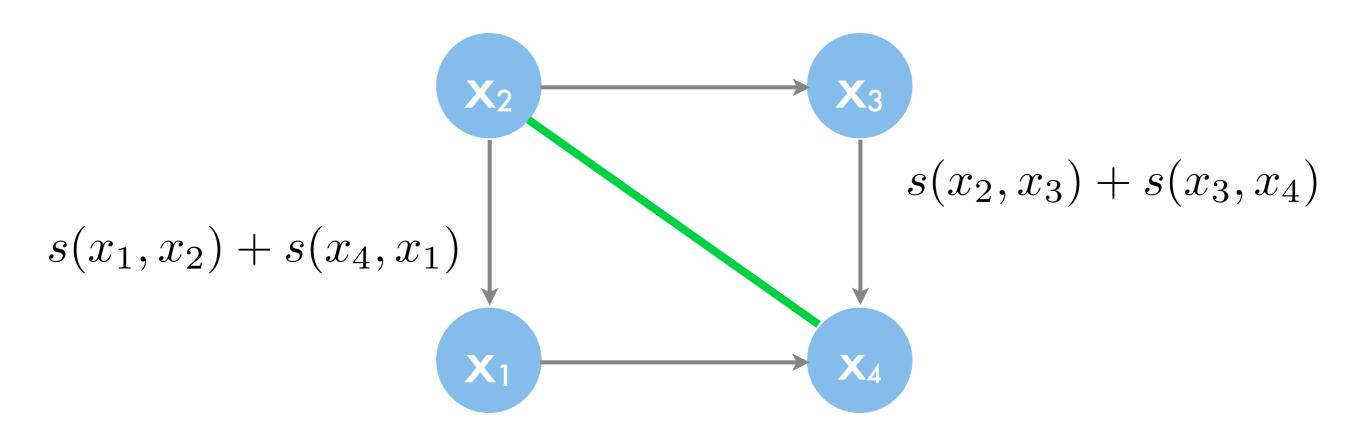
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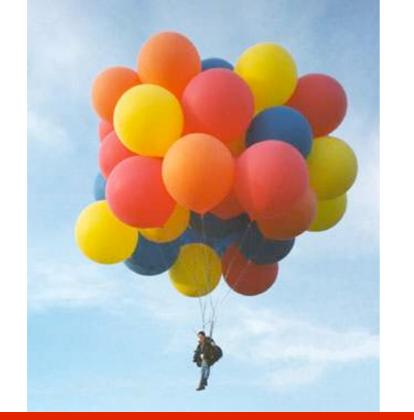


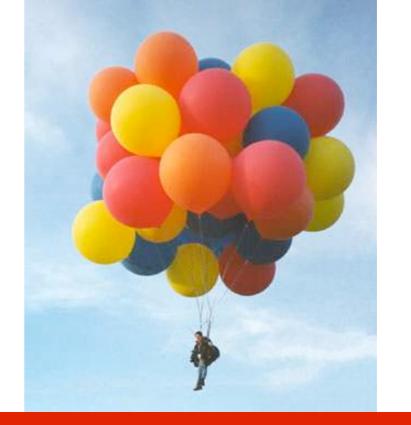
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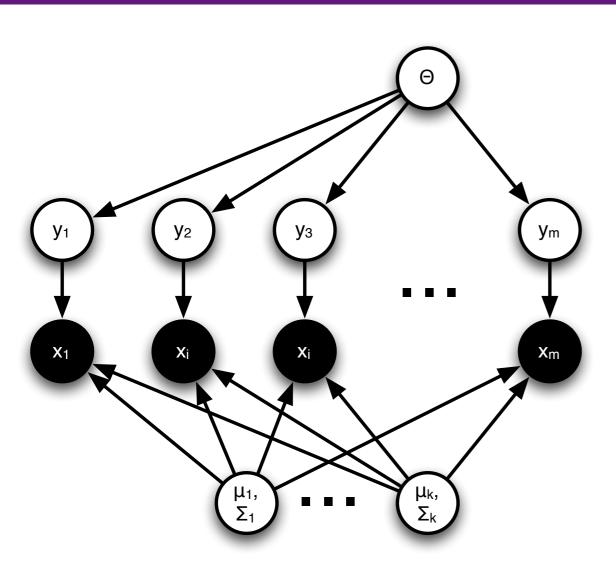


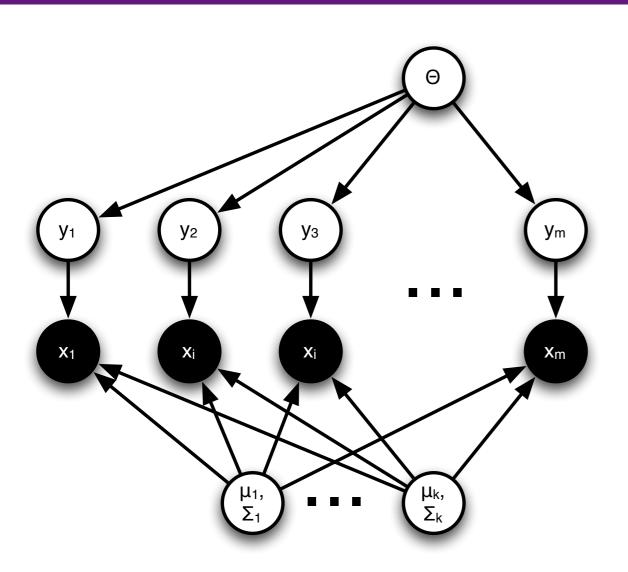


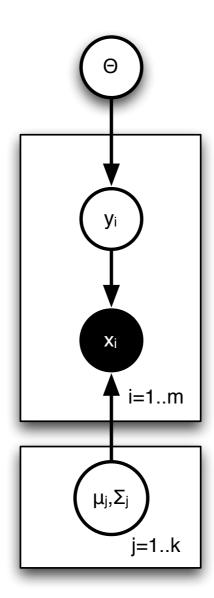


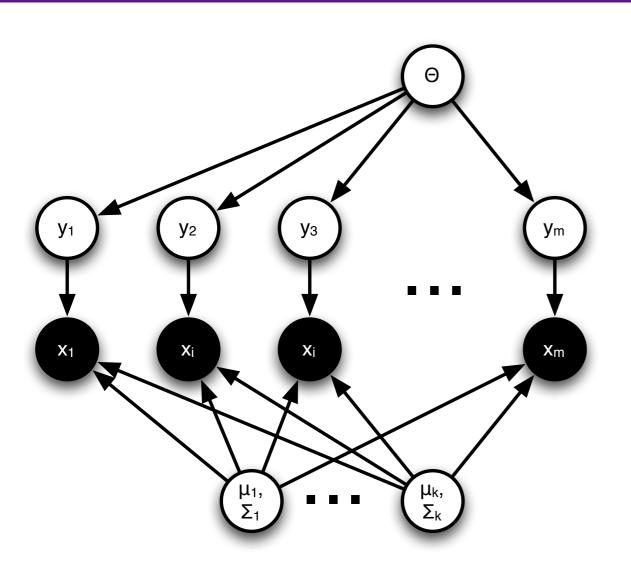
# Clustering

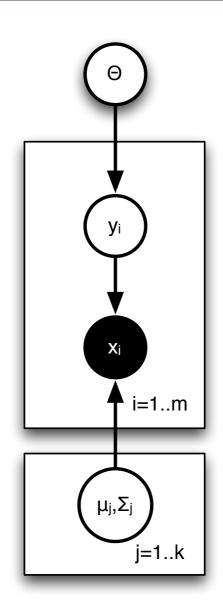


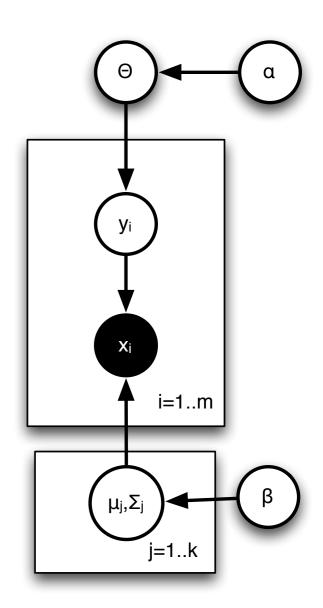


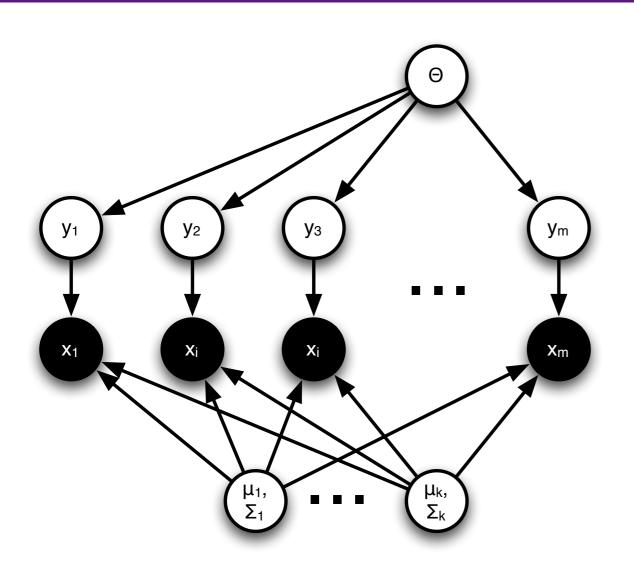


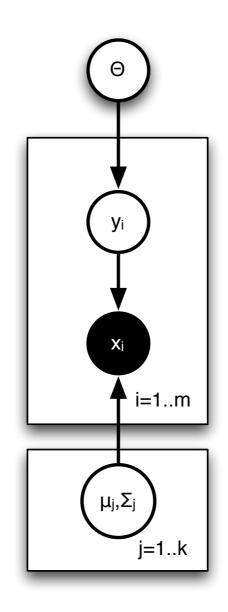


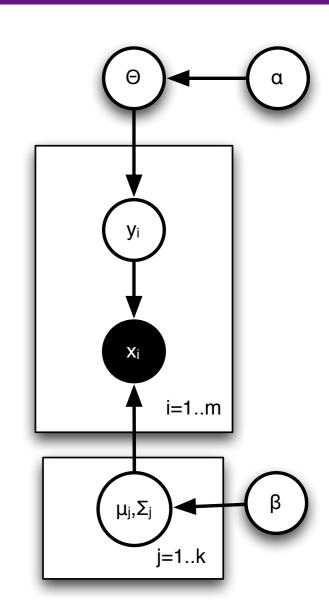








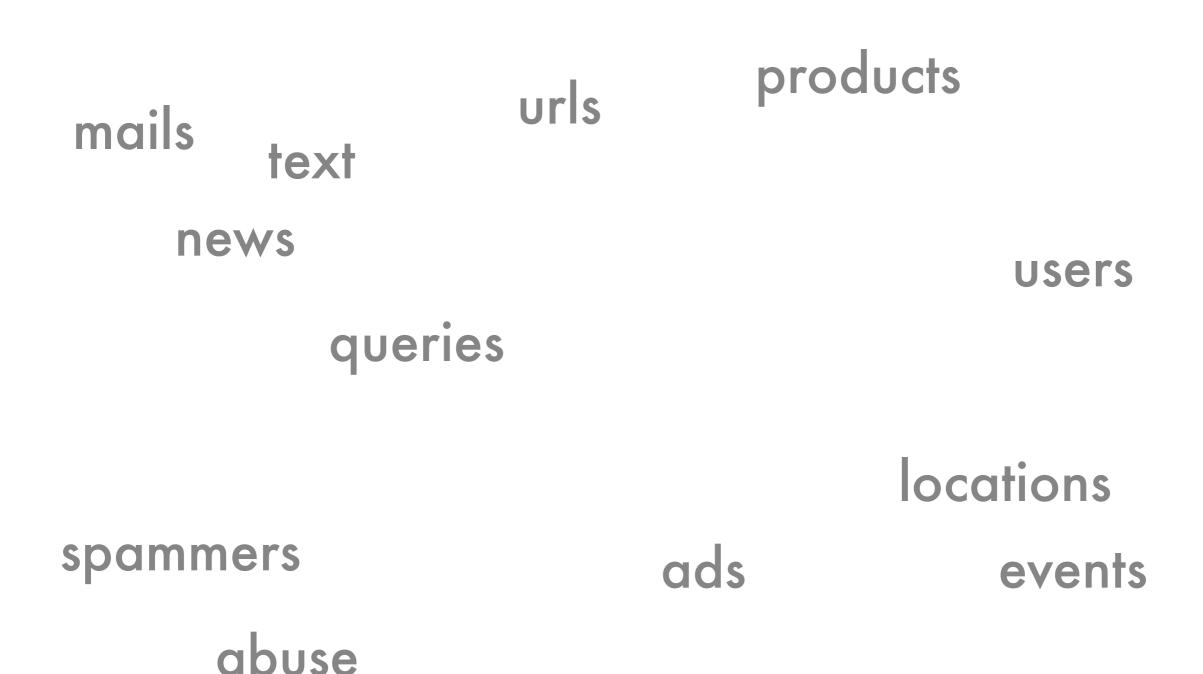




$$p(X, Y | \theta, \sigma, \mu) = \prod_{i=1}^{n} p(x_i | y_i, \sigma, \mu) p(y_i | \theta)$$

### What can we cluster?

### What can we cluster?



### Mixture of Gaussians

- Draw cluster label y from discrete distribution
- Draw data x from Gaussian for given cluster y
- Prior for discrete distribution Dirichlet
- Prior for Gaussians Gauss-Wishart distribution

- Problem: we don't know y
  - If we knew the parameters we could get y
  - If we knew y we could get the parameters

### k-means

- Fixed uniform variance for all Gaussians
- Fixed uniform distribution over clusters
- Initialize centers with random subset of points
- Find most likely cluster y for x

$$y_i = \underset{y}{\operatorname{argmax}} p(x_i|y, \sigma, \mu)$$

Find most likely center for given cluster

$$\mu_y = \frac{1}{n_y} \sum_i \{y_i = y\} x_i$$

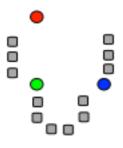
Repeat until converged

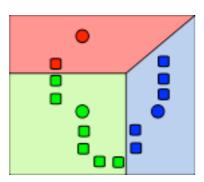
### k-means

- Pro
  - simple algorithm
  - can be implemented by MapReduce passes
- Con
  - no proper probabilistic representation
  - can get stuck easily in local minima

#### k-means

#### partitioning



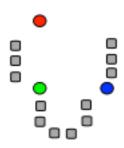


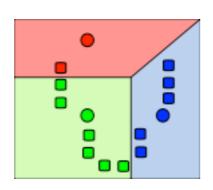
initialization

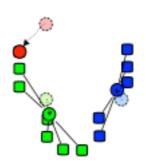
#### k-means

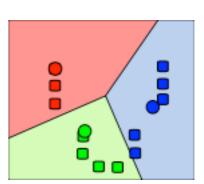
partitioning

partitioning









initialization

update

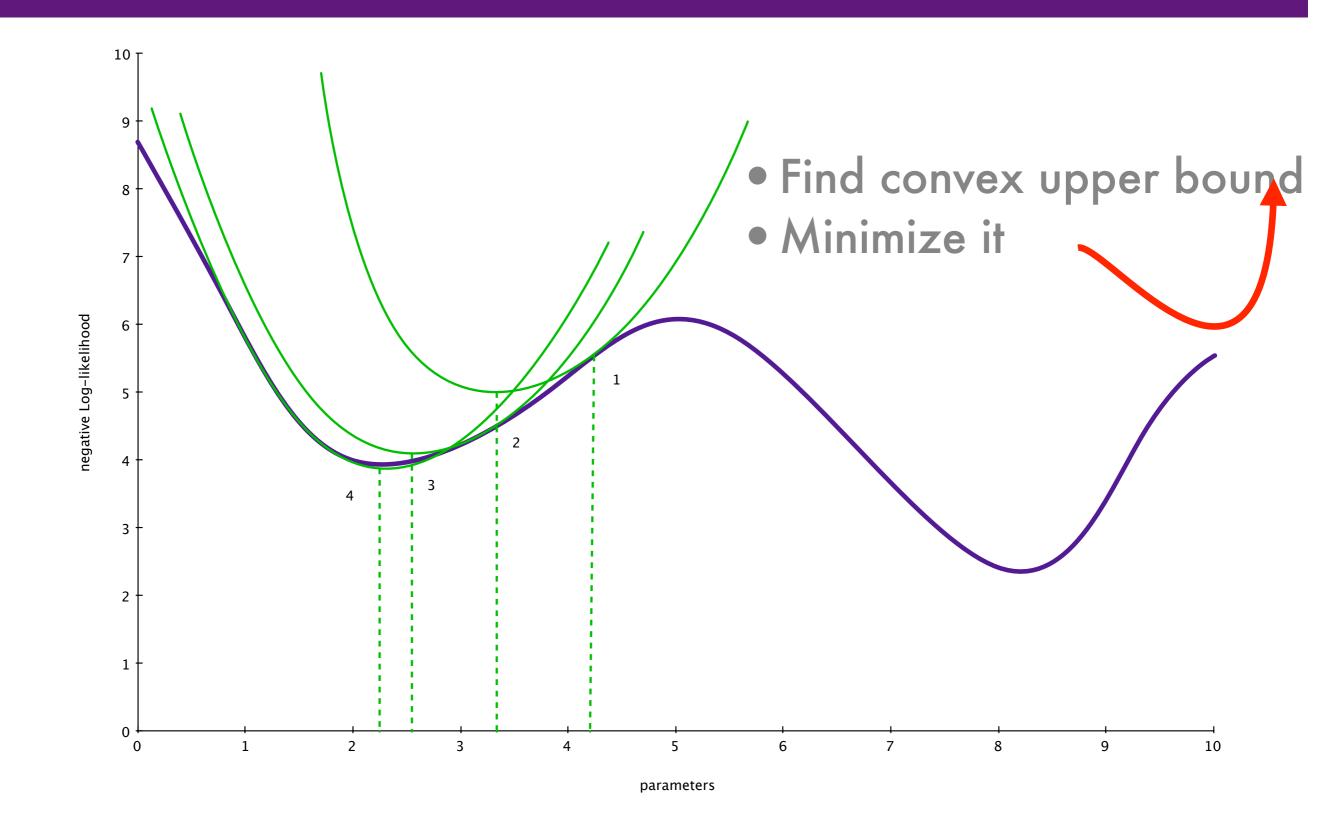
Optimization Problem

$$\underset{\theta,\mu,\sigma}{\text{maximize}} p(X|\theta,\sigma,\mu) = \underset{\theta,\mu,\sigma}{\text{maximize}} \sum_{Y} \prod_{i=1} p(x_i|y_i,\sigma,\mu) p(y_i|\theta)$$

This problem is nonconvex and difficult to solve

Key idea
 If we knew p(y|x) we could estimate the remaining parameters easily and vice versa

#### Nonconvex Minimization



#### Variational Bound

$$\log p(x;\theta) \ge \log p(x;\theta) - D(q(y)||p(y|x;\theta))$$

$$= \int dq(y) \left[\log p(x;\theta) + \log p(y|x;\theta) - \log q(y)\right]$$

$$= \int dq(y) \log p(x,y;\theta) - \int dq(y) \log q(y)$$

This inequality is tight for p(y|x) = q(y)

#### Variational Bound

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#### This inequality is tight for p(y|x) = q(y)

Expectation step

$$q(y) = p(y|x;\theta)$$

Variational Bound

$$\log p(x;\theta) \ge \log p(x;\theta) - D(q(y)||p(y|x;\theta))$$

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This inequality is tight for p(y|x) = q(y)

Expectation step

find bound

$$q(y) = p(y|x;\theta)$$

Variational Bound

$$\log p(x;\theta) \ge \log p(x;\theta) - D(q(y)||p(y|x;\theta))$$

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This inequality is tight for p(y|x) = q(y)

Expectation step

find bound

$$q(y) = p(y|x;\theta)$$

Maximization step

$$\theta^* = \underset{\theta}{\operatorname{argmax}} \int dq(y) \log p(x, y; \theta)$$

#### Variational Bound

$$\log p(x;\theta) \ge \log p(x;\theta) - D(q(y)||p(y|x;\theta))$$

$$= \int dq(y) \left[\log p(x;\theta) + \log p(y|x;\theta) - \log q(y)\right]$$

$$= \int dq(y) \log p(x,y;\theta) - \int dq(y) \log q(y)$$

#### This inequality is tight for p(y|x) = q(y)

Expectation step

find bound

$$q(y) = p(y|x;\theta)$$

Maximization step

maximize if 
$$\theta^* = \operatorname*{argmax}_{\theta} \int dq(y) \log p(x,y;\theta)$$

#### Expectation Step

Factorizing distribution

$$q(Y) = \prod_{i} q_i(y)$$

E-Step

$$q_i(y) \propto p(x_i|y_i, \mu, \sigma)p(y_i|\theta)$$
 hence

$$m_{iy} := \frac{1}{(2\pi)^{\frac{d}{2}} |\Sigma_y|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x_i - \mu_y)\Sigma_y^{-1}(x_i - \mu_y)\right] p(y)$$

$$q_i(y) = \frac{m_{iy}}{\sum_{y'} m_{iy'}}$$

#### Maximization Step

#### Log-likelihood

$$\log p(X, Y | \theta, \mu, \sigma) = \sum_{i=1}^{n} \log p(x_i | y_i, \mu, \sigma) + \log p(y_i | \theta)$$

 Cluster distribution (weighted Gaussian MLE)

$$n_y = \sum_i q_i(y)$$

$$\mu_{y} = \frac{1}{n_{y}} \sum_{i=1}^{n} q_{i}(y) x_{i}$$

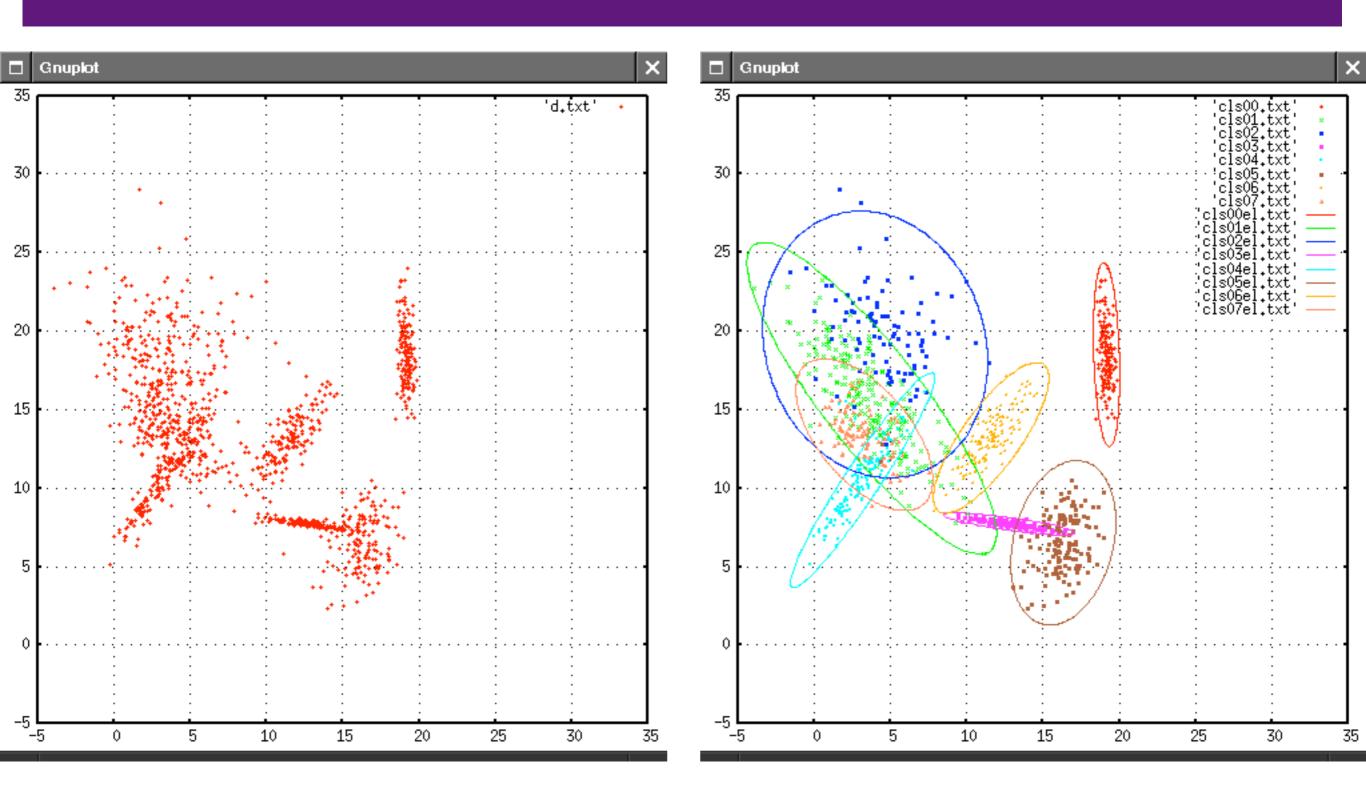
$$\Sigma_{y} = \frac{1}{n_{y}} \sum_{i=1}^{n} q_{i}(y) x_{i} x_{i}^{\top} - \mu_{y} \mu_{y}^{\top}$$

Cluster probabilities

$$\theta^* = \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{\infty} \sum_{y} q_i(y) \log p(y_i|\theta) \text{ hence } p(y|\theta) = \frac{n_y}{n}$$

$$p(y|\theta) = \frac{n_y}{n}$$

## EM Clustering in action



#### Problem

Estimates will diverge (infinite variance, zero probability, tiny clusters)

#### Solution

- Use priors for  $\mu, \sigma, \theta$ 
  - Dirichlet distribution for cluster probabilities
  - Gauss-Wishart for Gaussian
- Cluster distribution

$$n_y = n_0 + \sum_i q_i(y)$$

$$\mu_y = \frac{1}{n_y} \sum_{i=1}^n q_i(y) x_i$$

• Cluster distribution 
$$\mu_y = \frac{1}{n_y} \sum_{i=1}^n q_i(y) x_i$$
 
$$n_y = \frac{1}{n_0} + \sum_i q_i(y)$$
 
$$\sum_y = \frac{1}{n_y} \sum_{i=1}^n q_i(y) x_i x_i^\top + \frac{n_0}{n_y} \mathbf{1} - \mu_y \mu_y^\top$$
 • Cluster probabilities 
$$p(y|\theta) = \frac{n_y}{n + k \cdot n_0}$$

$$p(y|\theta) = \frac{n_y}{n + k \cdot n_0}$$

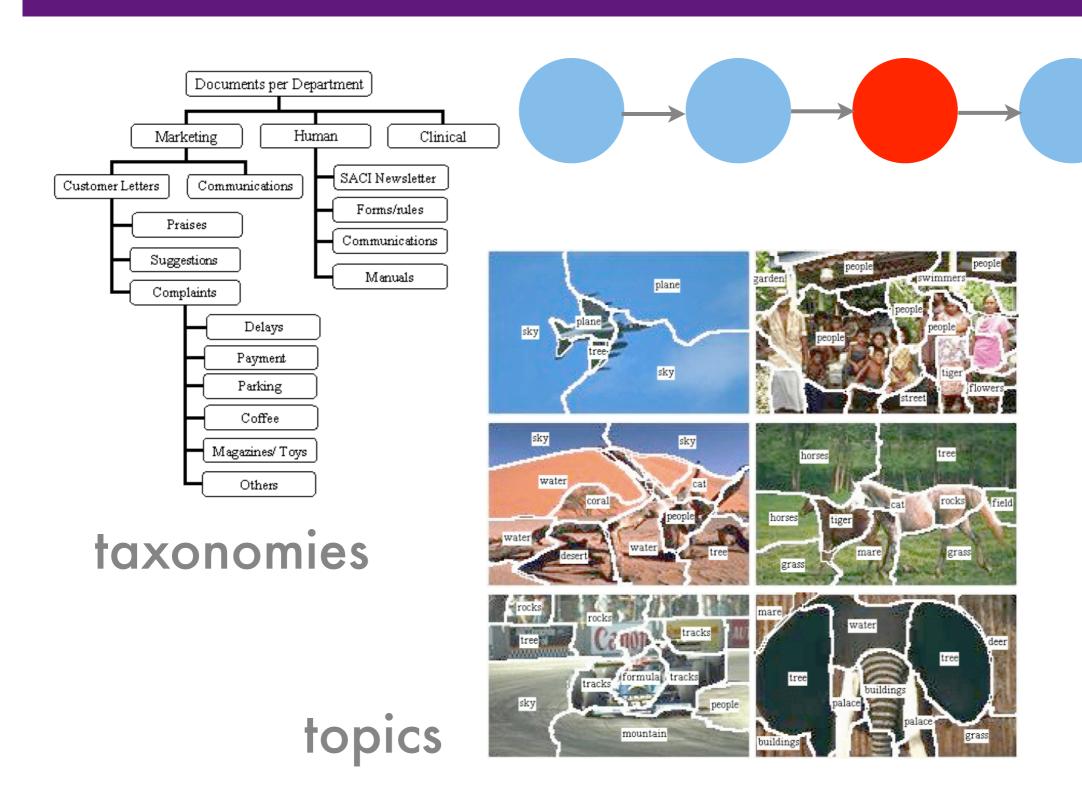
## Variational Approximation

Lower bound on log-likelihood

$$\log p(x;\theta) \ge \int dq(y) \log p(x,y;\theta) - \int dq(y) \log q(y)$$

- Key insight inequality holds for any q
  - Find q within subset Q to tighten inequality
  - Find parameters to maximize for fixed q
- Inference for graphical models where joint probability computation is infeasible

# Beyond mixtures

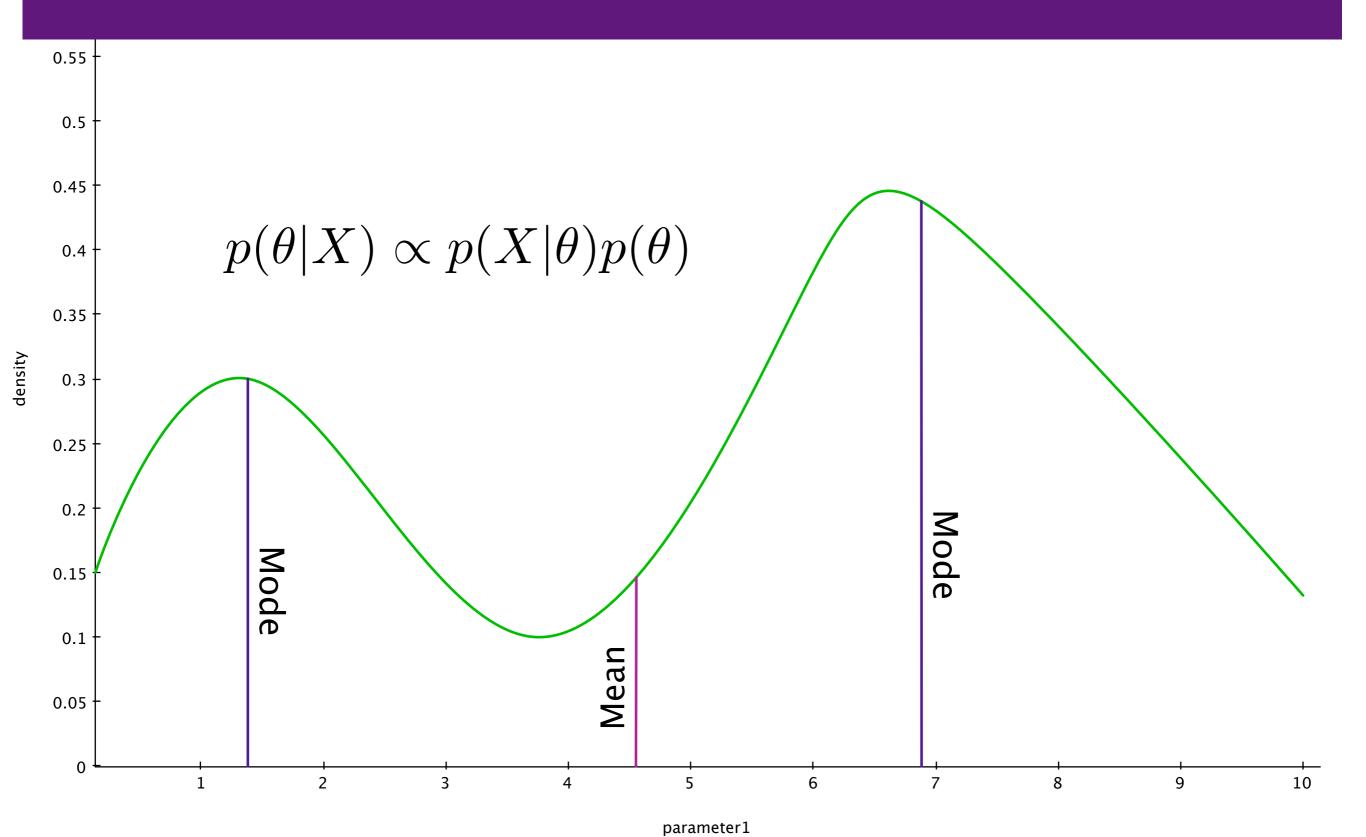


chains

# Sampling



#### Is maximization (always) good?



## Sampling

- Key idea
  - Want accurate distribution of the posterior
  - Sample from posterior distribution rather than maximizing it
- Problem direct sampling is usually intractable
- Solutions
  - Markov Chain Monte Carlo (complicated)
  - Gibbs Sampling (somewhat simpler)

) 
$$x \sim p(x|x')$$
 and then  $x' \sim p(x'|x)$ 

- Gibbs sampling:
  - In most cases direct sampling not possible
  - Draw one set of variables at a time

0.45	0.05
0.05	0.45

- Gibbs sampling:
  - In most cases direct sampling not possible
  - Draw one set of variables at a time

0.45	0.05
0.05	0.45

(b,g) - draw p(.,g)

- Gibbs sampling:
  - In most cases direct sampling not possible
  - Draw one set of variables at a time

0.45	0.05
0.05	0.45

- Gibbs sampling:
  - In most cases direct sampling not possible
  - Draw one set of variables at a time

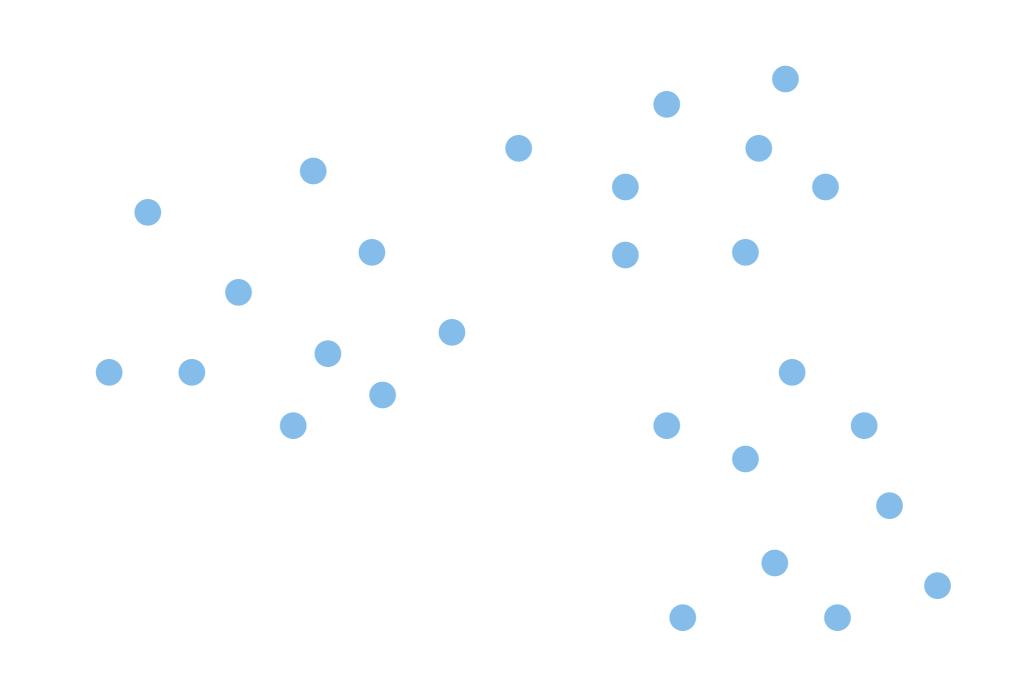
0.45	0.05
0.05	0.45

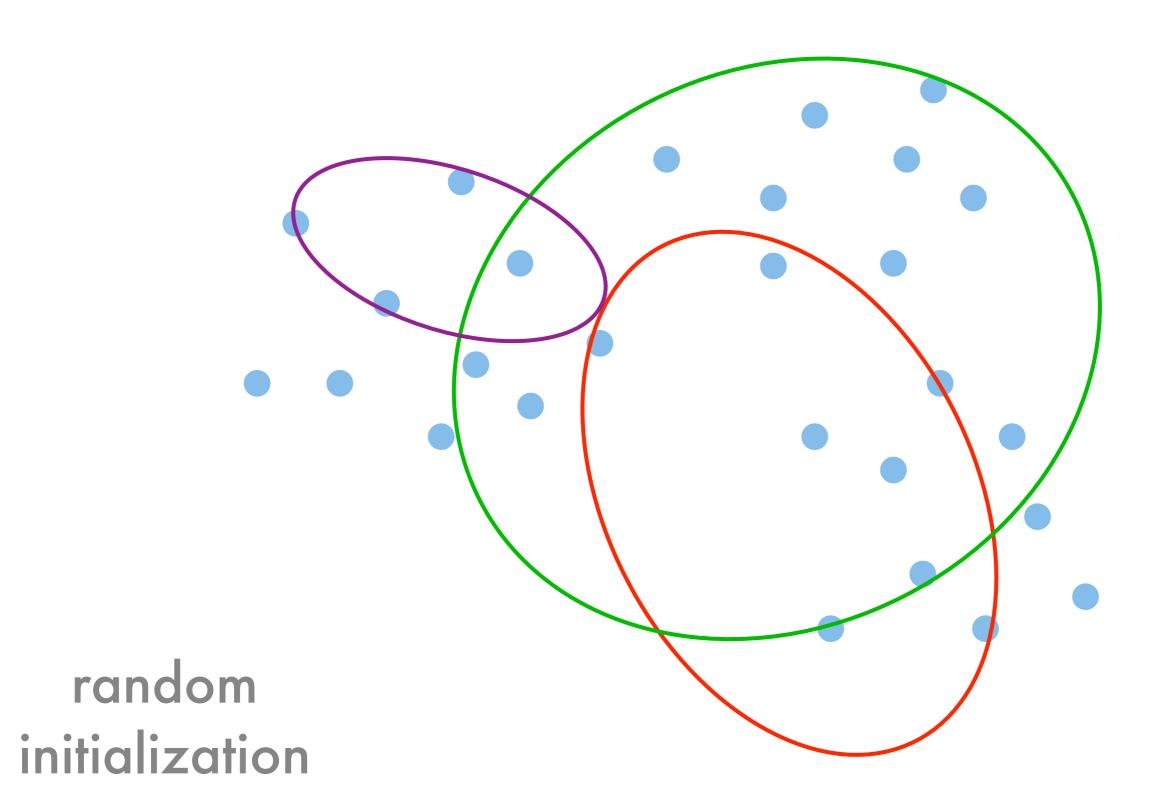
- Gibbs sampling:
  - In most cases direct sampling not possible
  - Draw one set of variables at a time

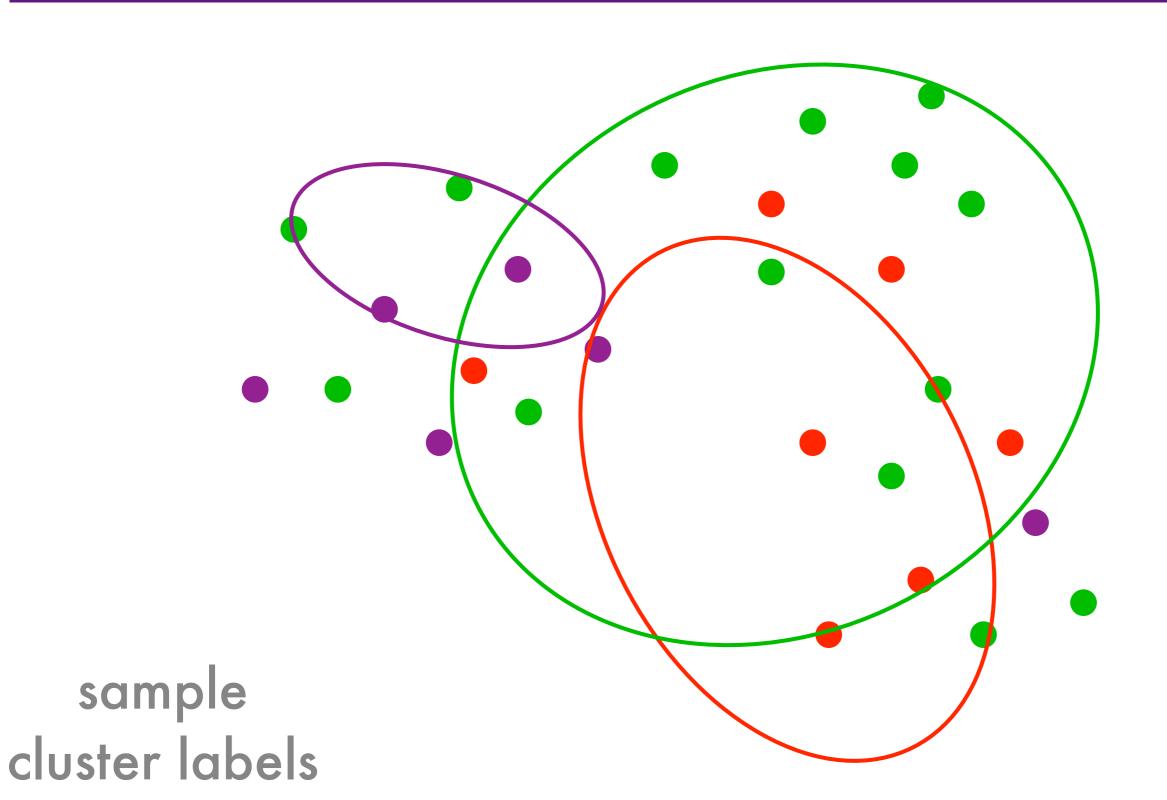
0.45	0.05
0.05	0.45

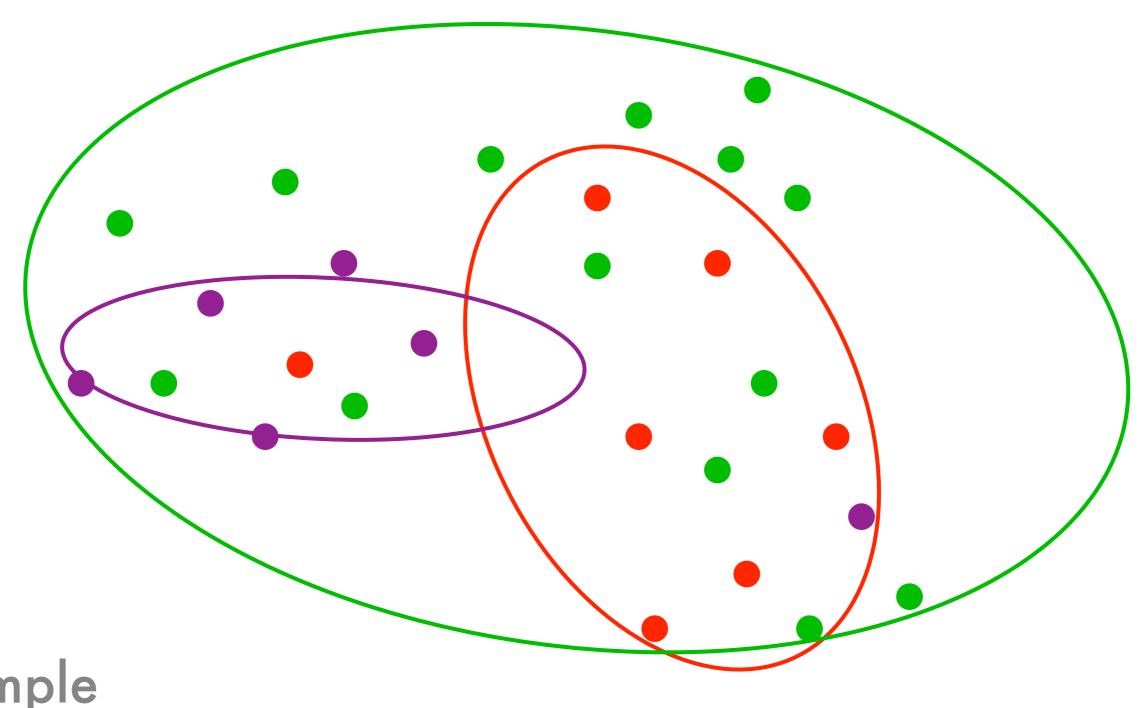
- Gibbs sampling:
  - In most cases direct sampling not possible
  - Draw one set of variables at a time

0.45	0.05
0.05	0.45

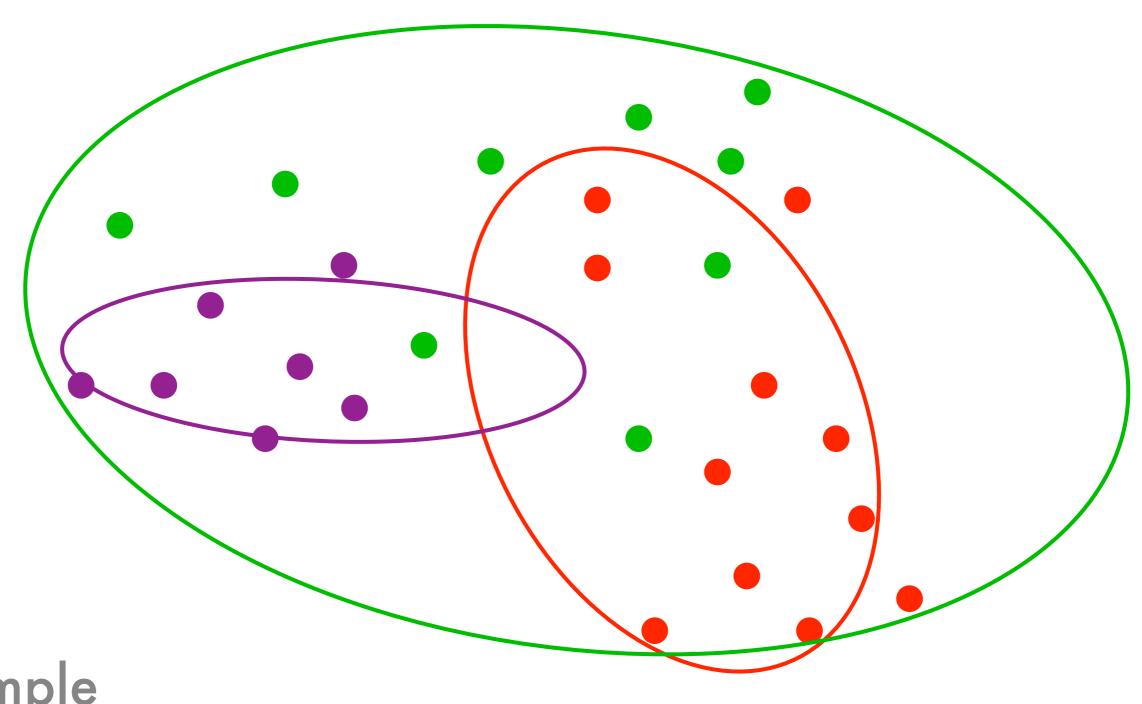




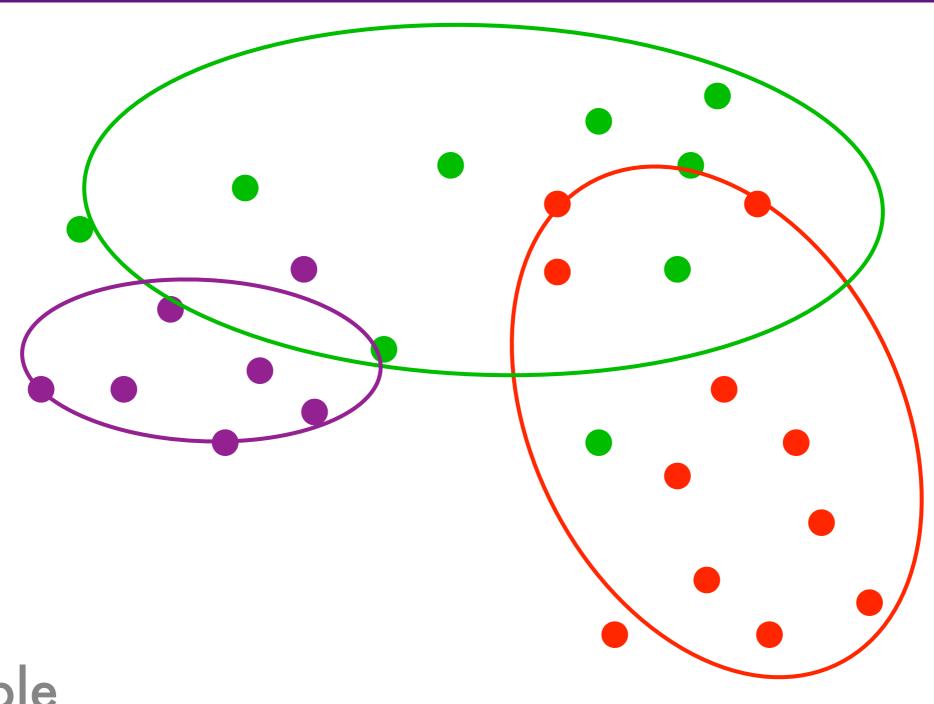




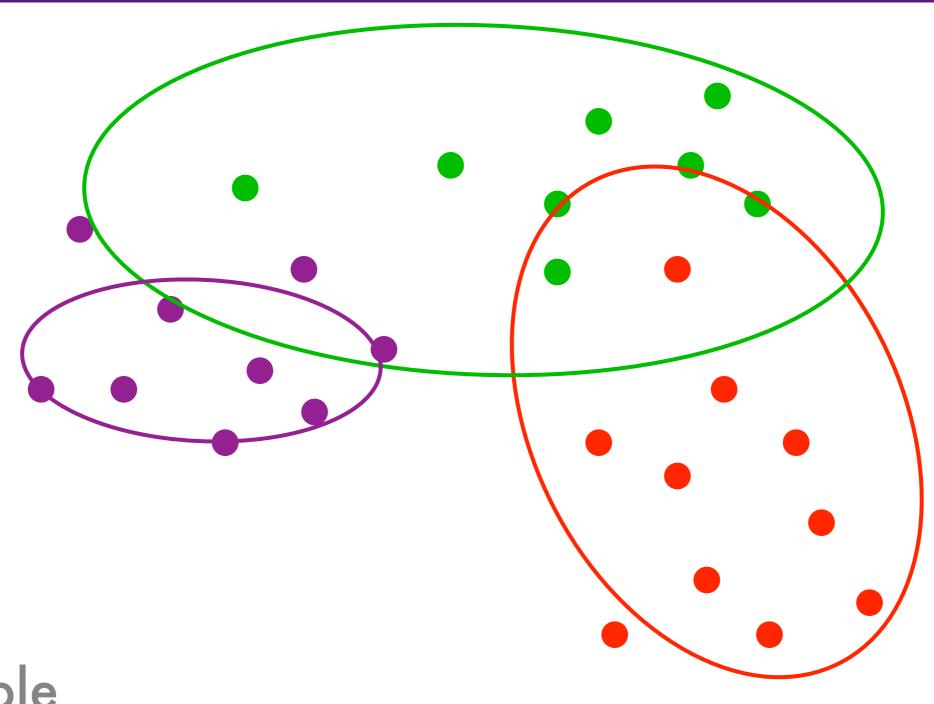
resample cluster model



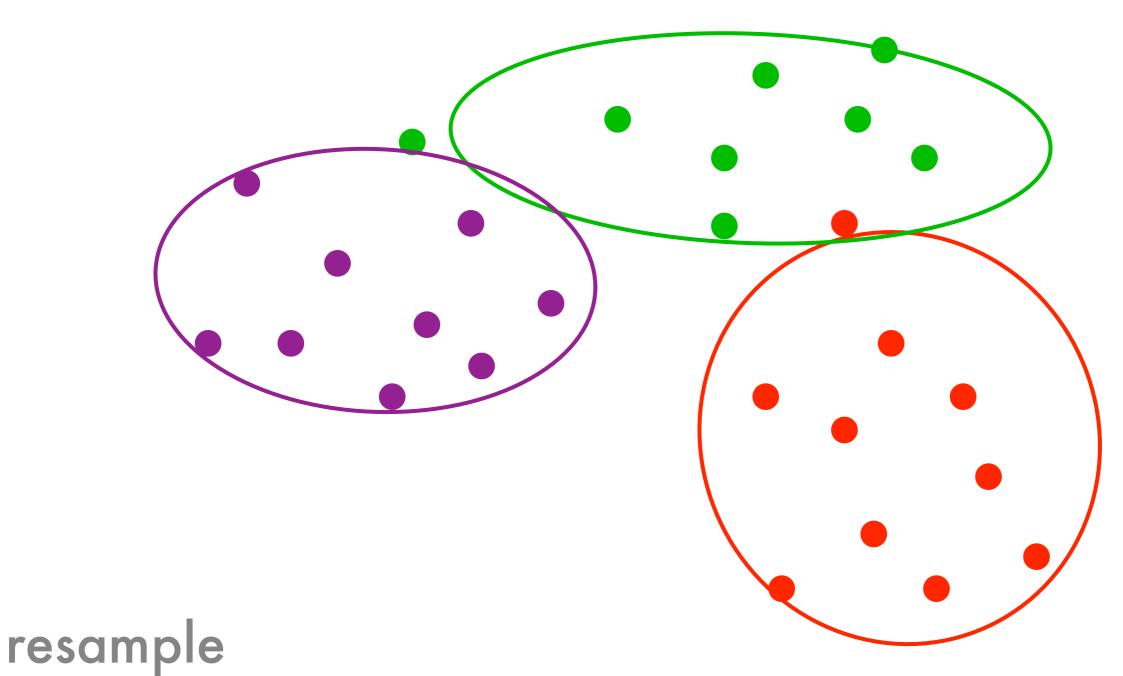
resample cluster labels



resample cluster model



resample cluster labels



cluster model e.g. Mahout Dirichlet Process Clustering

## Inference Algorithm ≠ Model

### Inference Algorithm ≠ Model

Corollary: EM ≠ Clustering

# Graphical Models Zoology





Models

**Statistics** 

### YAHOO

Inference Methods

Efficient Computation

Models

11, 12 Priors

Conjugate Prior

**Statistics** 

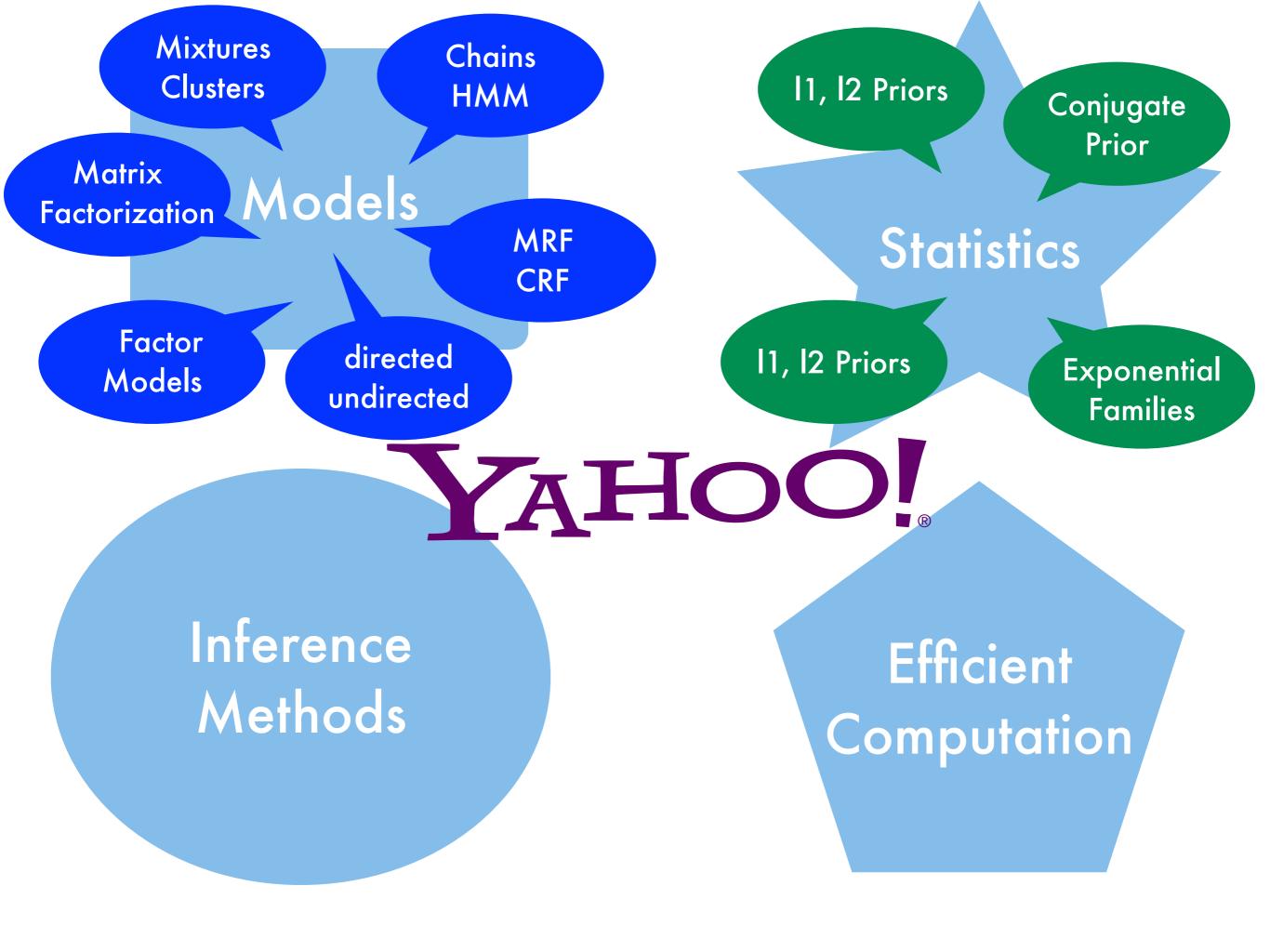
11, 12 Priors

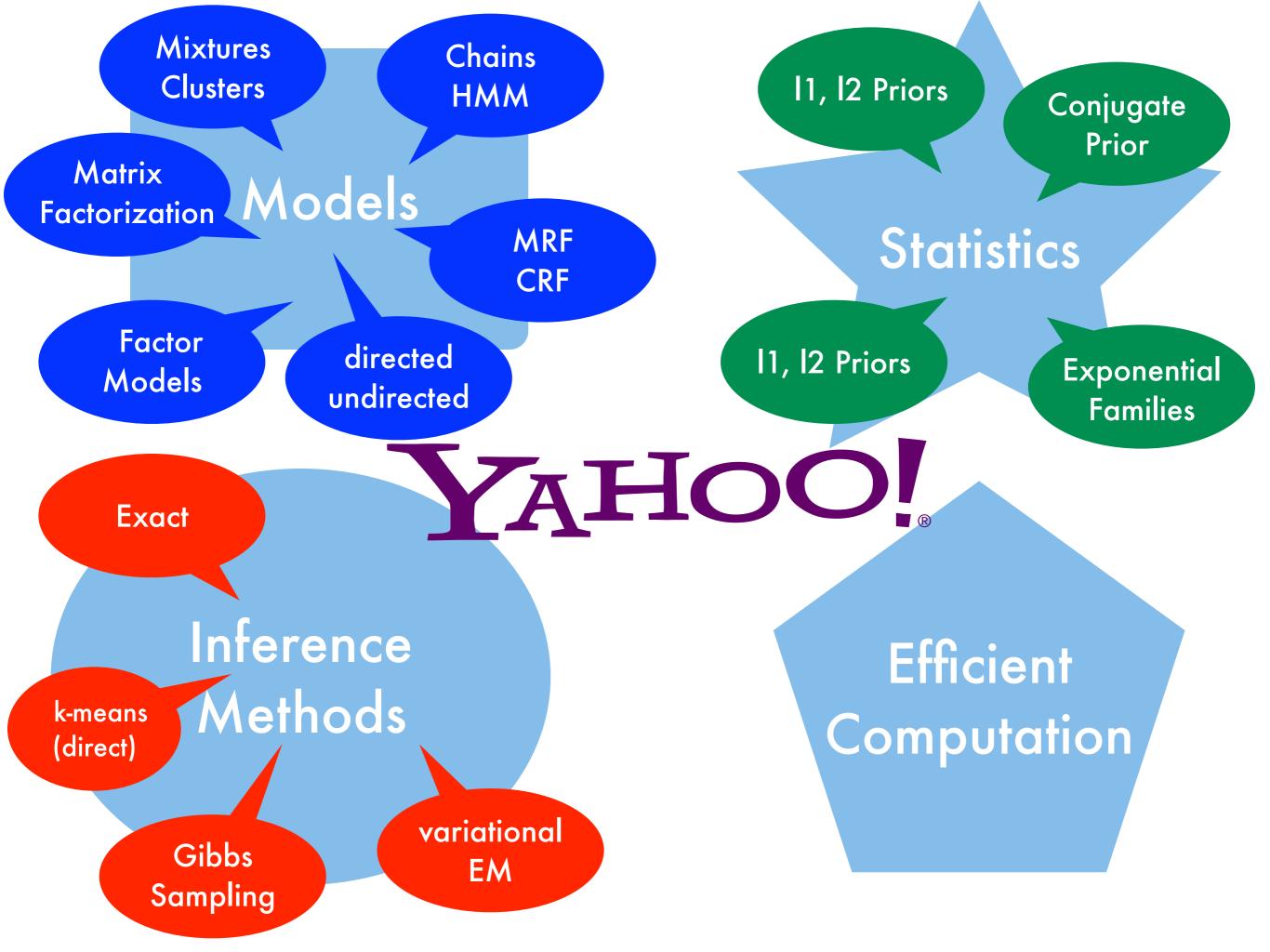
Exponential Families

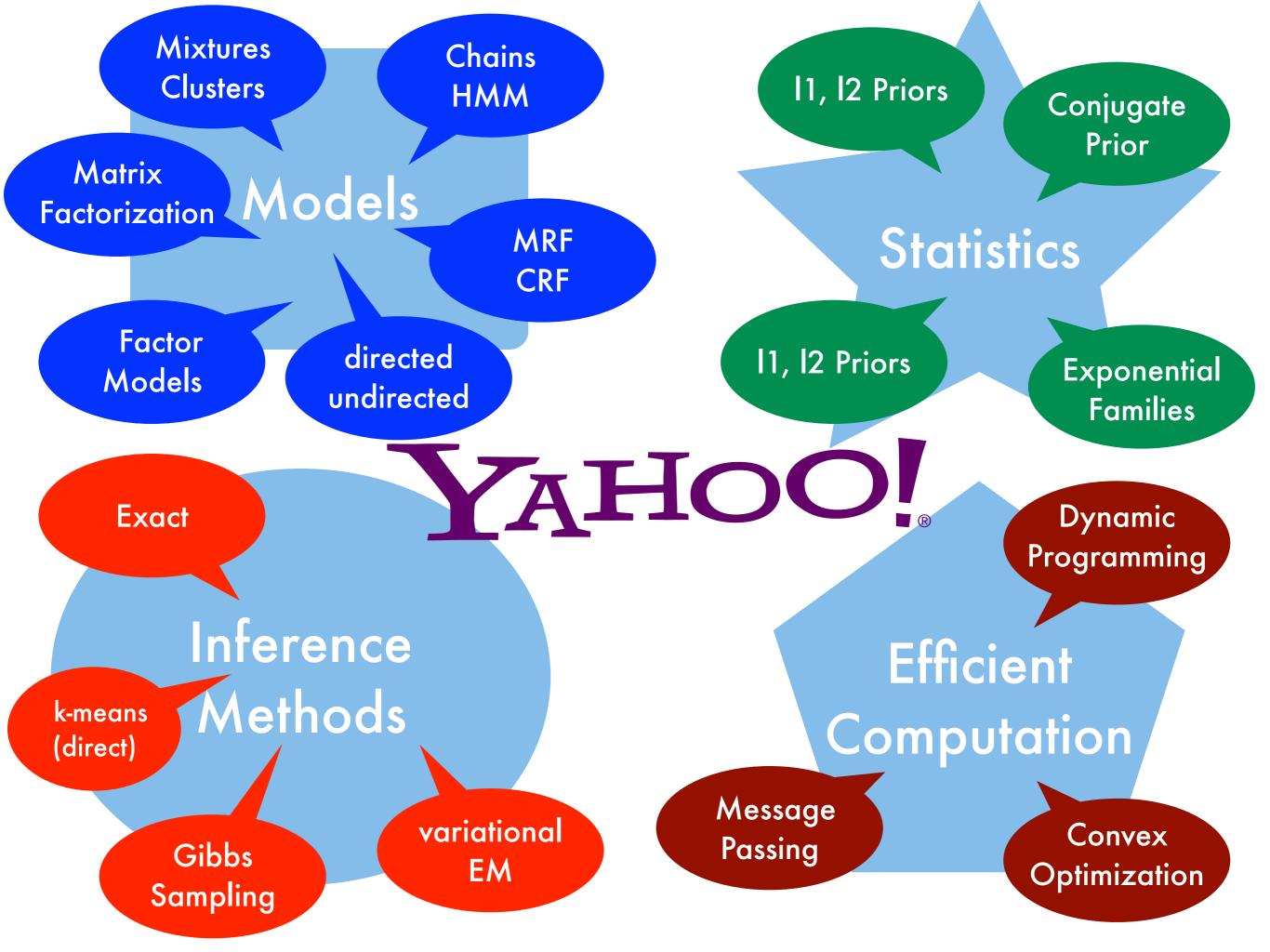
YAHOO!

Inference Methods

Efficient Computation





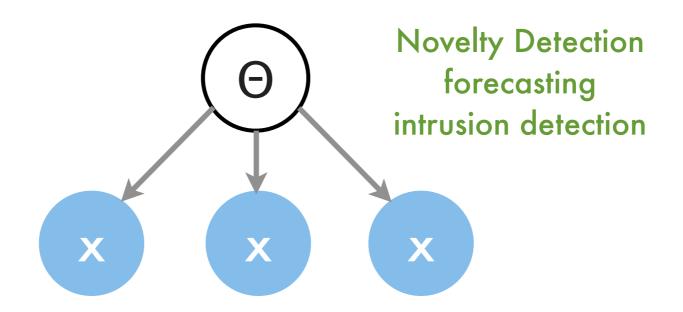


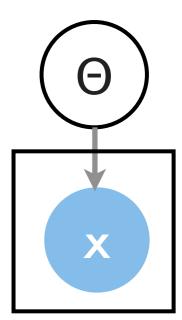




### 'Unsupervised' Models

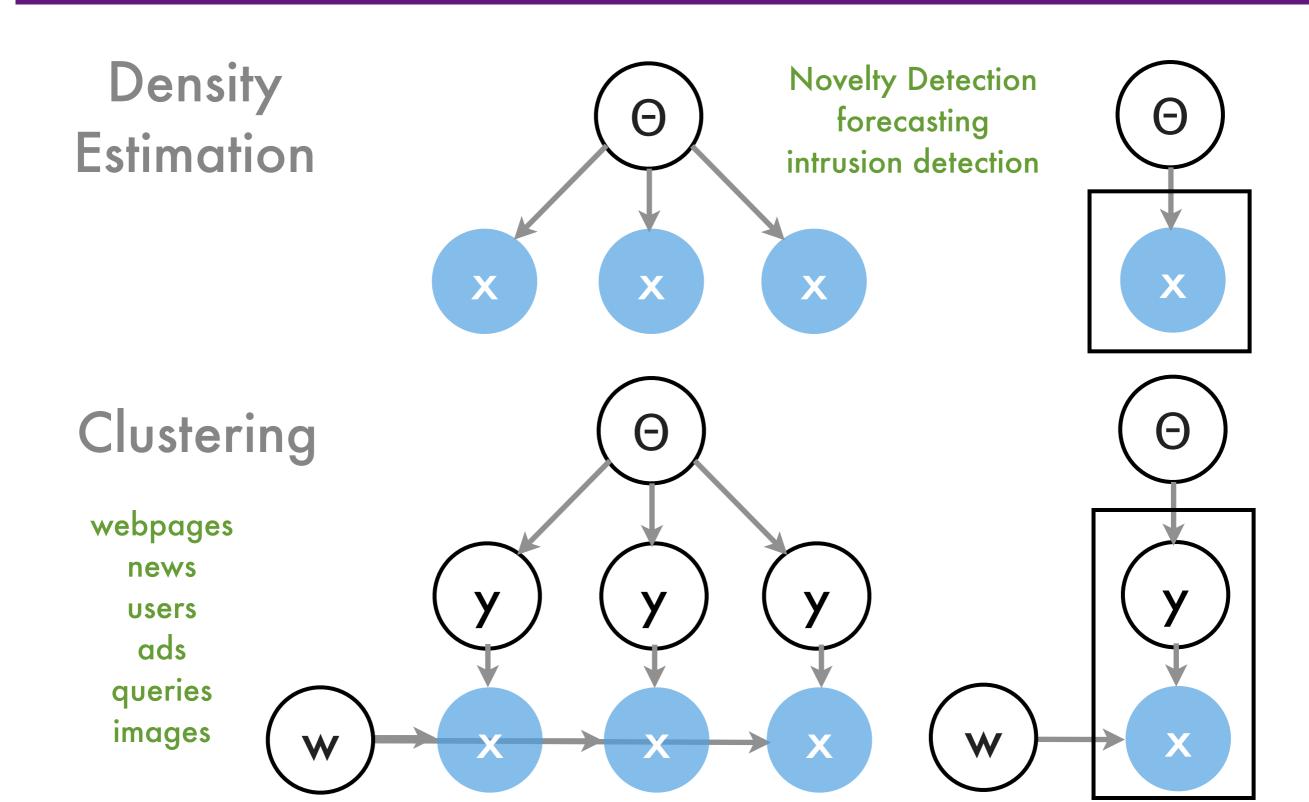
**Density Estimation** 





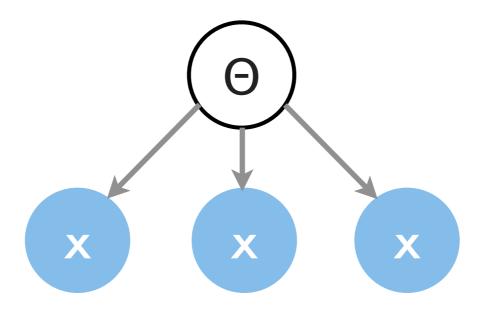
```
webpages
news
users
ads
queries
images
```

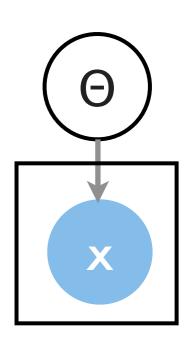
### 'Unsupervised' Models



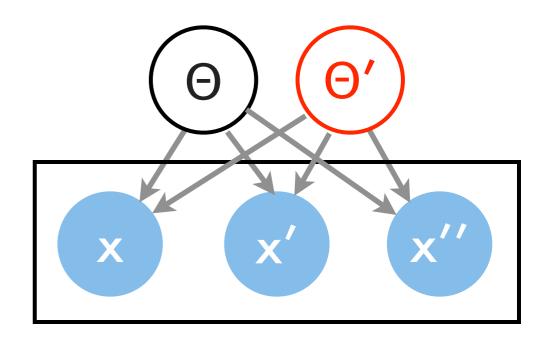
### 'Unsupervised' Models

**Density Estimation** 





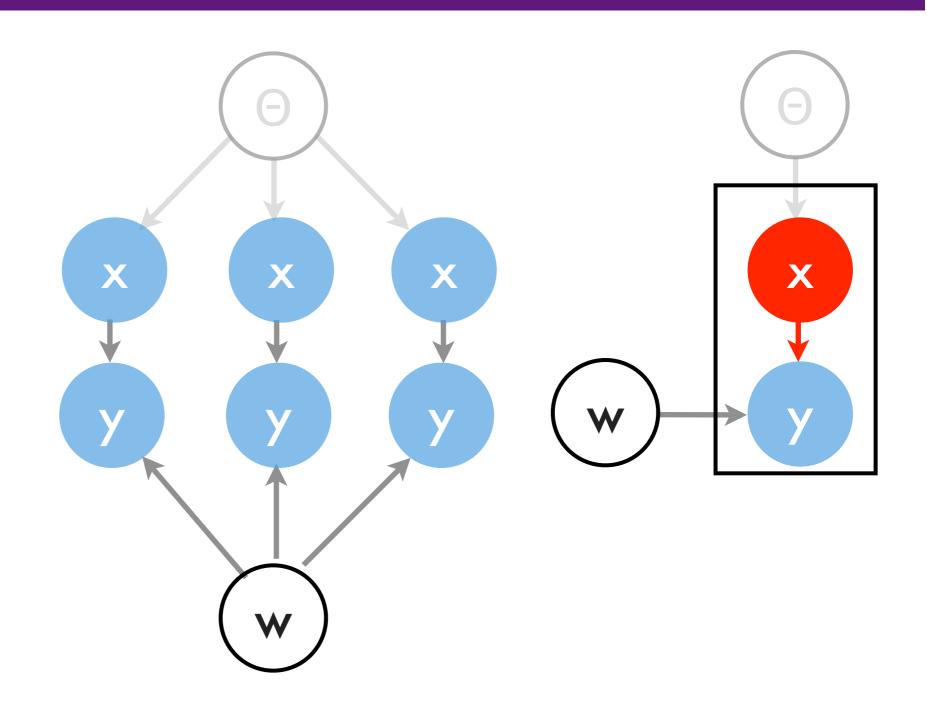
Factor Analysis



### 'Supervised' Models

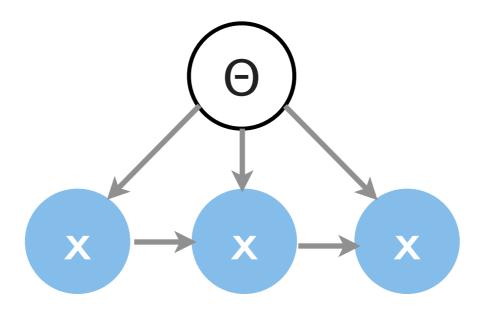
#### Classification Regression

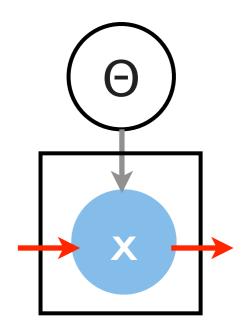
spam filtering
tiering
crawling
categorization
bid estimation
tagging



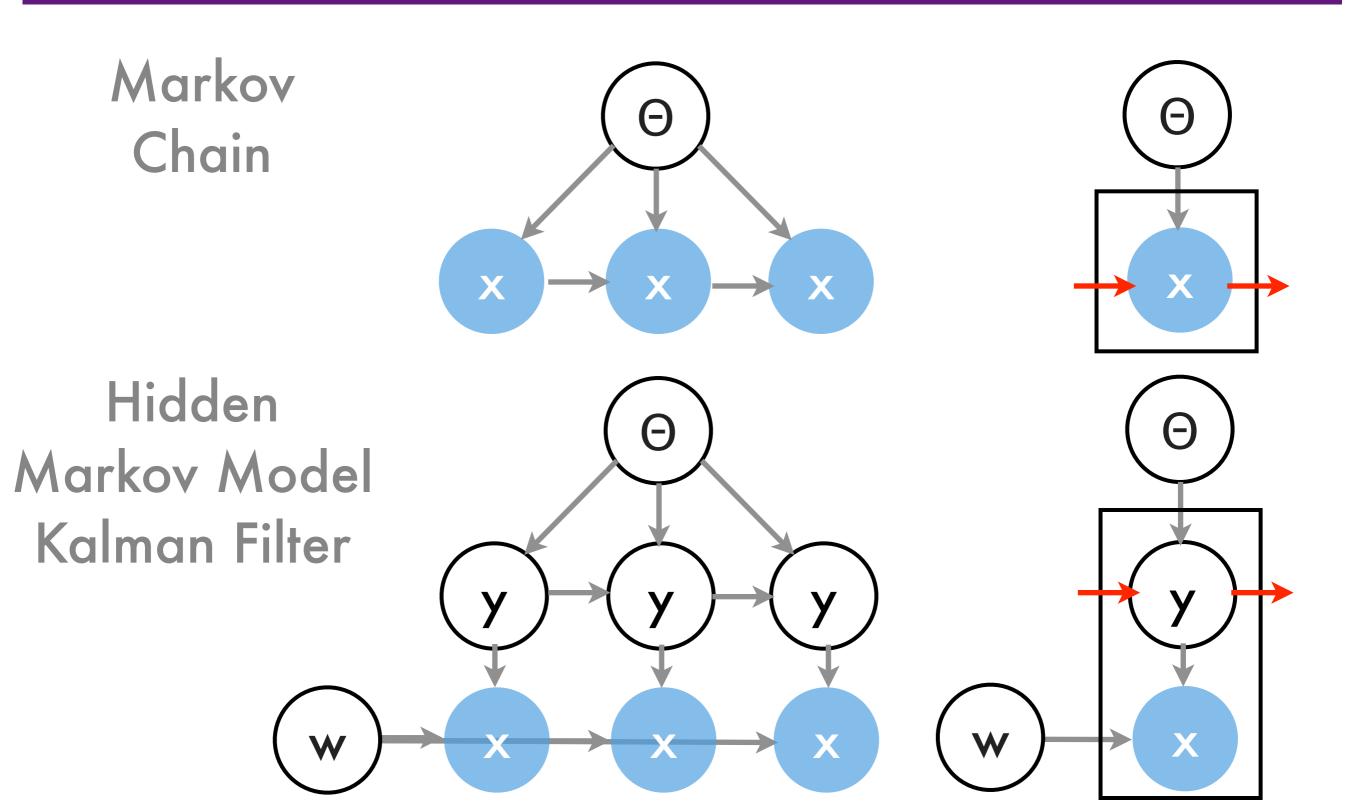
### Chains

Markov Chain

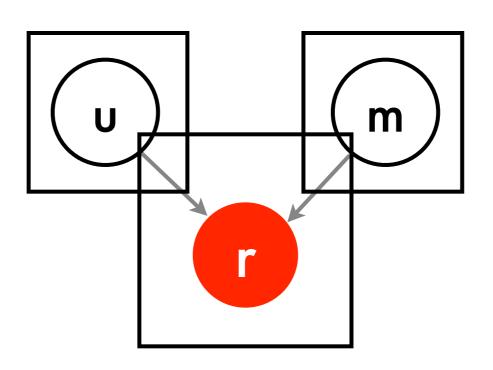




### Chains

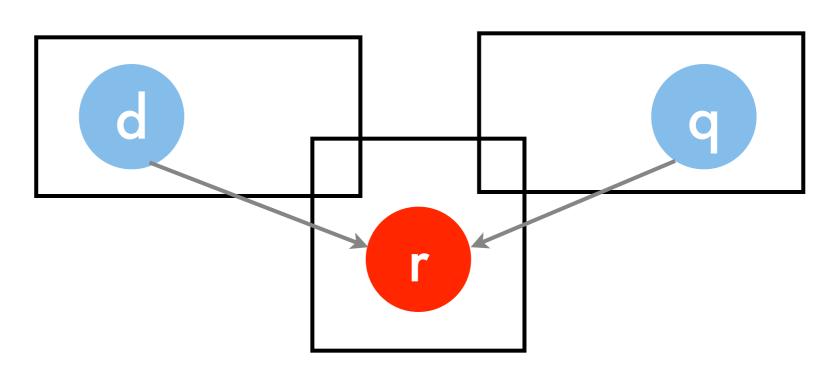


Collaborative Filtering

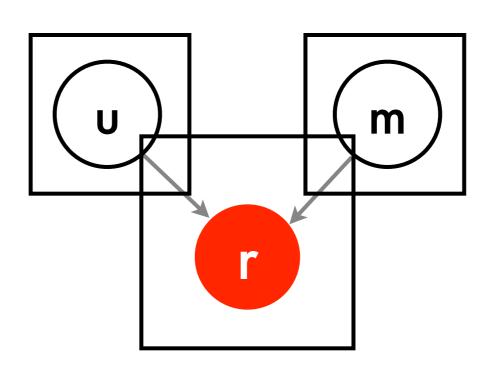


Collaborative Filtering

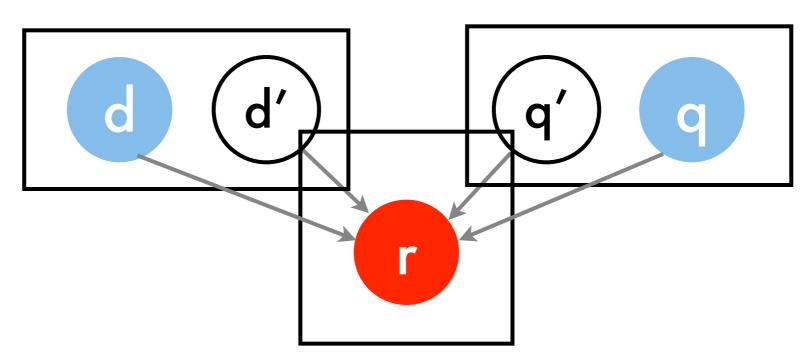
Current Webpage Ranking

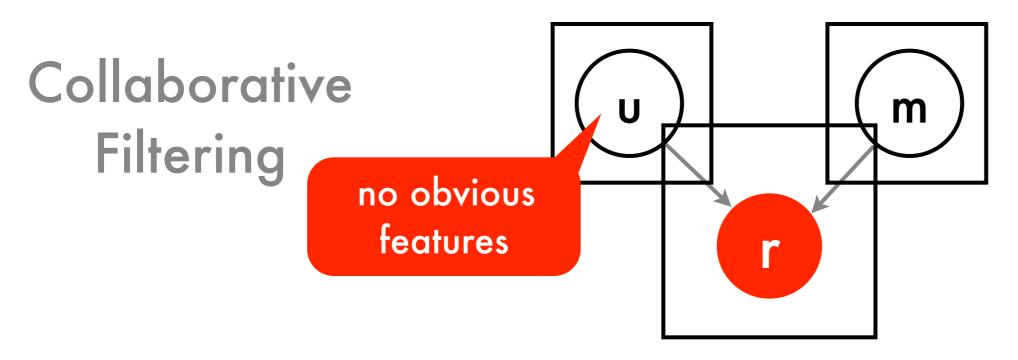


Collaborative Filtering

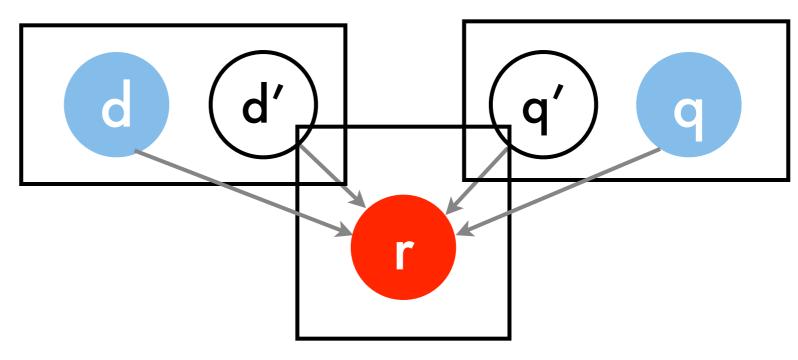


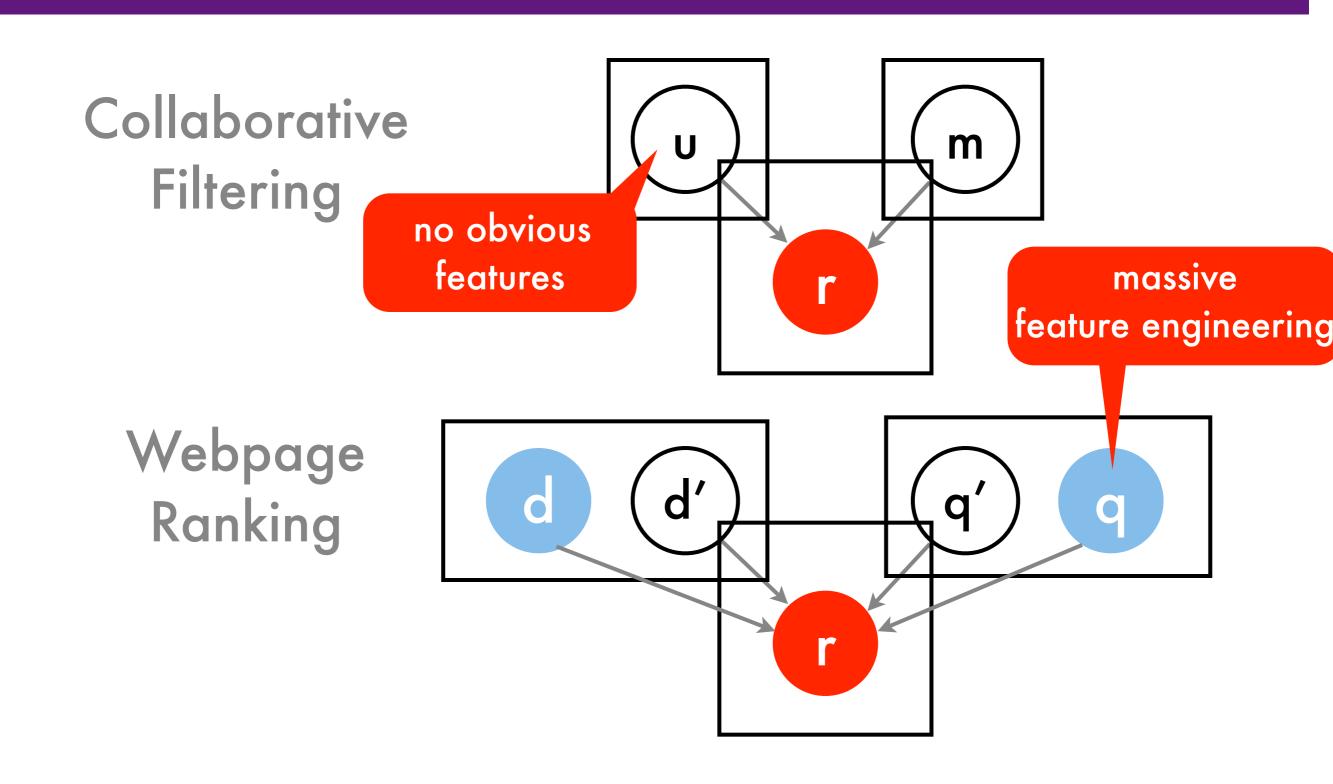
Webpage Ranking

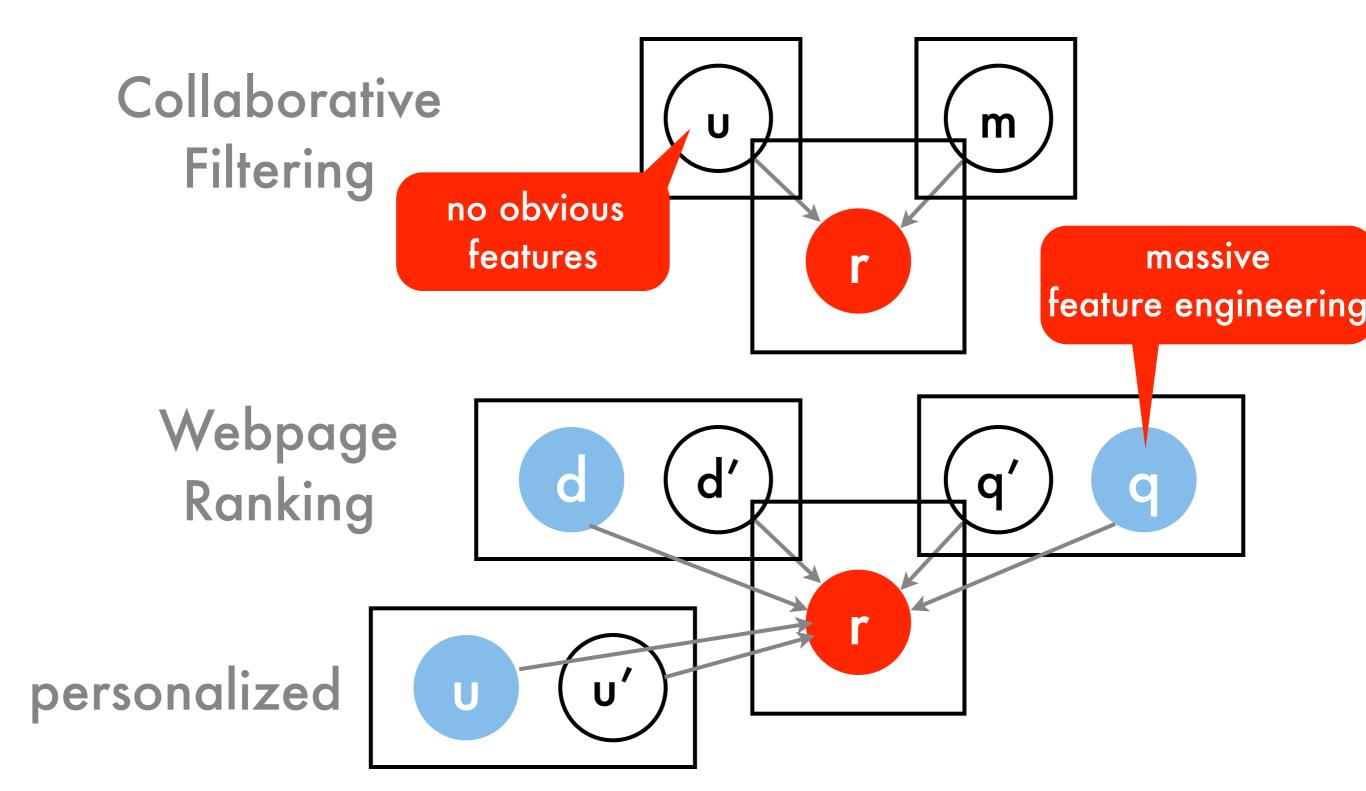


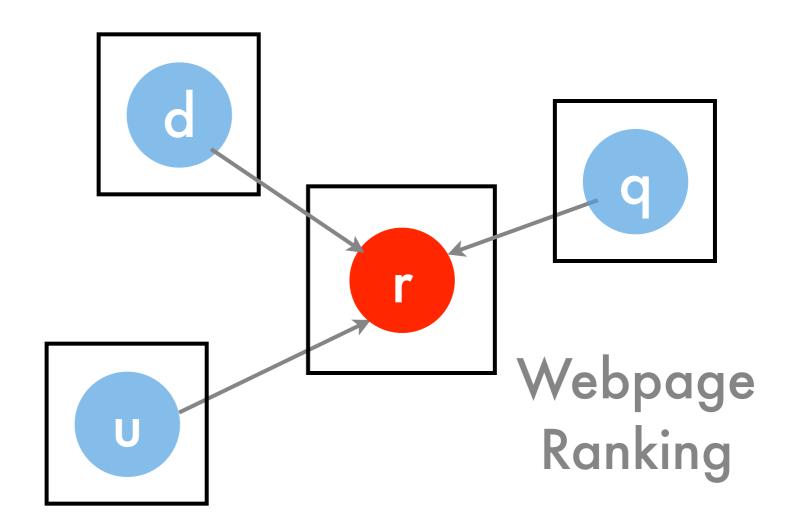


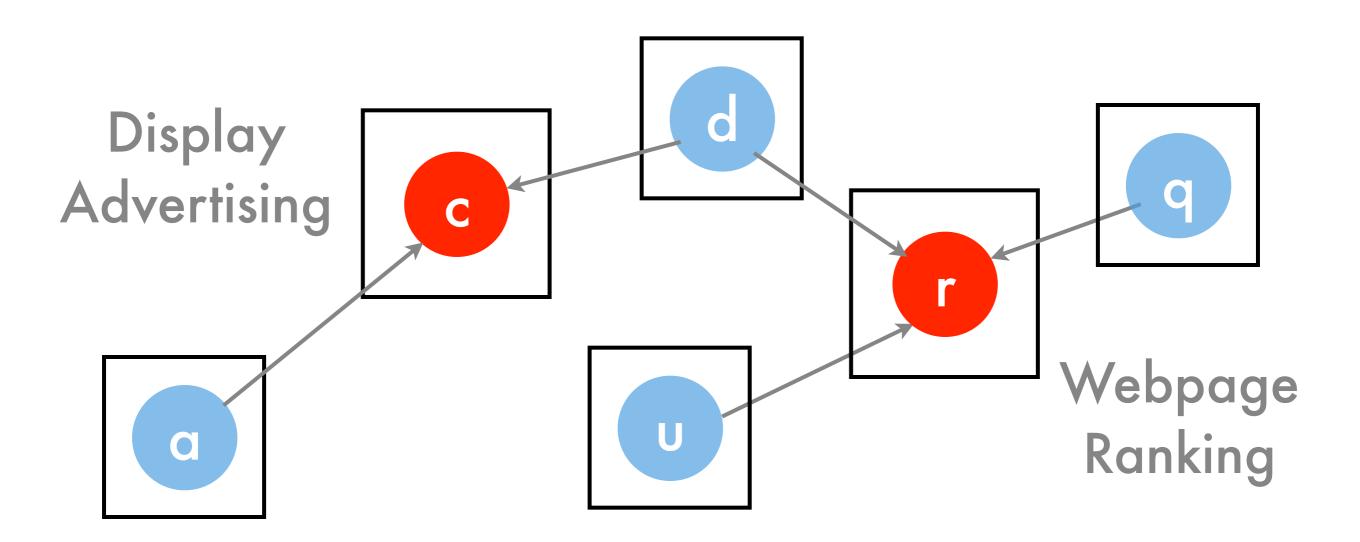
Webpage Ranking

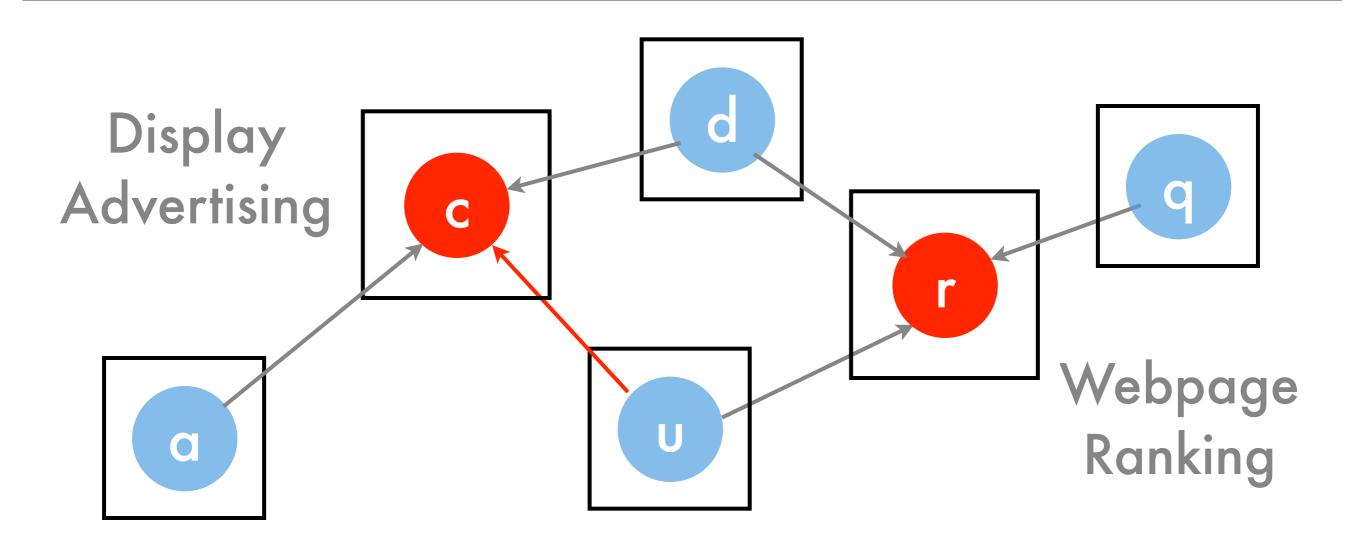


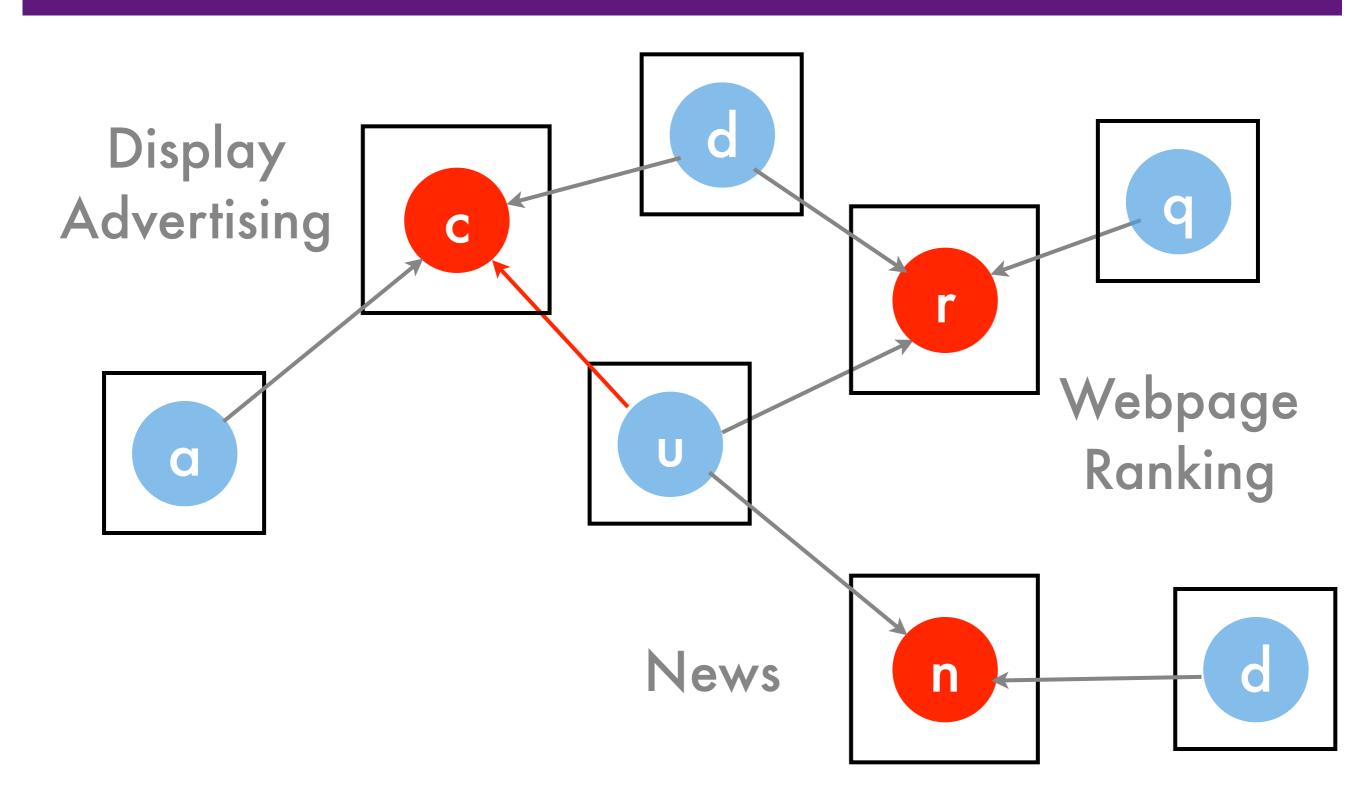


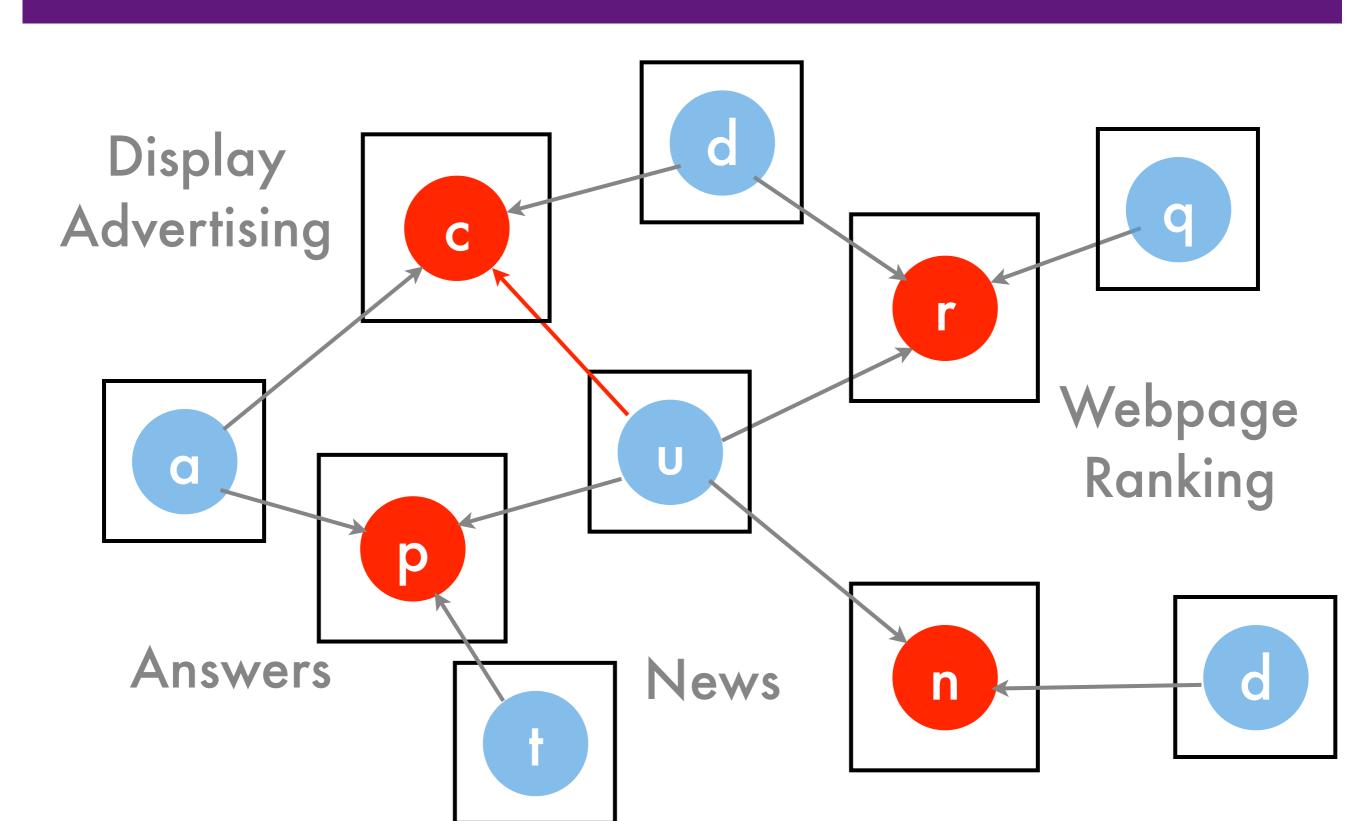




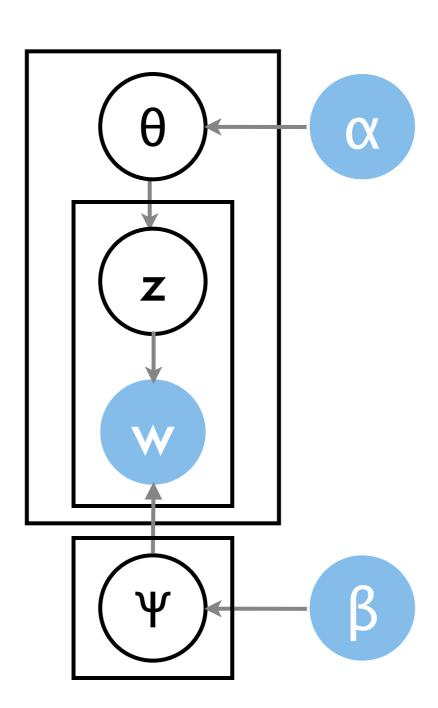






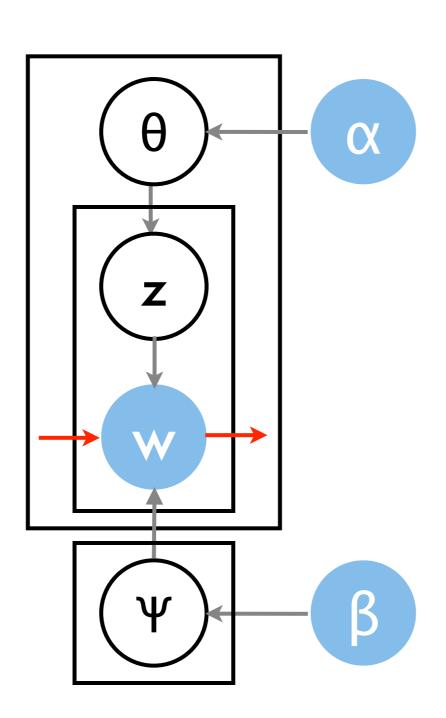


Topic Models



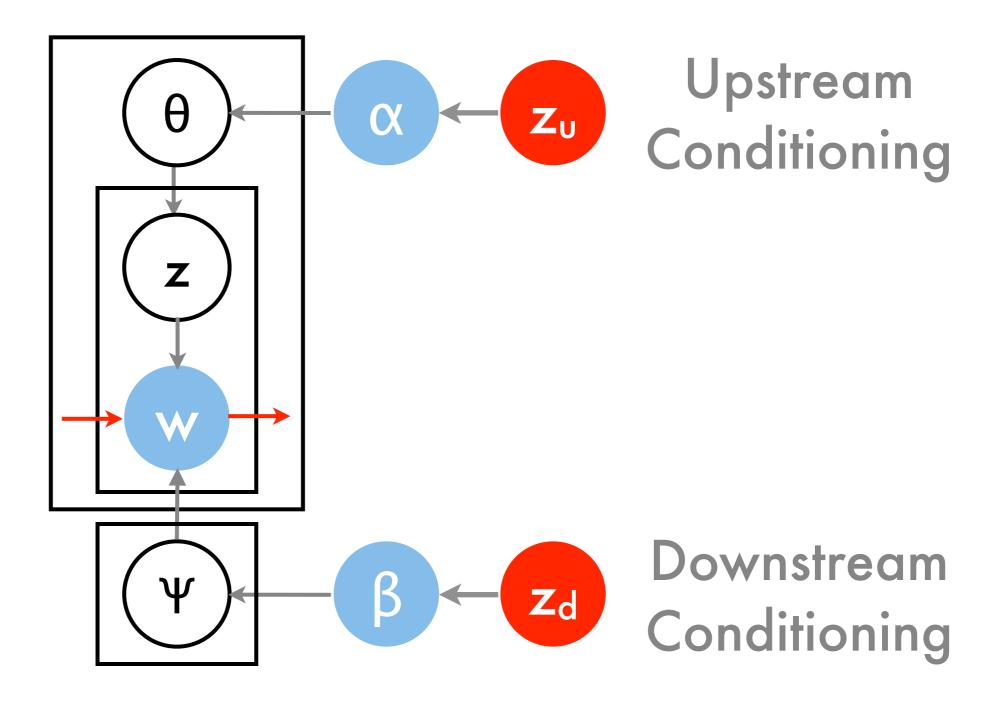
Topic Models

Simplical Mixtures



Topic Models

Simplical Mixtures



### Part 5 - Scalable Topic Models

## Grouping objects



# Grouping objects

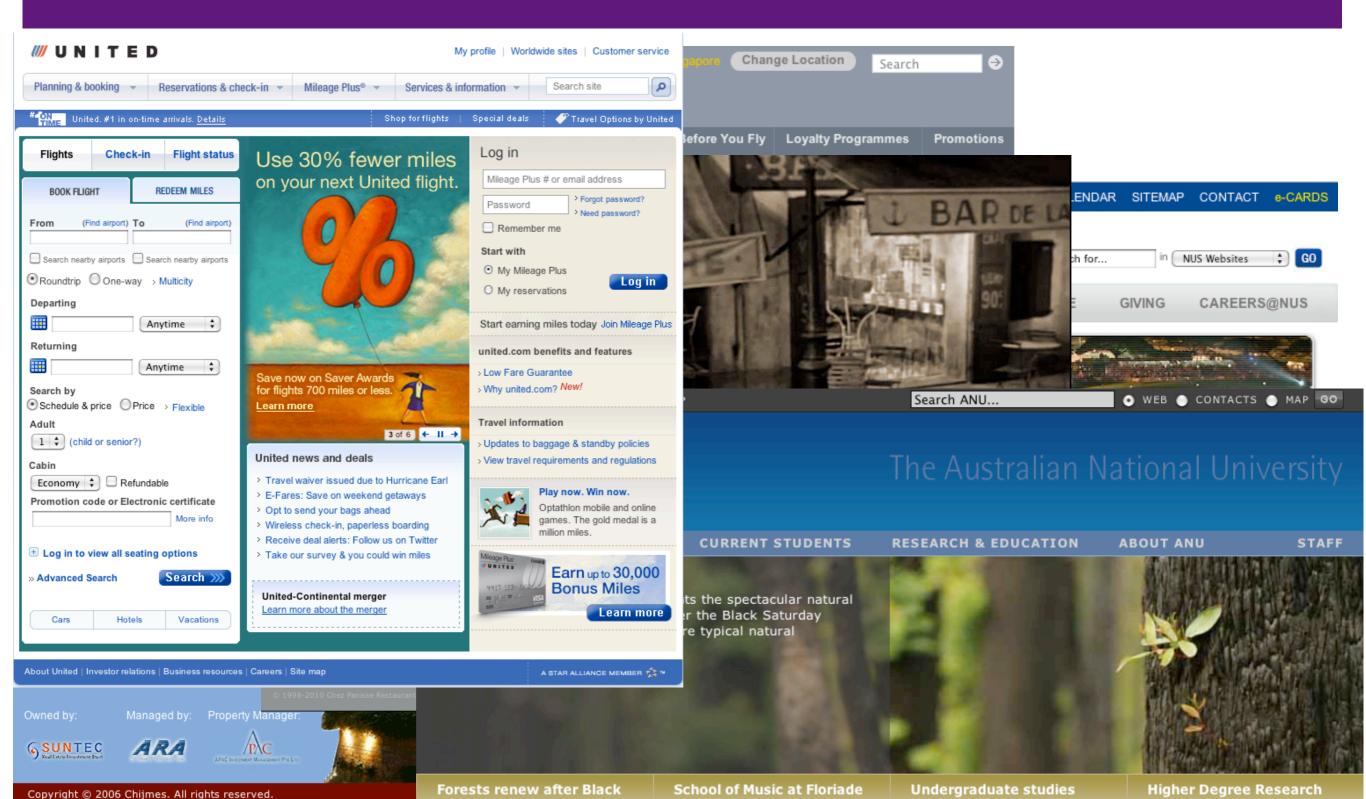


Feedback | Terms & Conditions

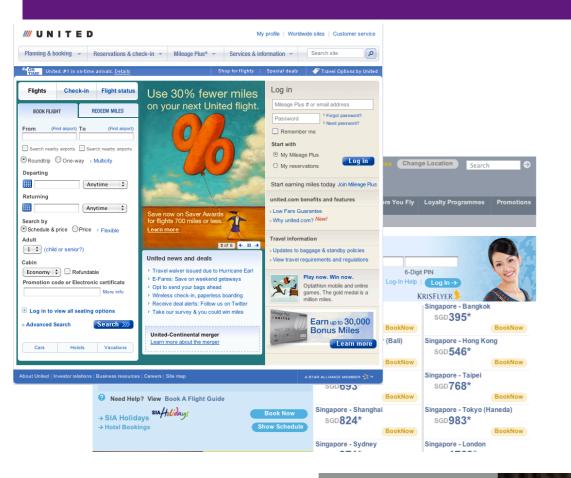
SUNTEC

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Saturday fires









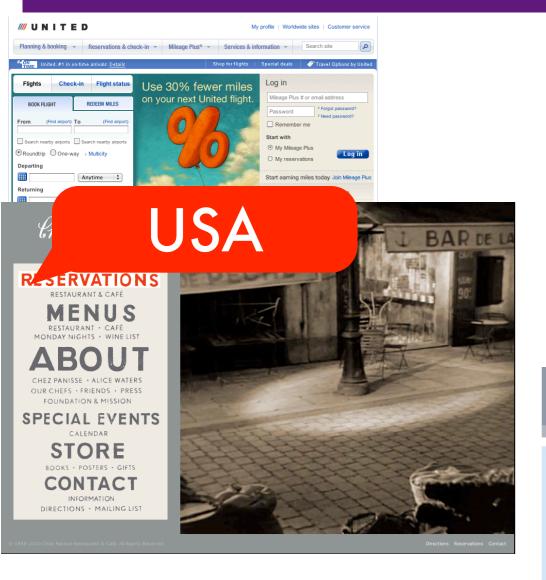










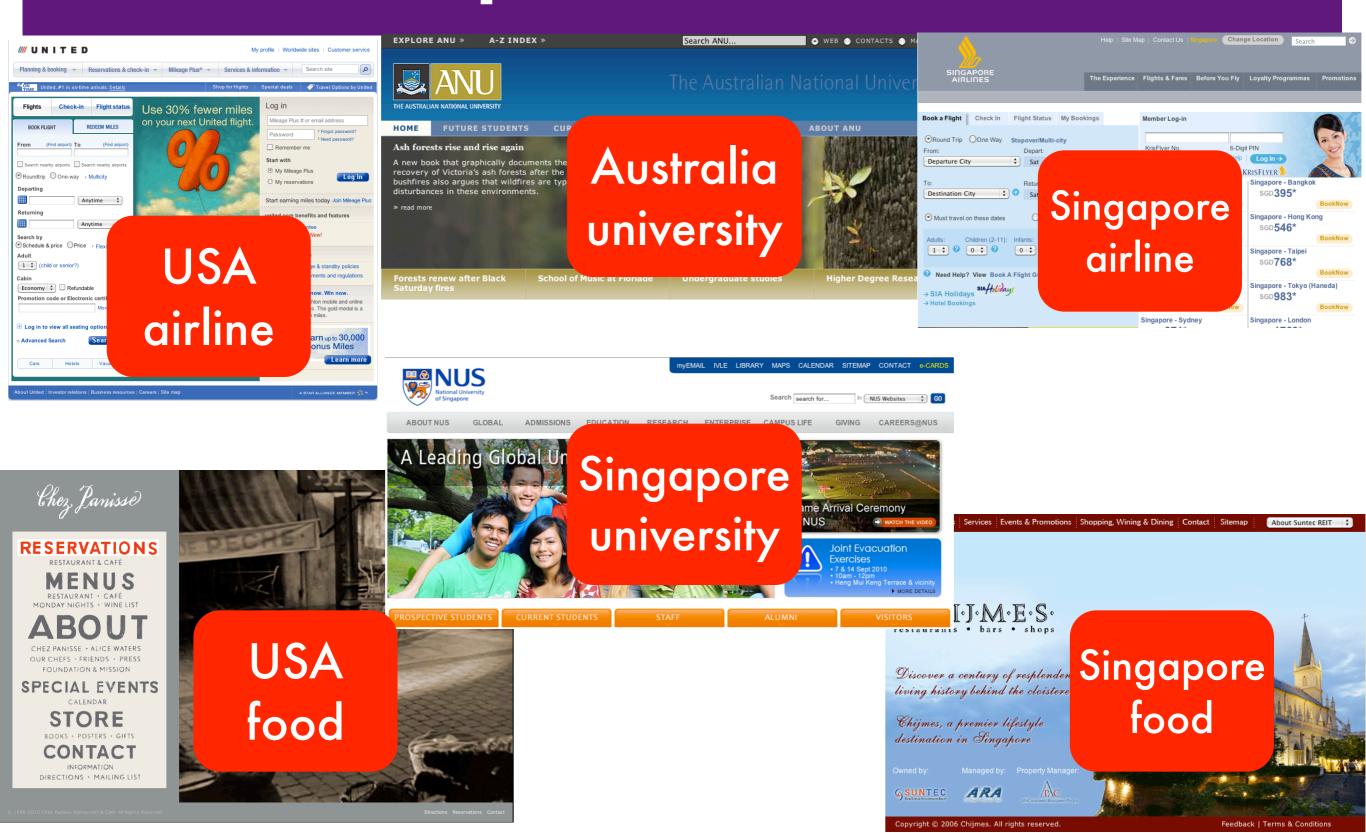




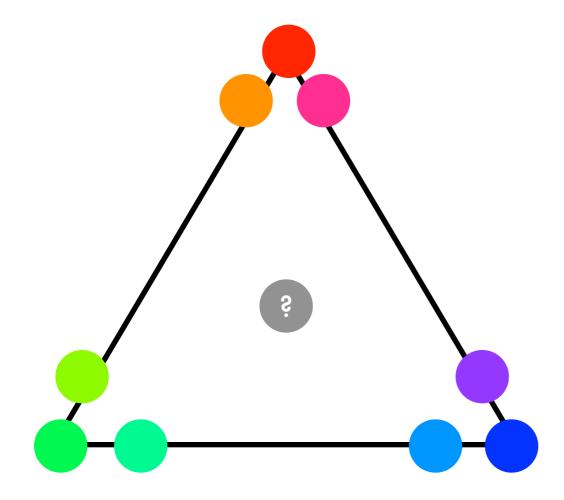




# Topic Models

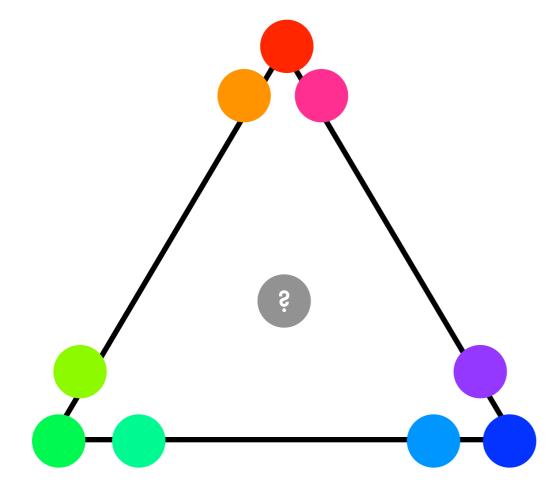


Clustering

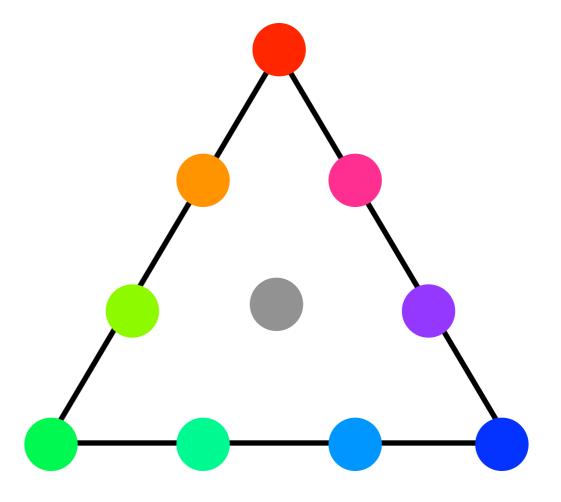


group objects by prototypes

Clustering



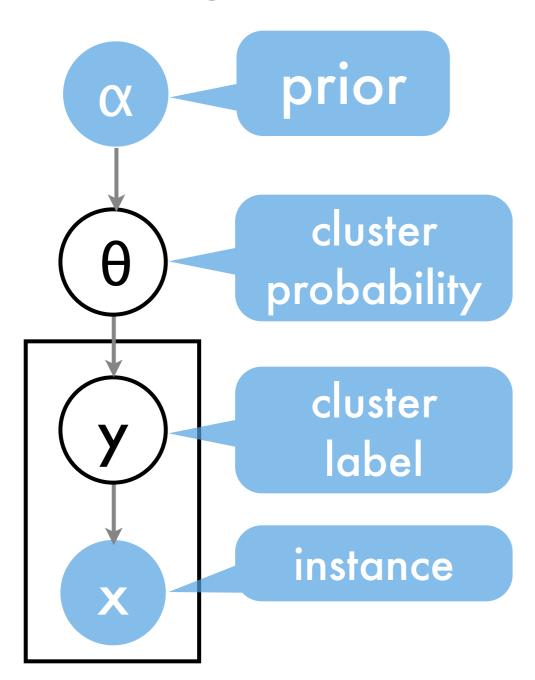
group objects by prototypes **Topics** 

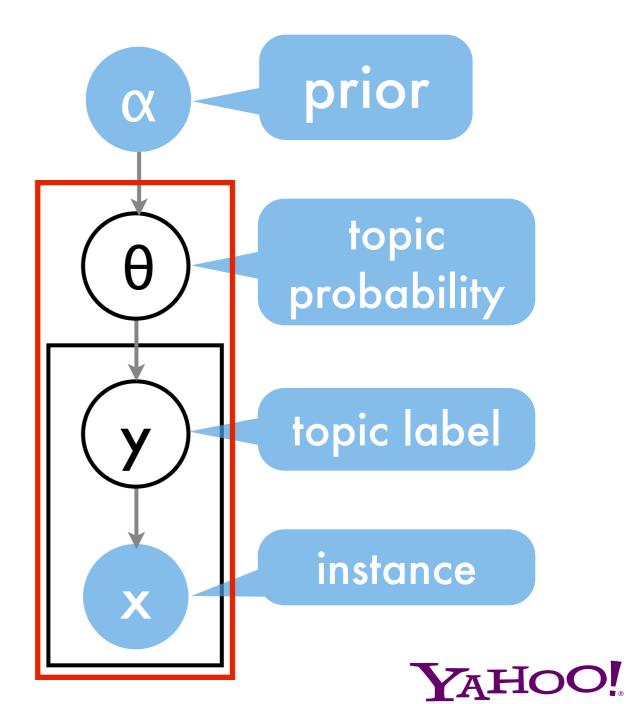


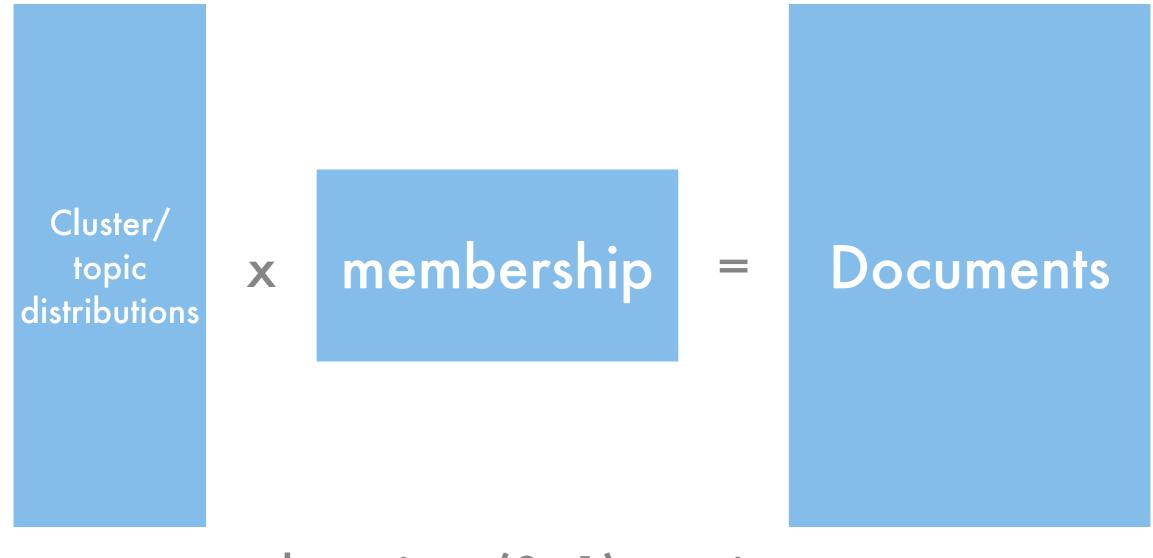
decompose objects into prototypes YAHOO!

clustering

Latent Dirichlet Allocation







clustering: (0, 1) matrix topic model: stochastic matrix LSI: arbitrary matrices



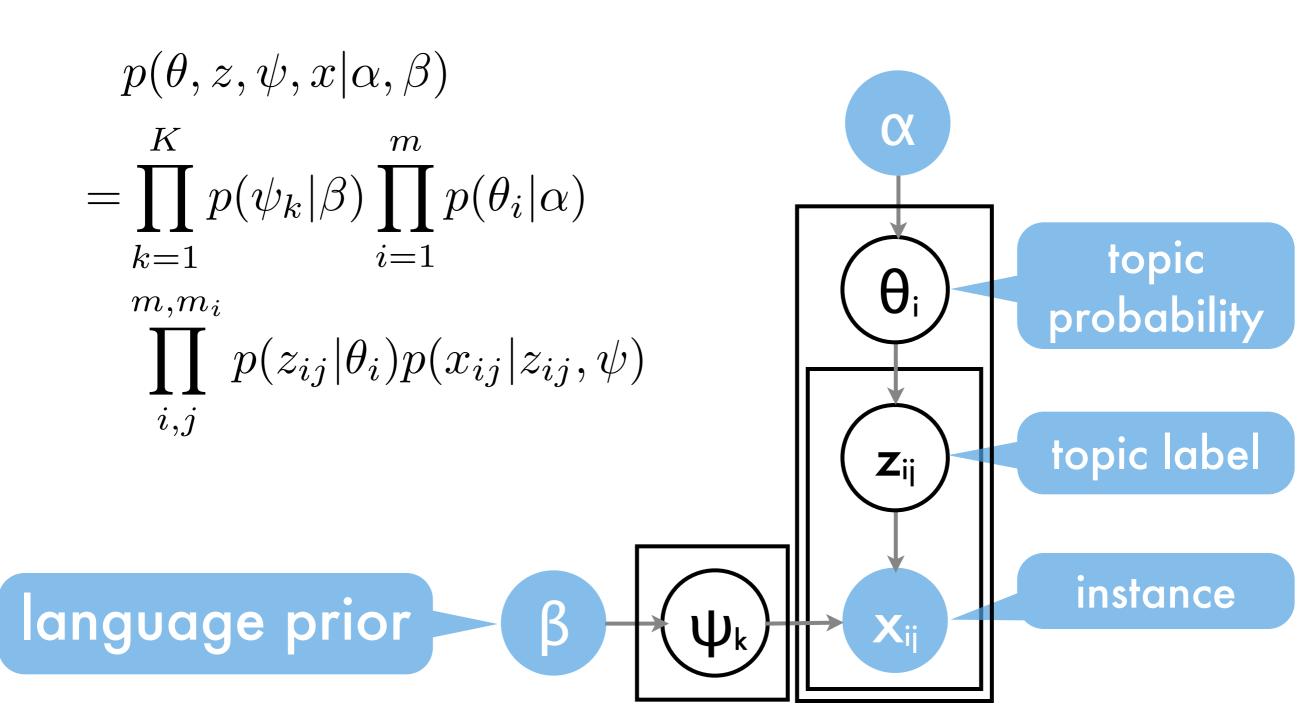
## Topics in text

The William Randolph Hearst Foundation will give \$1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be \$200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive \$400,000 each. The Juilliard School, where music and the performing arts are taught, will get \$250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual \$100,000 donation, too.

Latent Dirichlet Allocation; Blei, Ng, Jordan, JMLR 2003

# Collapsed Gibbs Sampler

## Joint Probability Distribution



# Joint Probability Distribution

sample Ψ independently

$$p(\theta, z, \psi, x | \alpha, \beta)$$

sample  $\theta$ independently

$$p(\theta, z, \psi, x | \alpha, \beta) \quad \text{index}$$

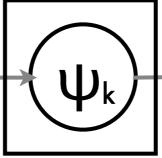
$$= \prod_{k=1}^{K} p(\psi_k | \beta) \prod_{i=1}^{m} p(\theta_i | \alpha)$$

 $m,m_i$ 

$$\prod_{i,j} p(z_{ij}|\theta_i)p(x_{ij}|z_{ij},\psi)$$

sample z independently

language prior



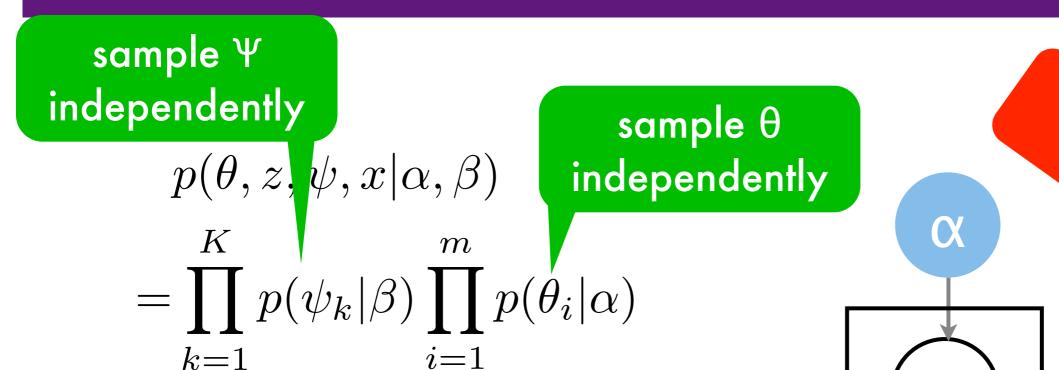
Zij

topic probability

topic label

instance

# Joint Probability Distribution

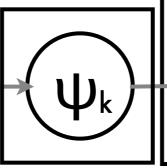


 $m,m_i$ 

 $\prod_{i,j} p(z_{ij}|\theta_i)p(x_{ij}|z_{ij},\psi)$ 

sample z independently

language prior



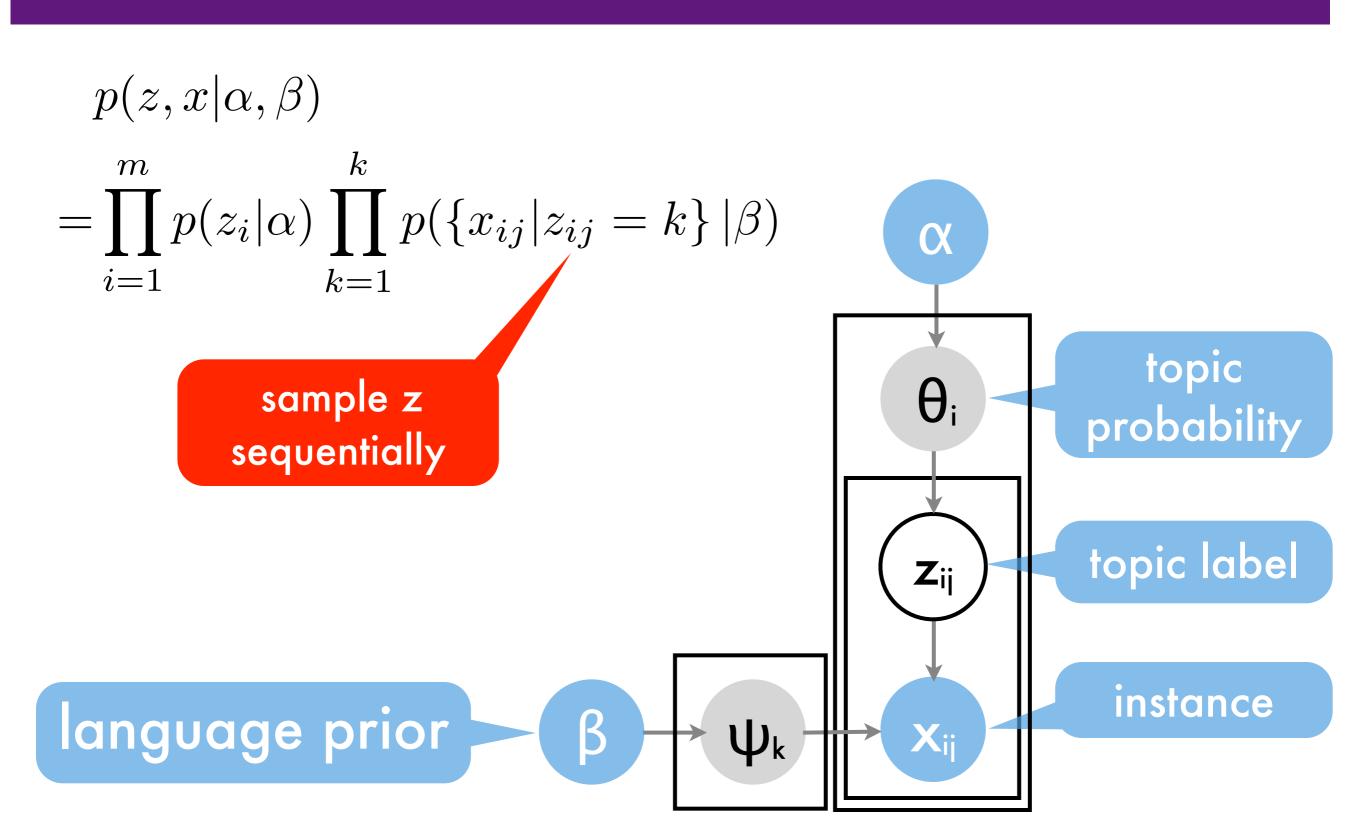
Zij

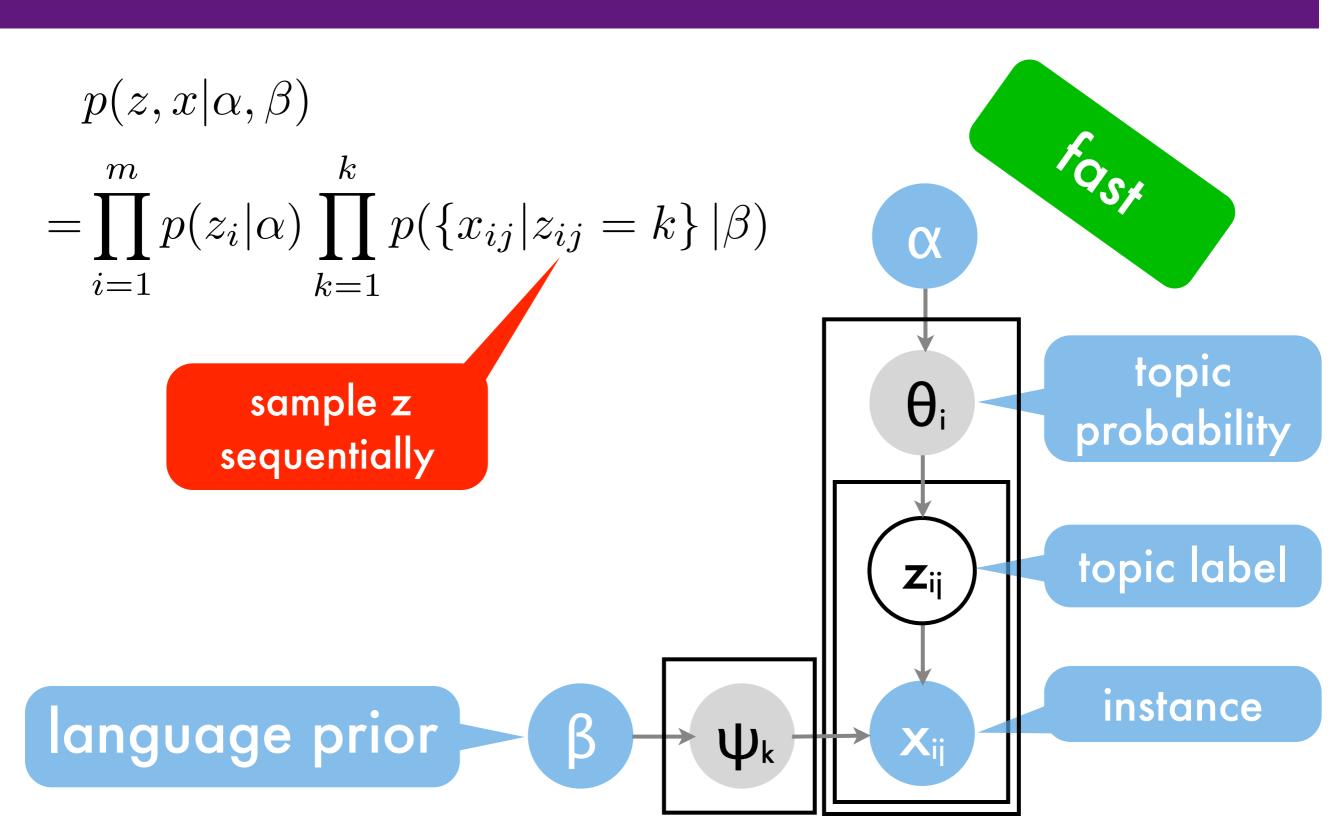
topic probability

topic label

instance

$$p(z,x|\alpha,\beta) = \prod_{i=1}^m p(z_i|\alpha) \prod_{k=1}^k p(\{x_{ij}|z_{ij}=k\} |\beta)$$
 
$$\theta_i$$
 topic probability 
$$\mathbf{z}_{ij}$$
 topic label instance

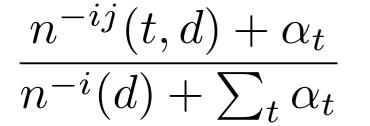




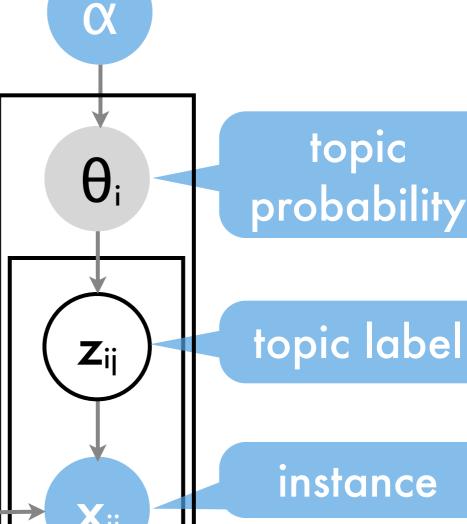
Griffiths & Steyvers, 2005

$$p(z, x | \alpha, \beta)$$

$$= \prod_{i=1}^{m} p(z_i | \alpha) \prod_{k=1}^{\kappa} p(\{x_{ij} | z_{ij} = k\} | \beta)$$



$$\frac{n^{-ij}(t,w) + \beta_t}{n^{-i}(t) + \sum_t \beta_t}$$

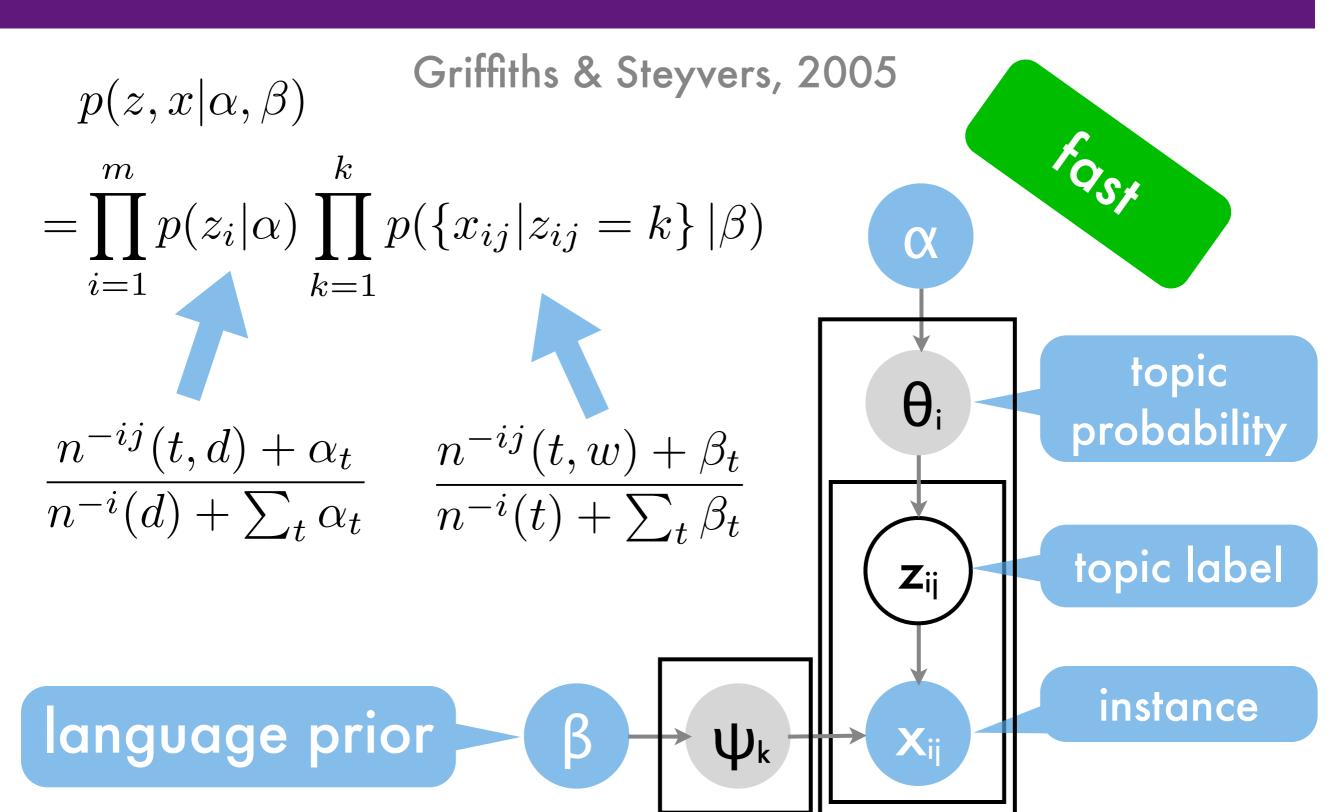


language prior









### Sequential Algorithm (Gibbs sampler)

- For 1000 iterations do
  - For each document do
    - For each word in the document do
      - Resample topic for the word
      - Update local (document, topic) table
      - Update CPU local (word, topic) table
      - Update global (word, topic) table



### Sequential Algorithm (Gibbs sampler)

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      - Update global (word, topic) table

this kills parallelism



- For 1000 iterations do
  - For each document do
    - For each word in the document do
      - Resample topic for the word
      - Update local (document, topic) table
      - Update CPU local (word, topic) table
  - Update global (word, topic) table

$$p(t|w_{ij}) \propto \beta_w \frac{\alpha_t}{n(t) + \bar{\beta}} + \beta_w \frac{n(t, d=i)}{n(t) + \bar{\beta}} + \frac{n(t, w=w_{ij}) [n(t, d=i) + \alpha_t]}{n(t) + \bar{\beta}}$$



- For 1000 iterations do
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    - For each word in the document do
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      - Update local (document, topic) table
      - Update CPU local (word, topic) table
  - Update global (word, topic) table

#### changes rapidly

$$p(t|w_{ij}) \propto \beta_w \frac{\alpha_t}{n(t) + \bar{\beta}} + \beta_w \frac{n(t, d=i)}{n(t) + \bar{\beta}} + \frac{n(t, w=w_{ij}) \left[n(t, d=i) + \alpha_t\right]}{n(t) + \bar{\beta}}$$





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#### changes rapidly

$$p(t|w_{ij}) \propto \beta_w \frac{\alpha_t}{n(t) + \bar{\beta}} + \beta_w \frac{n(t, d=i)}{n(t) + \bar{\beta}} + \frac{n(t, w=w_{ij}) \left[n(t, d=i) + \alpha_t\right]}{n(t) + \bar{\beta}}$$

slow

YAHOO!

- For 1000 iterations do
  - For each document do
    - For each word in the document do
      - Resample topic for the word
      - Update local (document, topic) table
      - Update CPU local (word, topic) table
  - Update global (word, topic) table

table out of sync

memory inefficient

blocking

network bound

#### changes rapidly

$$p(t|w_{ij}) \propto \beta_w \frac{\alpha_t}{n(t) + \bar{\beta}} + \beta_w \frac{n(t, d=i)}{n(t) + \bar{\beta}} + \frac{n(t, w=w_{ij}) [n(t, d=i) + \alpha_t]}{n(t) + \bar{\beta}}$$

slow

moderately fast



- For 1000 iterations do (independently per computer)
  - For each thread/core do
    - For each document do
      - For each word in the document do
        - Resample topic for the word
        - Update local (document, topic) table
        - Generate computer local (word, topic) message
    - In parallel update local (word, topic) table
  - In parallel update global (word, topic) table



- For 1000 iterations do (independently per computer)
  - For each thread/core do
    - For each document do
      - For each word in the document do
        - Resample topic for the word
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    - In parallel update local (word, topic) table
  - In parallel update global (word, topic) table

network bound

concurrent cpu hdd net



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network bound

concurrent cpu hdd net

memory inefficient

minimal view



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network bound

concurrent cpu hdd net

memory inefficient

minimal view

table out of sync

continuous sync



- For 1000 iterations do (independently per computer)
  - For each thread/core do
    - For each document do
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    - In parallel update local (word, topic) table
  - In parallel update global (word, topic) table

network bound

concurrent cpu hdd net

memory inefficient

minimal view

table out of sync

continuous sync

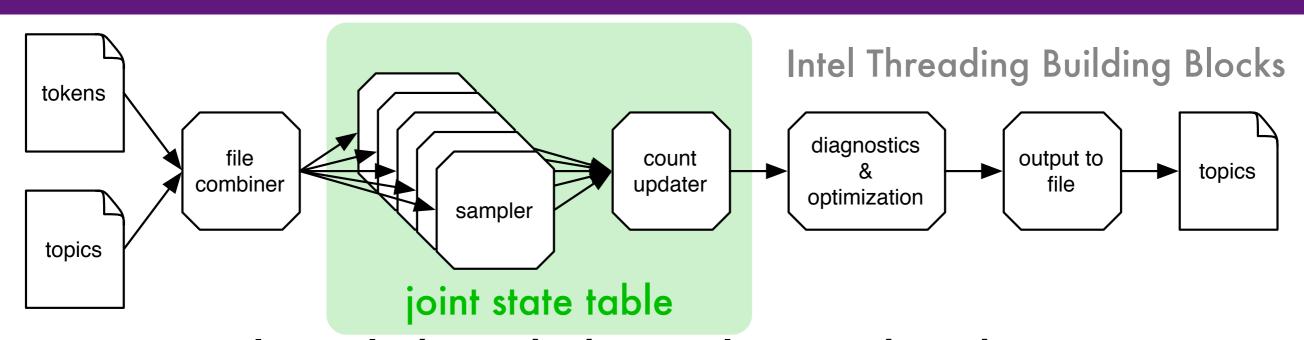
blocking

barrier free



### Architecture details

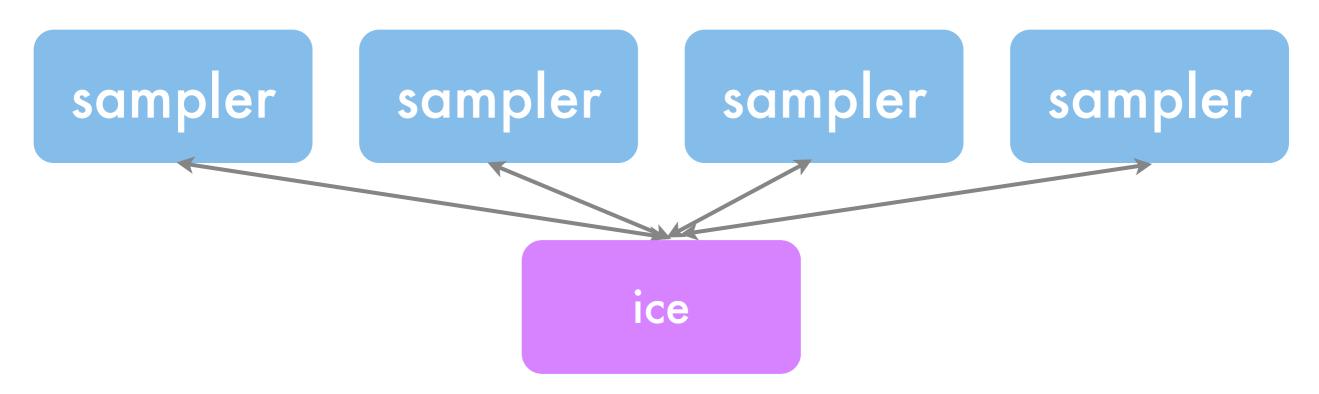
### Multicore Architecture



- Decouple multithreaded sampling and updating (almost) avoids stalling for locks in the sampler
- Joint state table
  - much less memory required
  - samplers syncronized (10 docs vs. millions delay)
- Hyperparameter update via stochastic gradient descent
- No need to keep documents in memory (streaming)

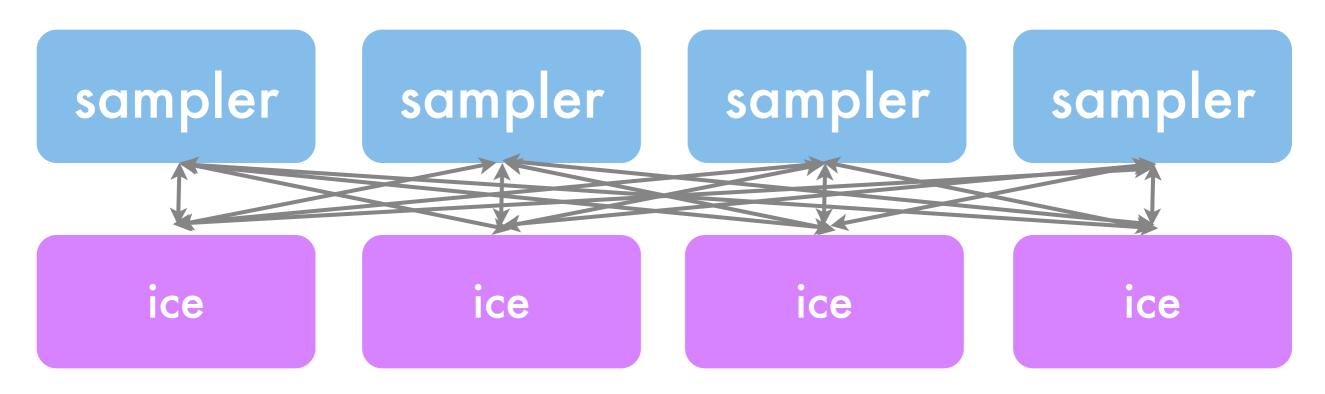


### Cluster Architecture



- Distributed (key, value) storage via memcached
- Background asynchronous synchronization
  - single word at a time to avoid deadlocks
  - no need to have joint dictionary
  - uses disk, network, cpu simultaneously

### Cluster Architecture



- Distributed (key, value) storage via ICE
- Background asynchronous synchronization
  - single word at a time to avoid deadlocks
  - no need to have joint dictionary
  - uses disk, network, cpu simultaneously

# Making it work

#### Startup

- Randomly initialize topics on each node (read from disk if already assigned - hotstart)
- Sequential Monte Carlo for startup much faster
- Aggregate changes on the fly

#### Failover

- State constantly being written to disk (worst case we lose 1 iteration out of 1000)
- Restart via standard startup routine
- Achilles heel: need to restart from checkpoint if even a single machine dies.



# Easily extensible

- Better language model (topical n-grams)
   can process millions of users (vs 1000s)
- Conditioning on side information (upstream)
   estimate topic based on authorship, source,
   joint user model ...
- Conditioning on dictionaries (downstream) integrate topics between different languages
- Time dependent sampler for user model approximate inference per episode



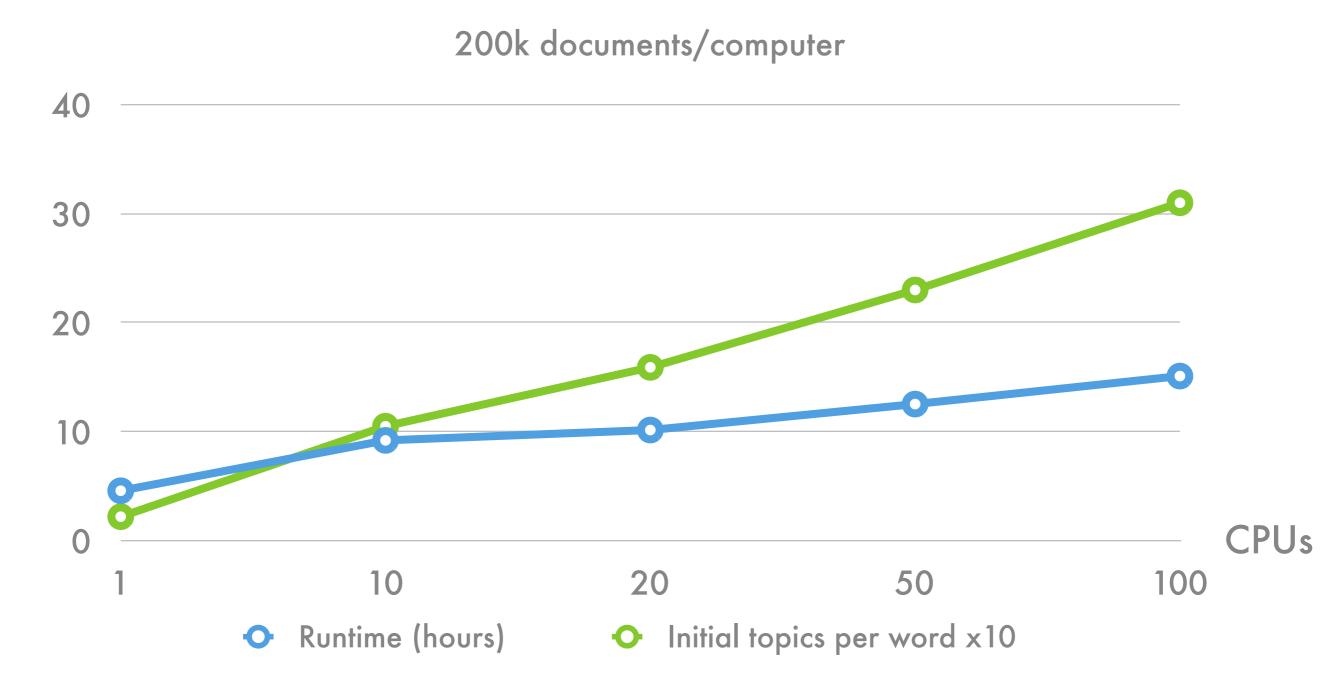
	Google LDA	Mallet	Irvine'08	Irvine'09	Yahoo LDA
Multicore	no	yes	yes	yes	yes
Cluster	MPI	no	MPI	point 2 point	memcached
State table	dictionary split	separate sparse	separate	separate	joint sparse
Schedule	synchronous exact	synchronous exact	synchronous exact	asynchronous approximate messages	asynchronous exact

# Speed

- 1M documents per day on 1 computer (1000 topics per doc, 1000 words per doc)
- 350k documents per day per node (context switches & memcached & stray reducers)
- 8 Million docs (Pubmed) (sampler does not burn in well - too short doc)
  - Irvine: 128 machines, 10 hours
  - Yahoo: 1 machine, 11 days
  - Yahoo: 20 machines, 9 hours
- 20 Million docs (Yahoo! News Articles)
  - Yahoo: 100 machines, 12 hours

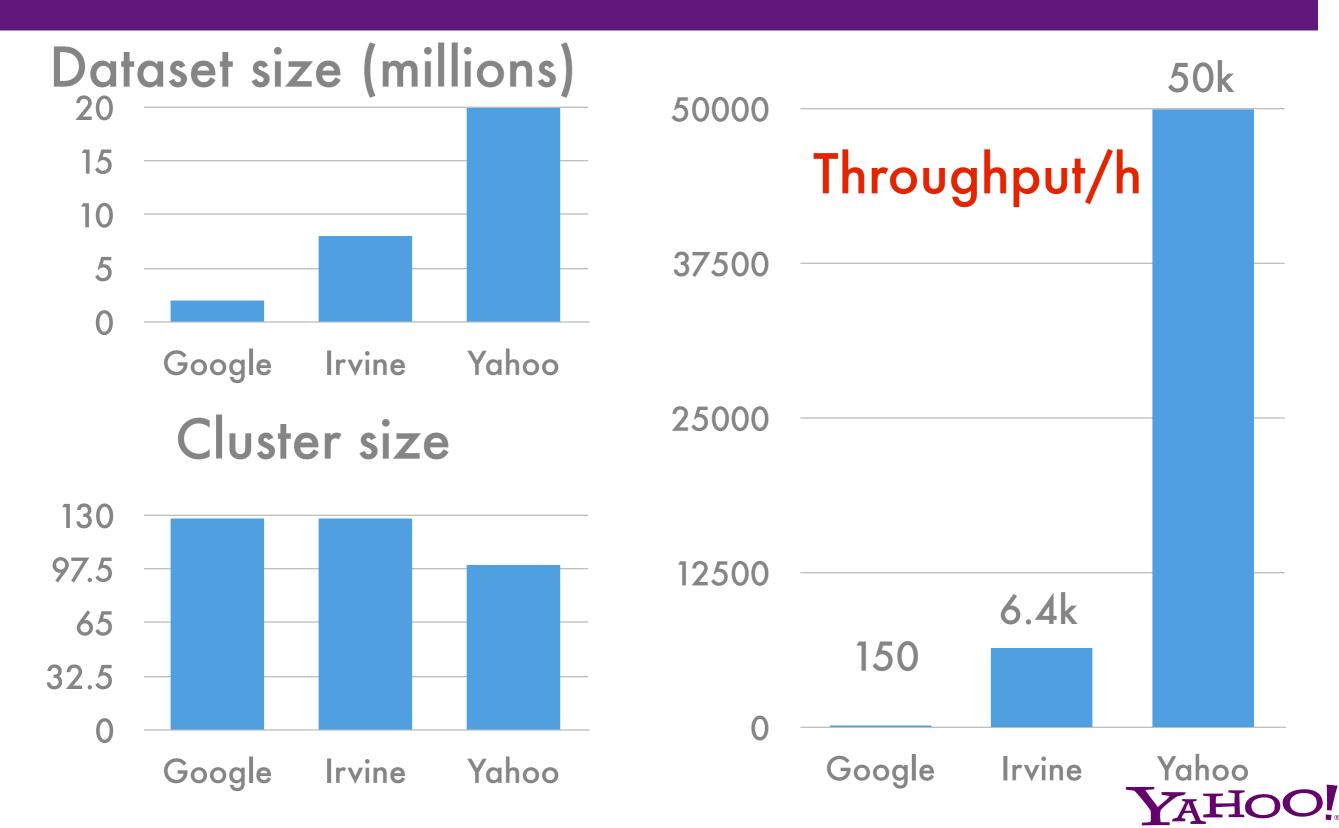


# Scalability



Likelihood even improves with parallelism! -3.295 (1 node) -3.288 (10 nodes) -3.287 (20 nodes)

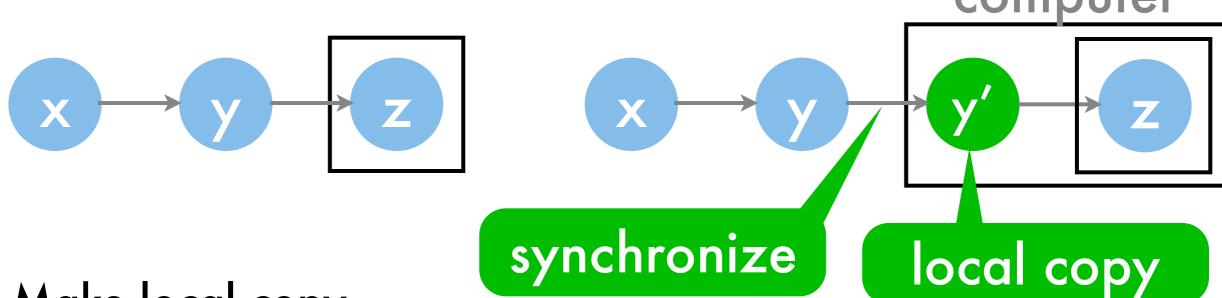
# The Competition



# Design Principles

# Variable Replication

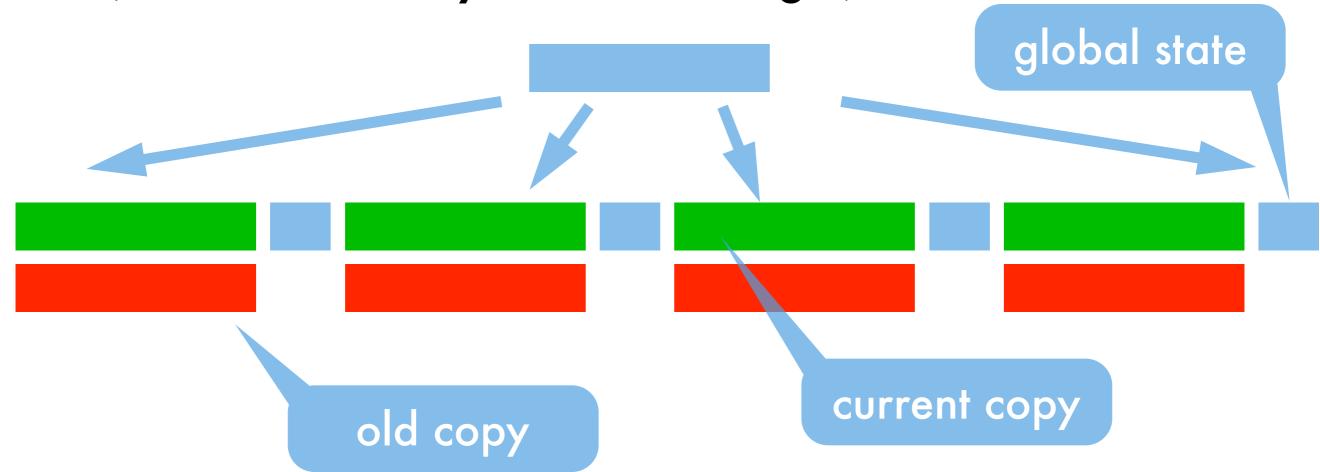
Global shared variable



- Make local copy
  - Distributed (key, value) storage table for global copy
  - Do all bookkeeping locally (store old versions)
  - Sync local copies asynchronously using message passing (no global locks are needed)
- This is an approximation!

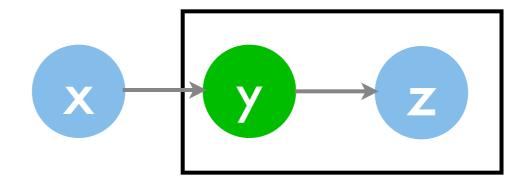
# Asymmetric Message Passing

- Large global shared state space (essentially as large as the memory in computer)
- Distribute global copy over several machines (distributed key, value storage)

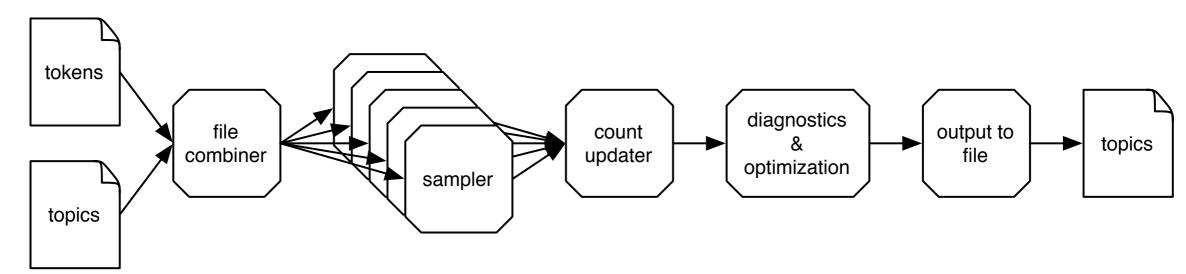


# Out of core storage

Very large state space



- Gibbs sampling requires us to traverse the data sequentially many times (think 1000x)
- Stream local data from disk and update coupling variable each time local data is accessed
- This is exact

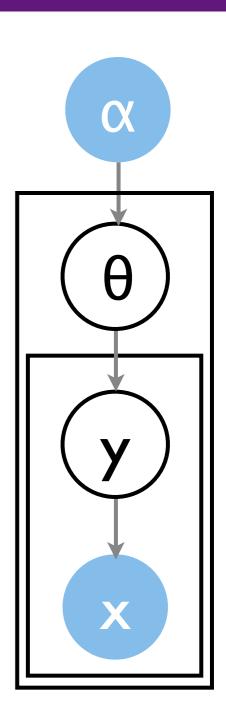


# Part 6 - Advanced Modeling

# Advances in Representation

# Extensions to topic models

- Prior over document topic vector
  - Usually as Dirichlet distribution
  - Use correlation between topics (CTM)
  - Hierarchical structure over topics
- Document structure
  - Bag of words
  - n-grams (Li & McCallum)
  - Simplical Mixture (Girolami & Kaban)
- Side information
  - Upstream conditioning (Mimno & McCallum)
  - Downstream conditioning (Petterson et al.)
  - Supervised LDA (Blei and McAulliffe 2007; Lacoste, Sha and Jordan 2008; Zhu, Ahmed and Xing 2009)



# Correlated topic models

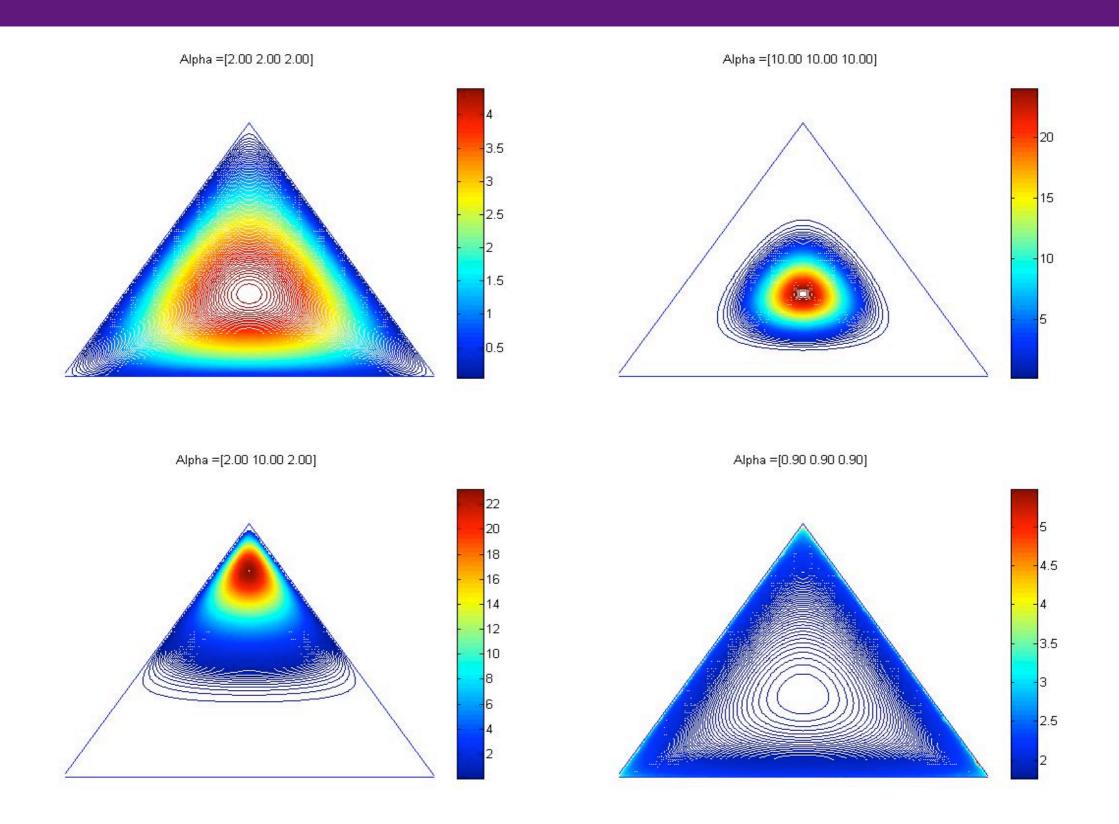
- Dirichlet distribution
  - Can only model which topics are hot
  - Does not model relationships between topics

# Correlated topic models

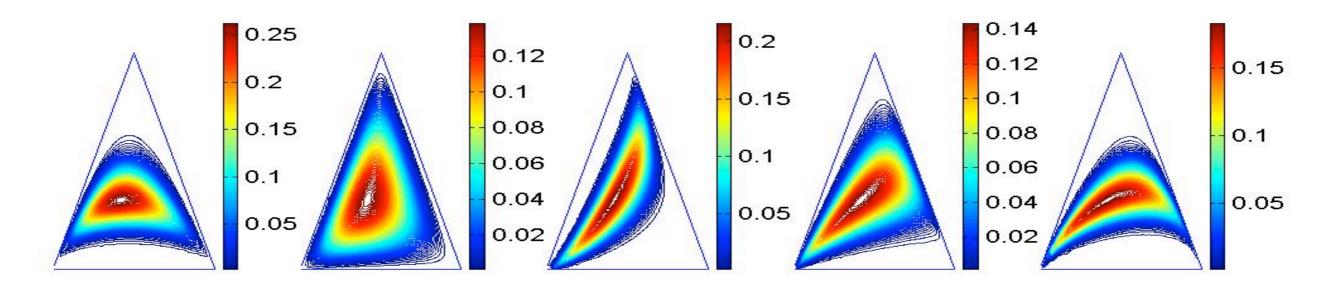
- Dirichlet distribution
  - Can only model which topics are hot
  - Does not model relationships between topics
- Key idea
  - We expect to see documents about sports and health but not about sports and politics
  - Uses a logistic normal distribution as a prior
- Conjugacy is no longer maintained
- Inference is harder than in LDA

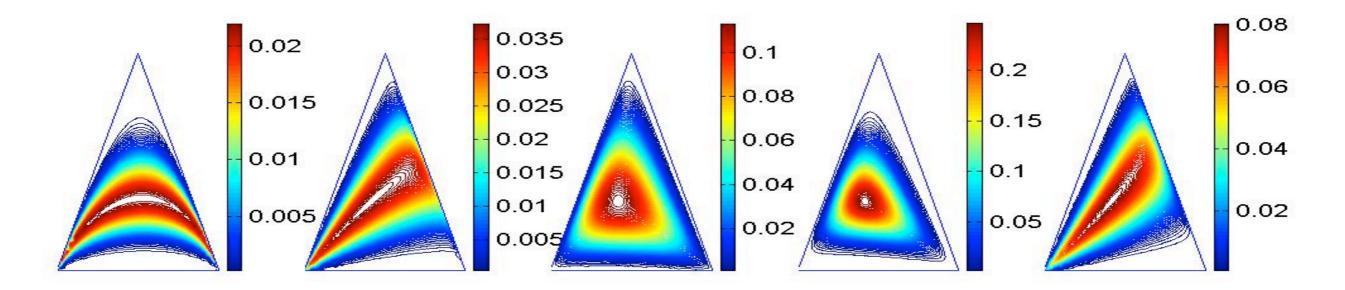
Blei & Lafferty 2005; Ahmed & Xing 2007

# Dirichlet prior on topics



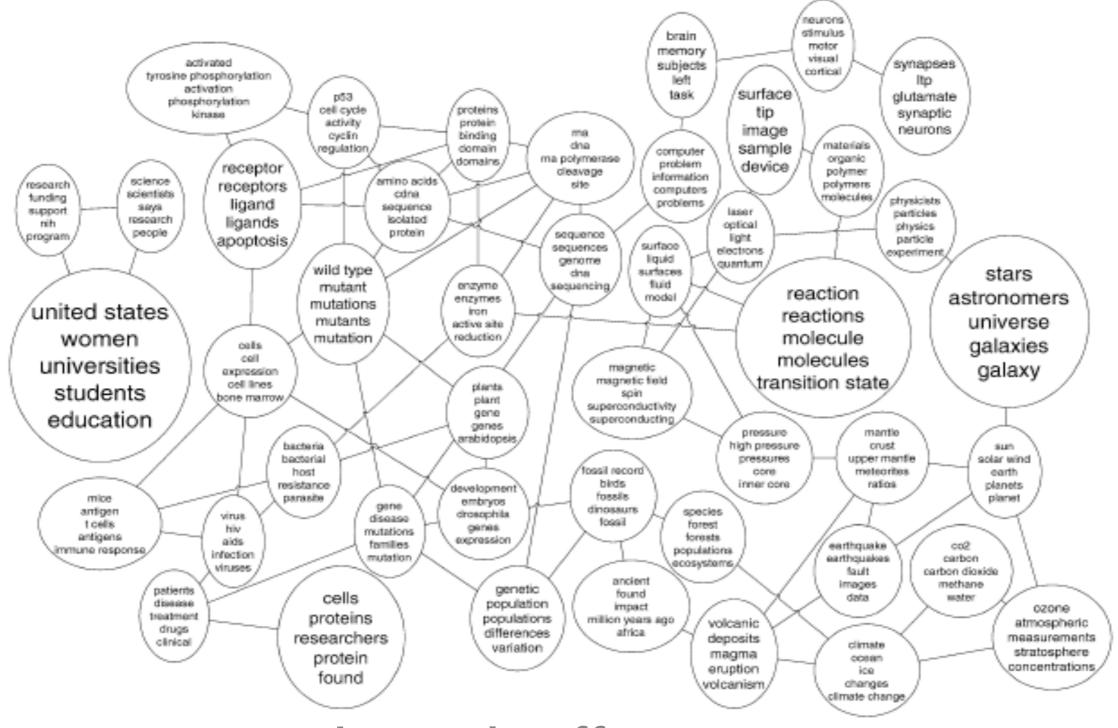
# Log-normal prior on topics





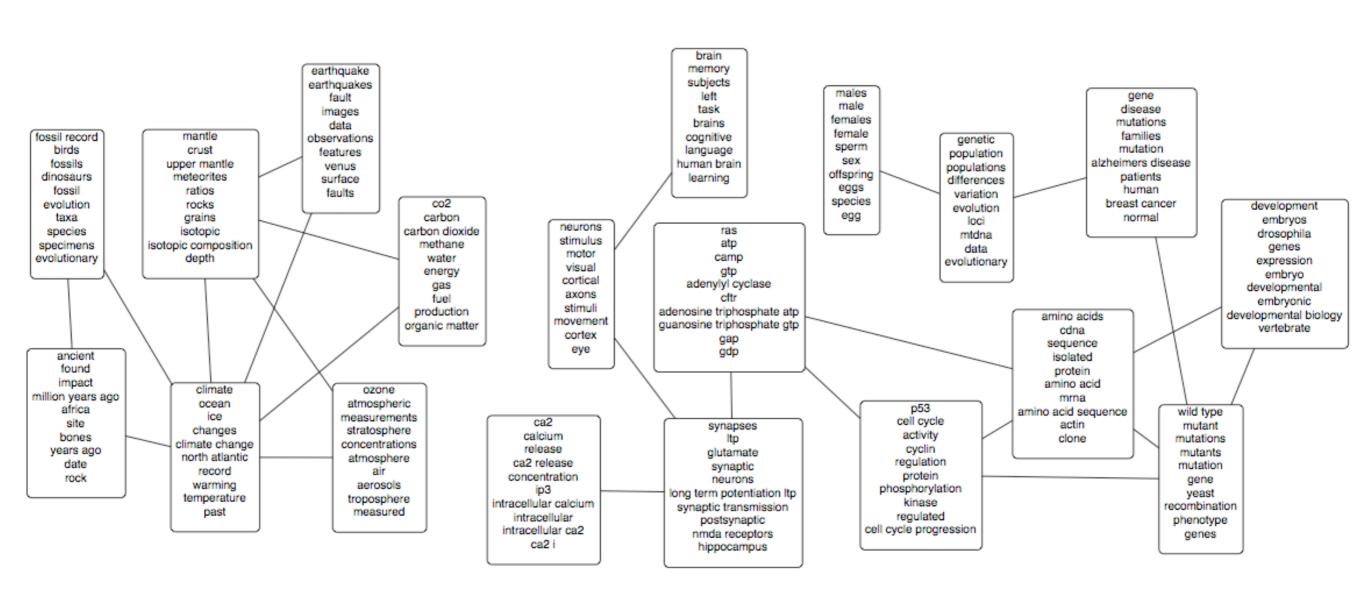
$$\theta = e^{\eta - g(\eta)} \ \, \text{with} \ \, \eta \sim \mathcal{N}(\mu, \Sigma)$$

# Correlated topics



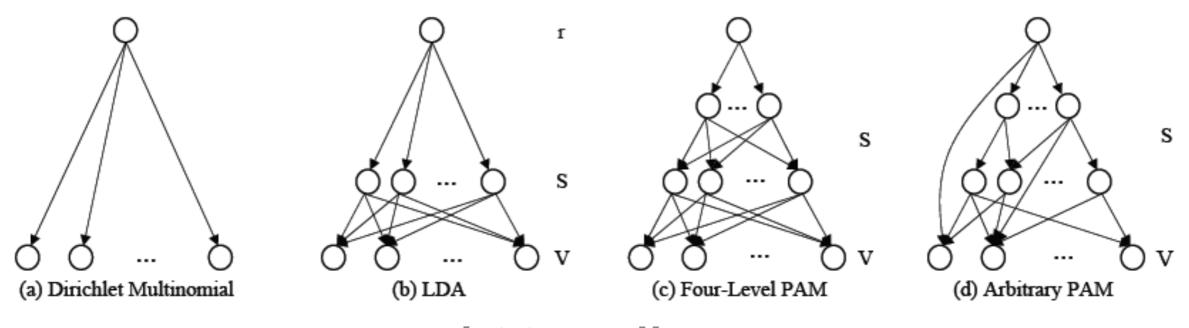
Blei and Lafferty 2005

# Correlated topics



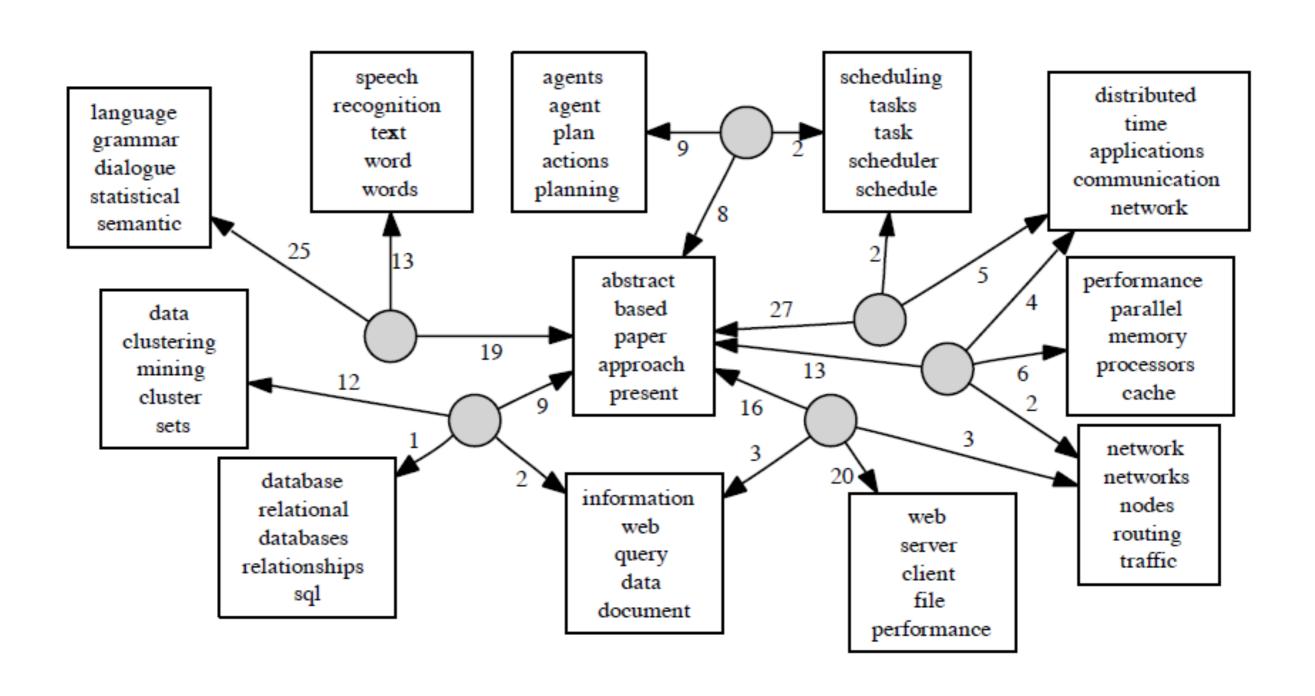
### Pachinko Allocation

- Model the prior as a Directed Acyclic Graph
- · Each document is modeled as multiple paths
- To sample a word, first select a path and then sample a word from the final topic
- The topics reside on the leaves of the tree



Li and McCallum 2006

### Pachinko Allocation



Li and McCallum 2006

# Topic Hierarchies

- Topics can appear anywhere in the tree
- Each document is modeled as
  - Single path over the tree (Blei et al., 2004)
  - Multiple paths over the tree (Mimno et al.,2007)

# Topic Hierarchies

the, of, a, to, and, in, is, for

Blei et al. 2004

neurons, visual, cells, cortex, synaptic, motion, response, processing algorithm, learning, training, method, we, new, problem, on

cell,
neuron,
circuit,
cells,
input,
i,
figure,
synapses

chip,
analog,
vlsi,
synapse,
weight,
digital,
cmos,
design

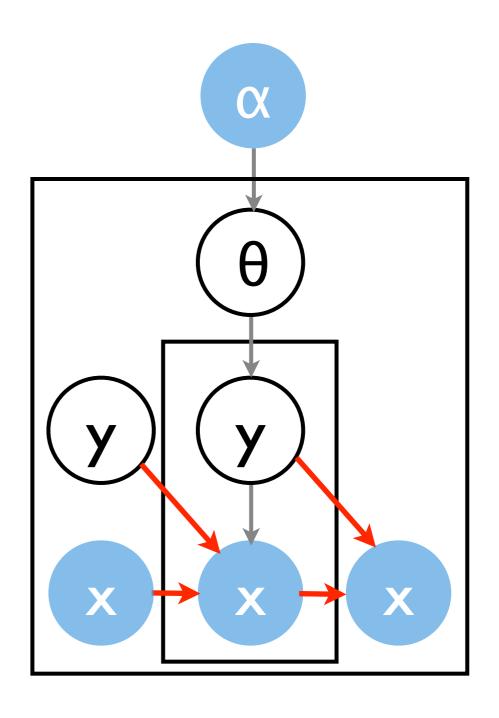
recognition,
speech,
character,
word,
system,
classification,
characters,
phonetic

b, x, e, n, p, any, if, training hidden,
units,
layer,
input,
output,
unit,
x,
vector

control,
reinforcement,
learning,
policy,
state,
actions,
value,
optimal

# Topical n-grams

- Documents as bag of words
- Exploit sequential structure
- N-gram models
  - Capture longer phrases
  - Switch variables to determine segments
  - Dynamic programming needed



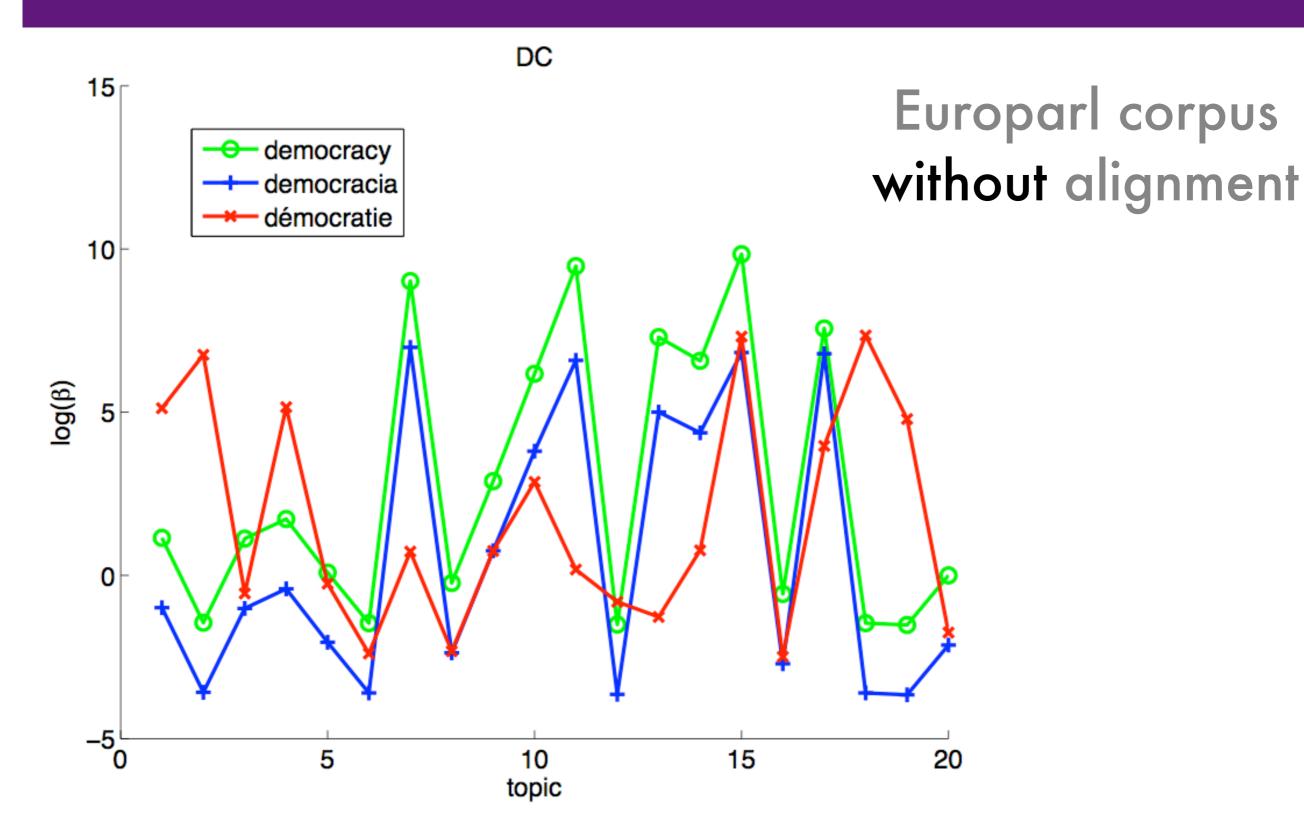
# Topic n-grams

	Speech Recognition	Support Vector Machines			
LDA	n-gram (2+)	n-gram (1)	LDA	n-gram (2+)	n-gram (1)
recognition	speech recognition	speech	kernel	support vectors	kernel
system	training data	word	linear	test error	training
word	neural network	training	vector	support vector machines	support
face	error rates	system	support	training error	margin
context	neural net	recognition	set	feature space	svm
character	hidden markov model	hmm	nonlinear	training examples	solution
hmm	feature vectors	speaker	data	decision function	kernels
based	continuous speech	performance	algorithm	cost functions	regularization
frame	training procedure	phoneme	space	test inputs	adaboost
segmentation	continuous speech recognition	acoustic	pca	kkt conditions	test
training	gamma filter	words	function	leave-one-out procedure	data
characters	hidden control	context	problem	soft margin	generalization
set	speech production	systems	margin	bayesian transduction	examples
probabilities	neural nets	frame	vectors	training patterns	cost
features	input representation	trained	solution	training points	convex
faces	output layers	sequence	training	maximum margin	algorithm
words	training algorithm	phonetic	svm	strictly convex	working
frames	test set	speakers	kernels	regularization operators	feature
database	speech frames	mlp	matrix	base classifiers	sv
mlp	speaker dependent	hybrid	machines	convex optimization	functions

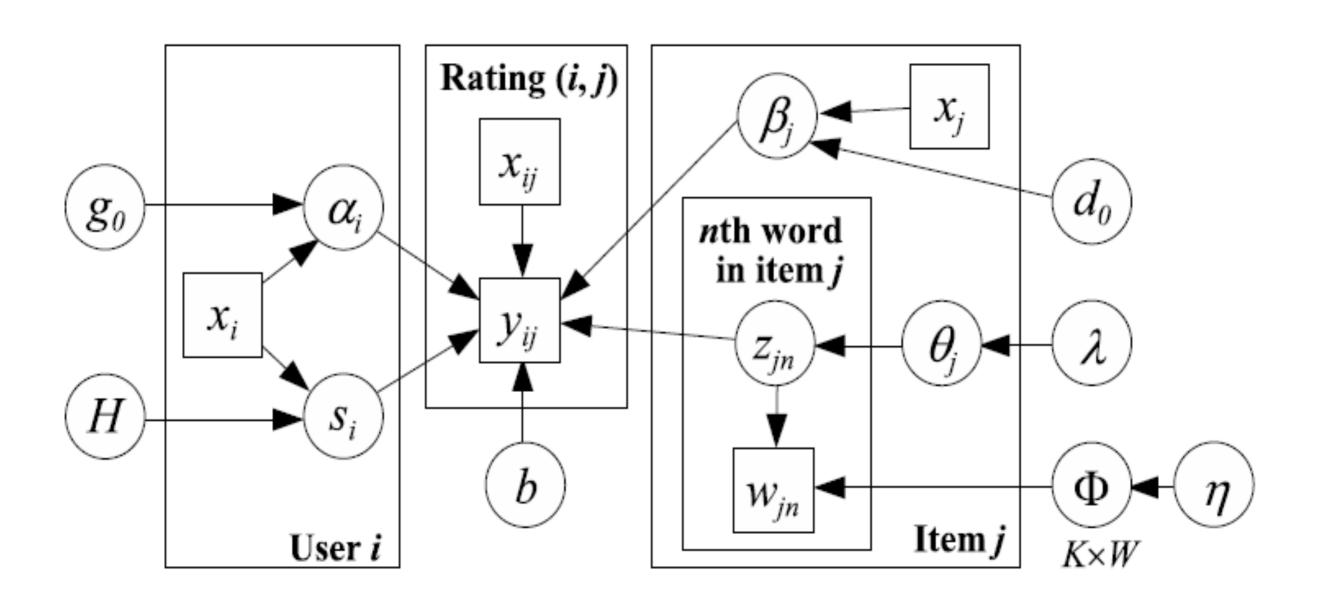
## Side information

- Upstream conditioning (Mimno et al., 2008)
  - Document features are informative for topics
  - Estimate topic distribution e.g. based on authors, links, timestamp
- Downstream conditioning (Petterson et al., 2010)
  - Word features are informative on topics
  - Estimate topic distribution for words e.g. based on dictionary, lexical similarity, distributional similarity
- Class labels (Blei and McAulliffe 2007; Lacoste, Sha and Jordan 2008; Zhu, Ahmed and Xing 2009)
  - Joint model of unlabeled data and labels
  - Joint likelihood semisupervised learning done right!

# Downstream conditioning

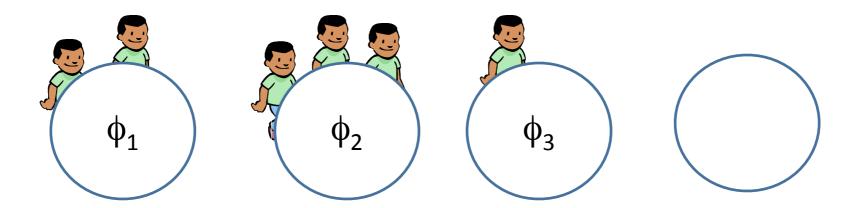


# Recommender Systems



Agarwal & Chen, 2010

### Chinese Restaurant Process



### Problem

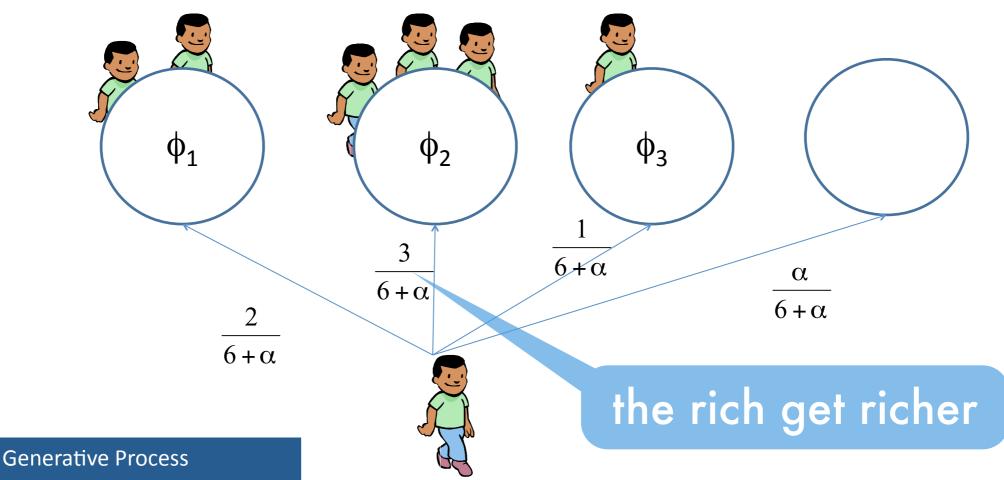
- How many clusters should we pick?
- How about a prior for infinitely many clusters?
- Finite model

$$p(y|Y,\alpha) = \frac{n(y) + \alpha_y}{n + \sum_{y'} \alpha_{y'}}$$

Infinite model
 Assume that the total smoother weight is constant

$$p(y|Y,\alpha) = \frac{n(y)}{n + \sum_{y'} \alpha_{y'}} \text{ and } p(\text{new}|Y,\alpha) = \frac{\alpha}{n + \alpha}$$

# Chinese Restaurant Metaphor

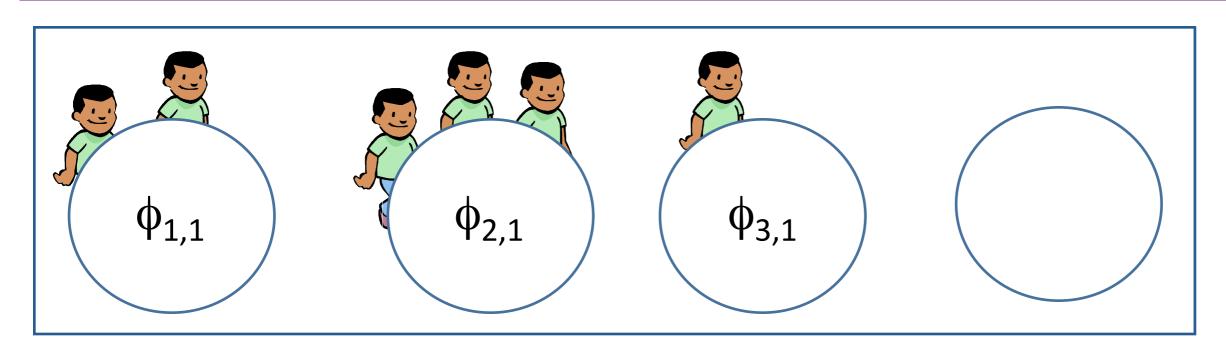


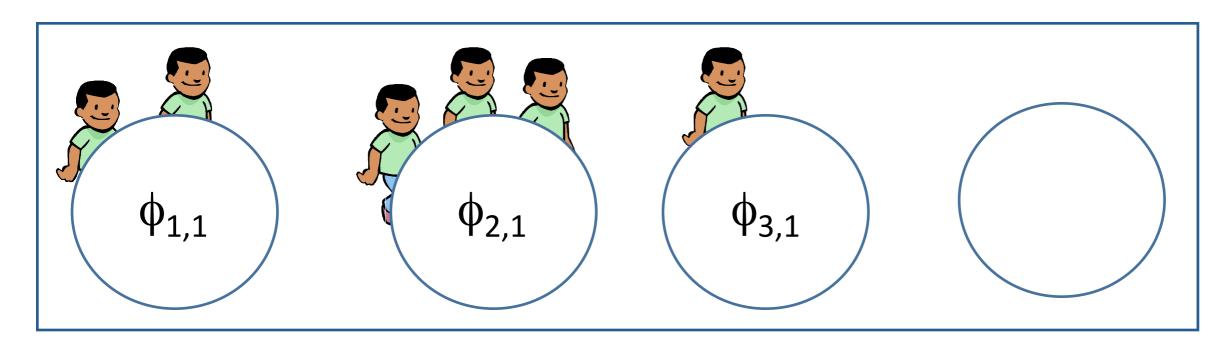
- -For data point x<sub>i</sub>
  - Choose table  $j \propto m_i$  and Sample  $x_{i} \sim f(\phi_i)$
  - Choose a new table K+1  $\propto \alpha$ 
    - Sample  $\phi_{K+1} \sim G_0$  and Sample  $x_i \sim f(\phi_{K+1})$

Pitman; Antoniak; Ishwaran; Jordan et al.; Teh et al.;

# Evolutionary Clustering

- Time series of objects, e.g. news stories
- Stories appear / disappear
- Want to keep track of clusters automatically

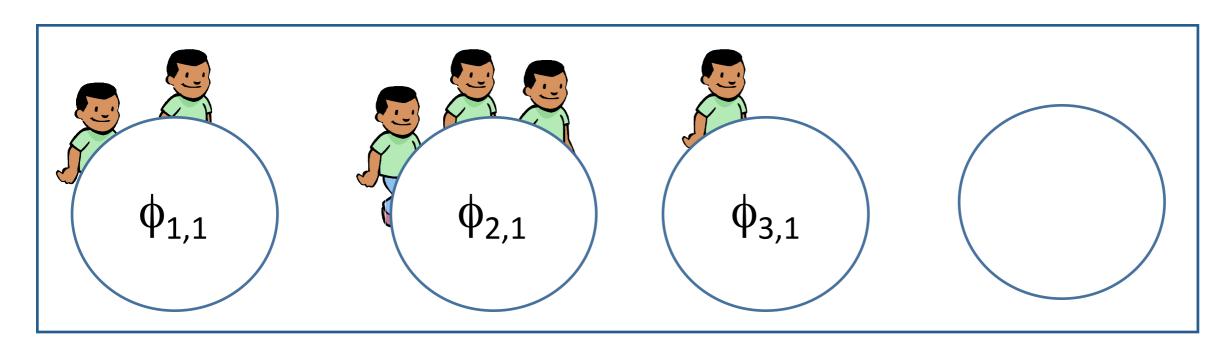




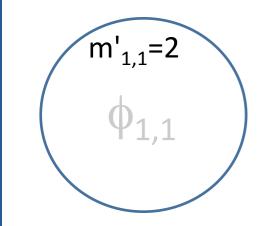
T=1

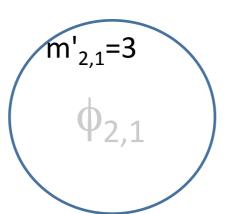
$$m'_{2,1}=3$$

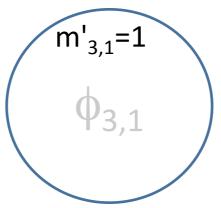
$$m'_{3,1}=1$$



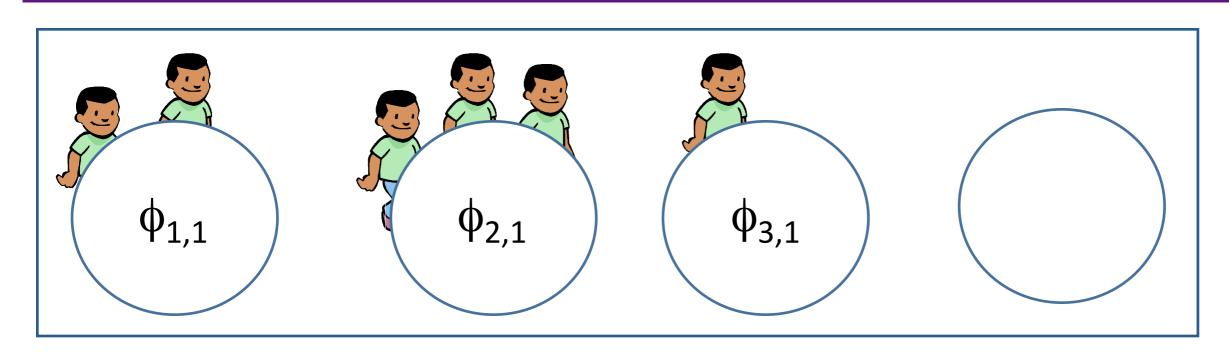
T=1



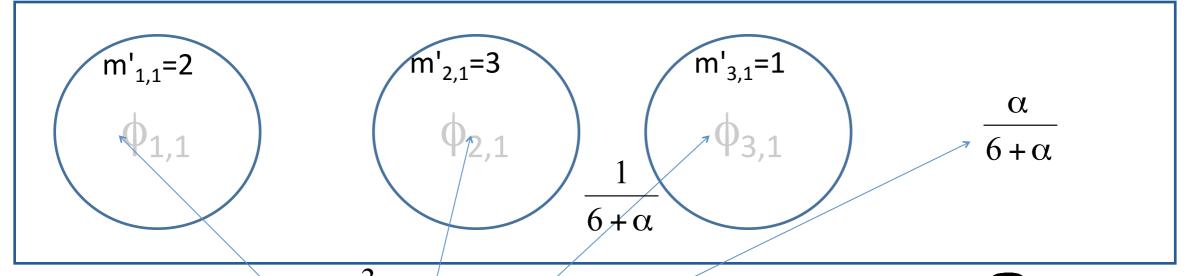


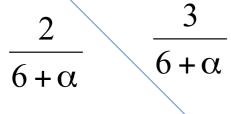




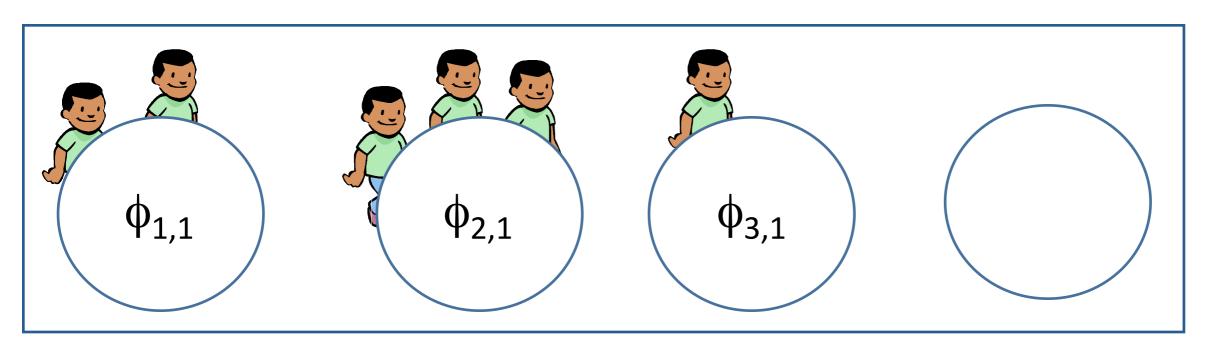


T=1

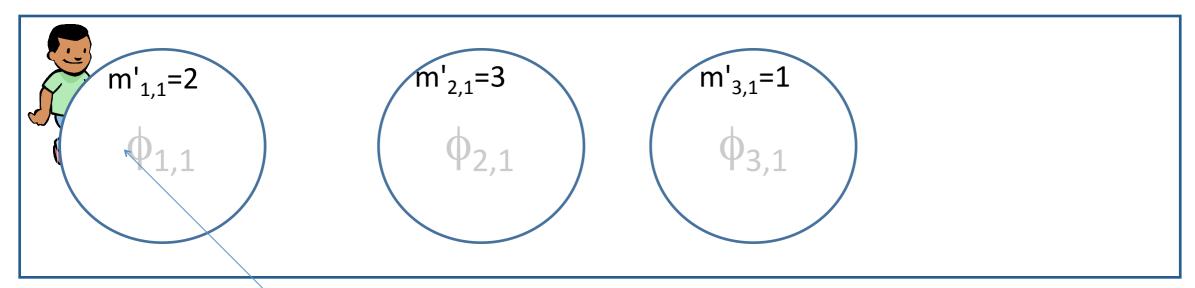






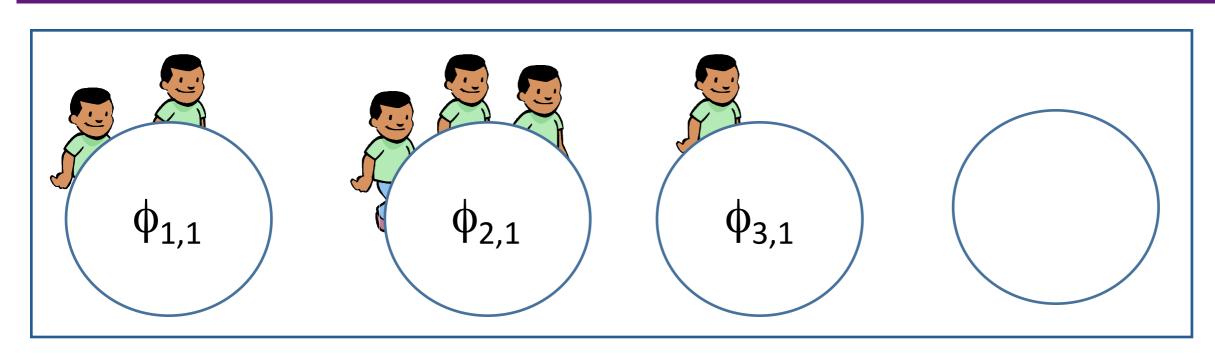


T=1

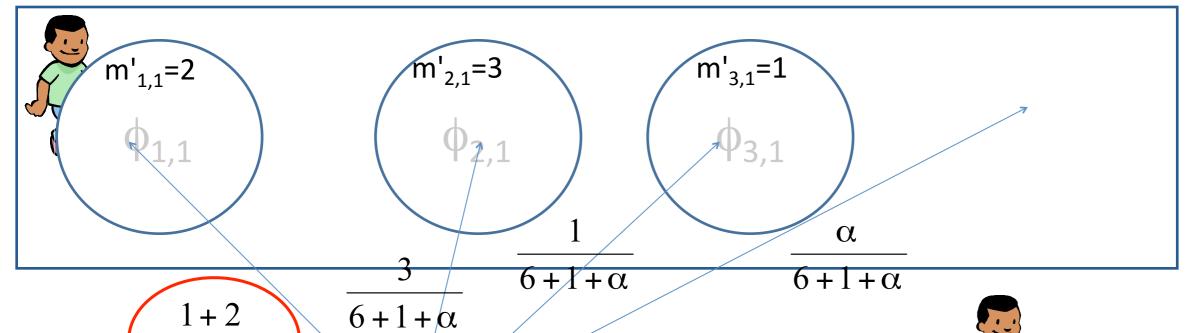


$$\frac{2}{6+\alpha}$$
Sample  $\phi_{1,2} \sim P(.|\phi_{1,1})$ 

### Recurrent Chinese Restaurant Process



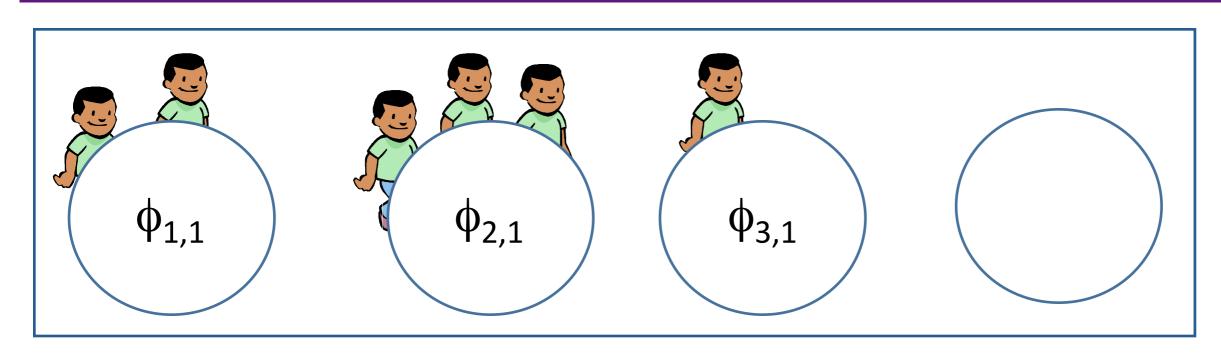
T=1



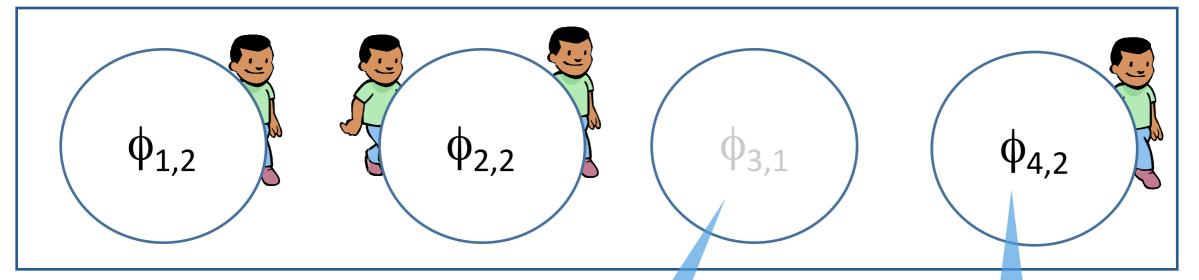
 $6+1+\alpha$ 

T=2

### Recurrent Chinese Restaurant Process



T=1

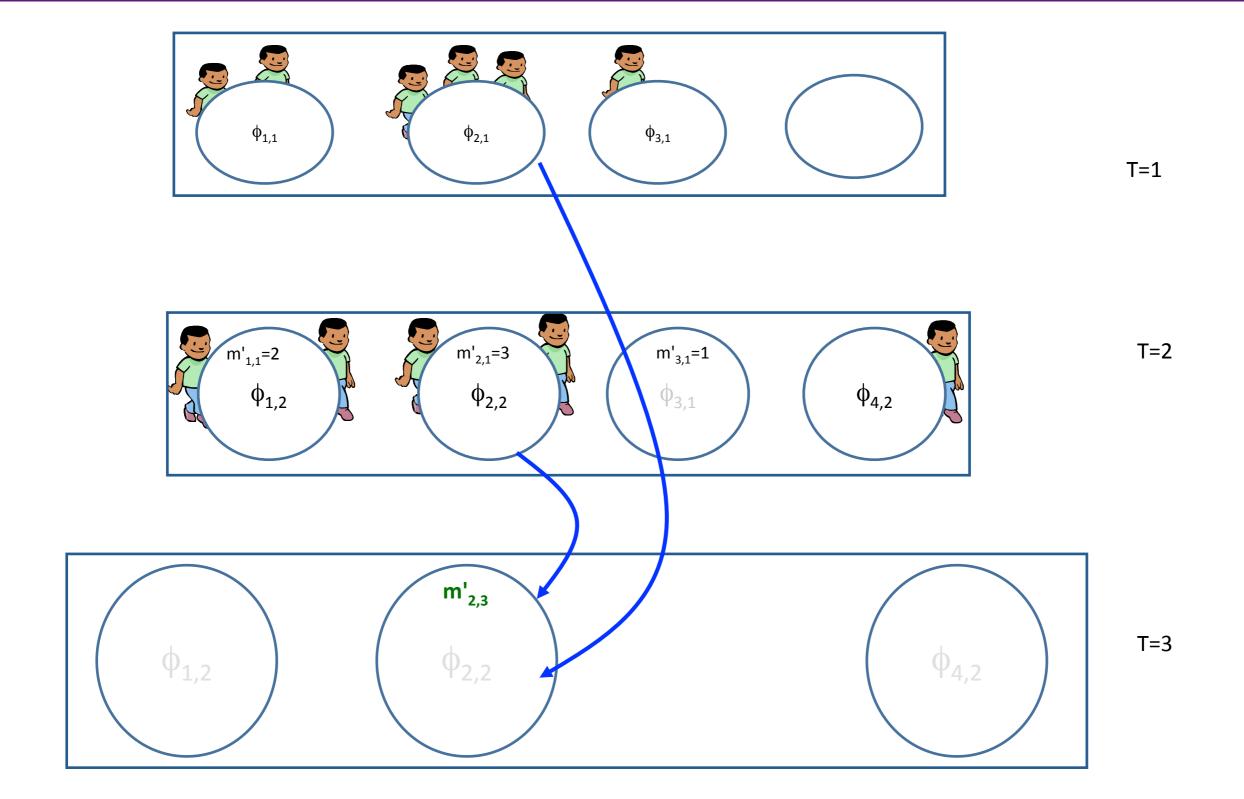


T=2

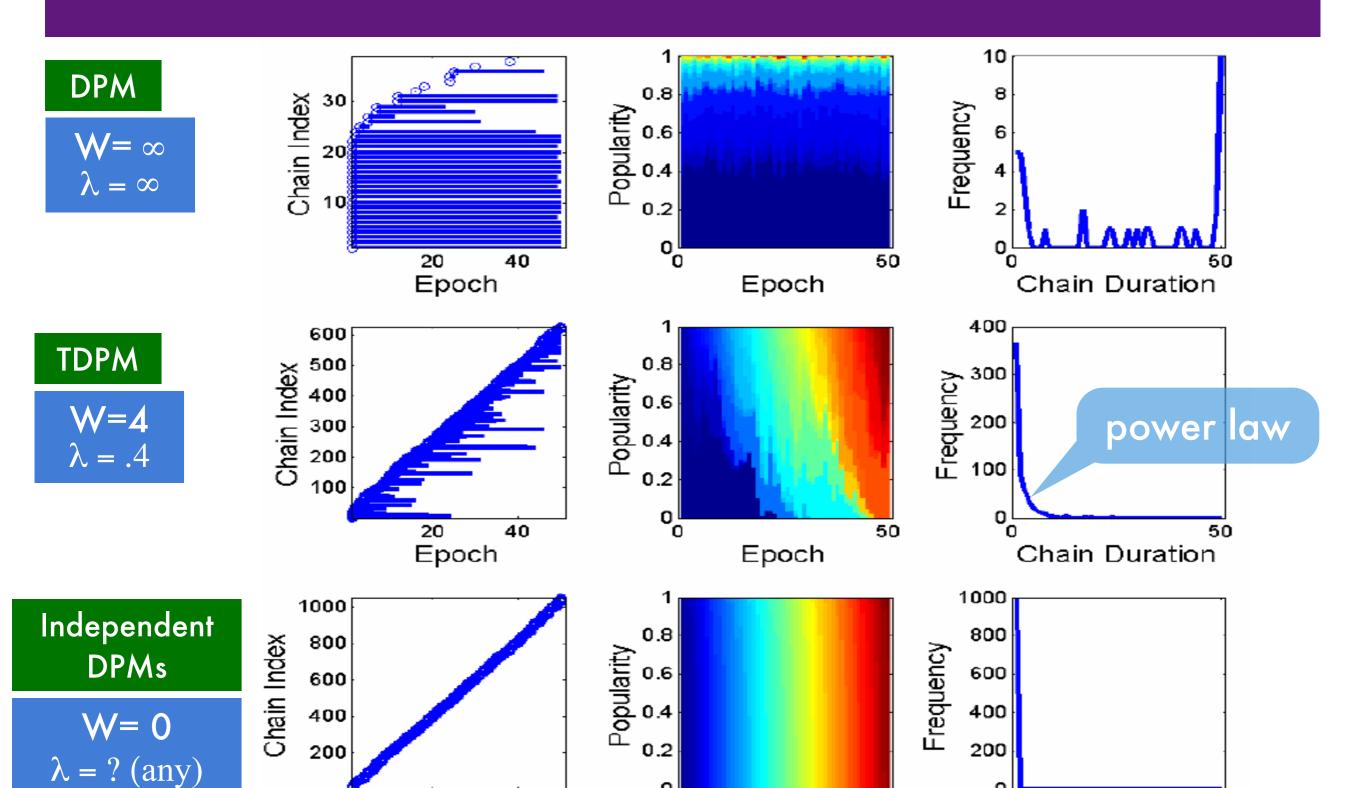
dead cluster

new cluster

# Longer History



### TDPM Generative Power



0,

50

**Epoch** 

20

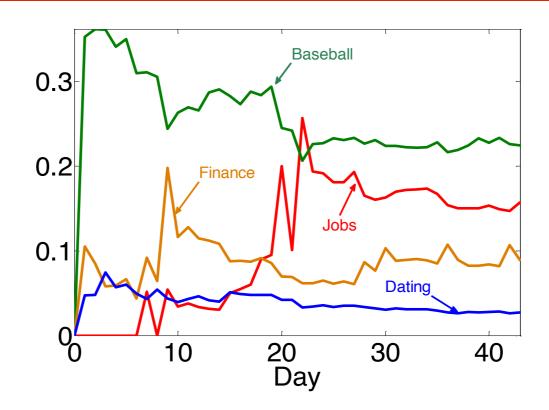
**Epoch** 

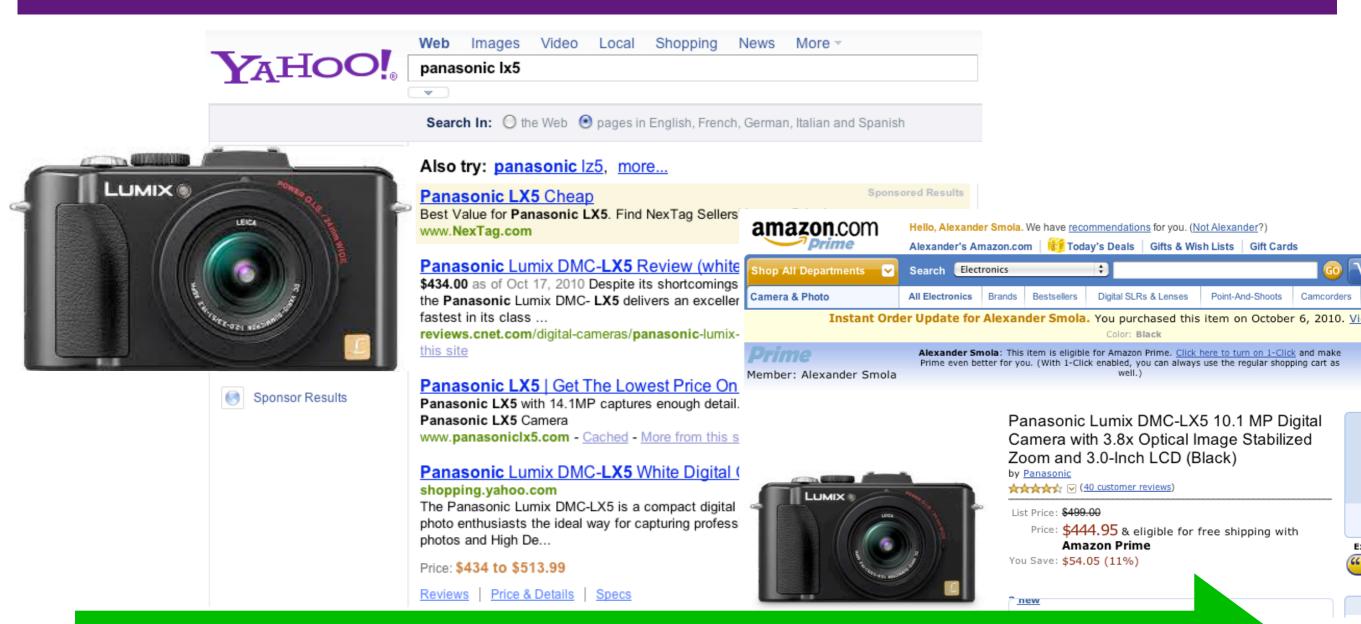
40

50

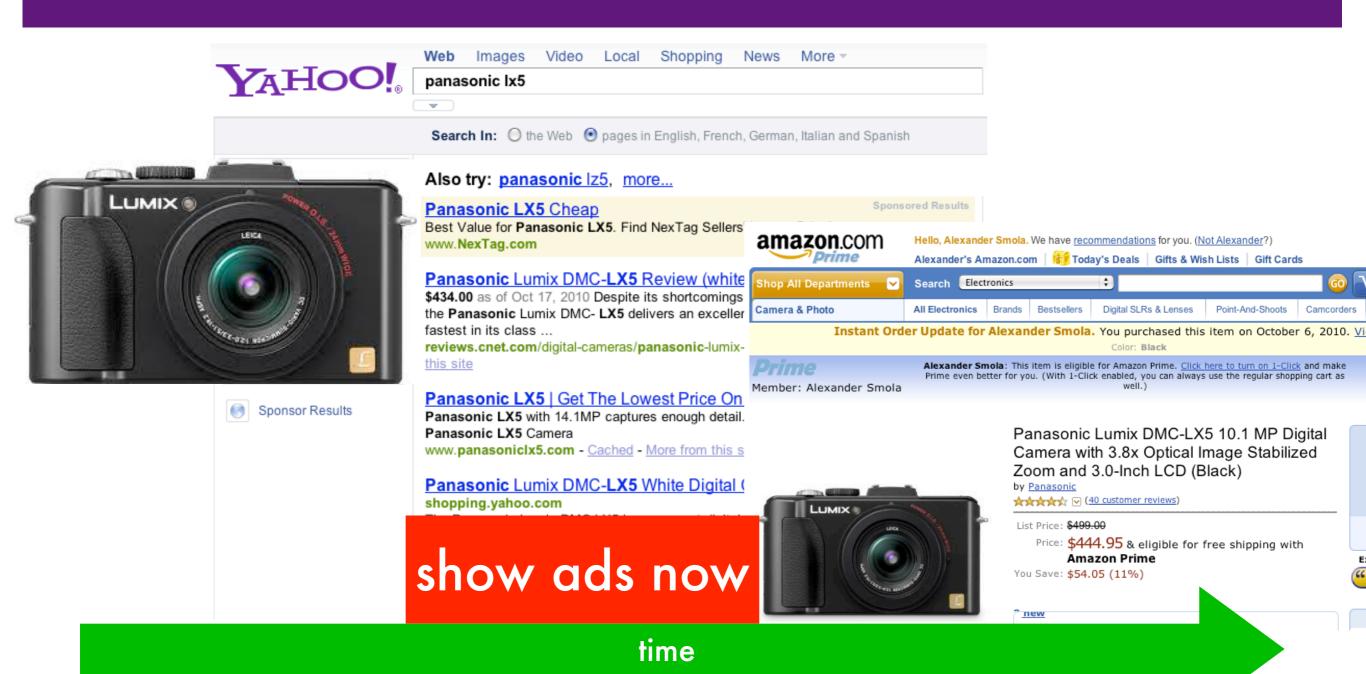
Chain Duration

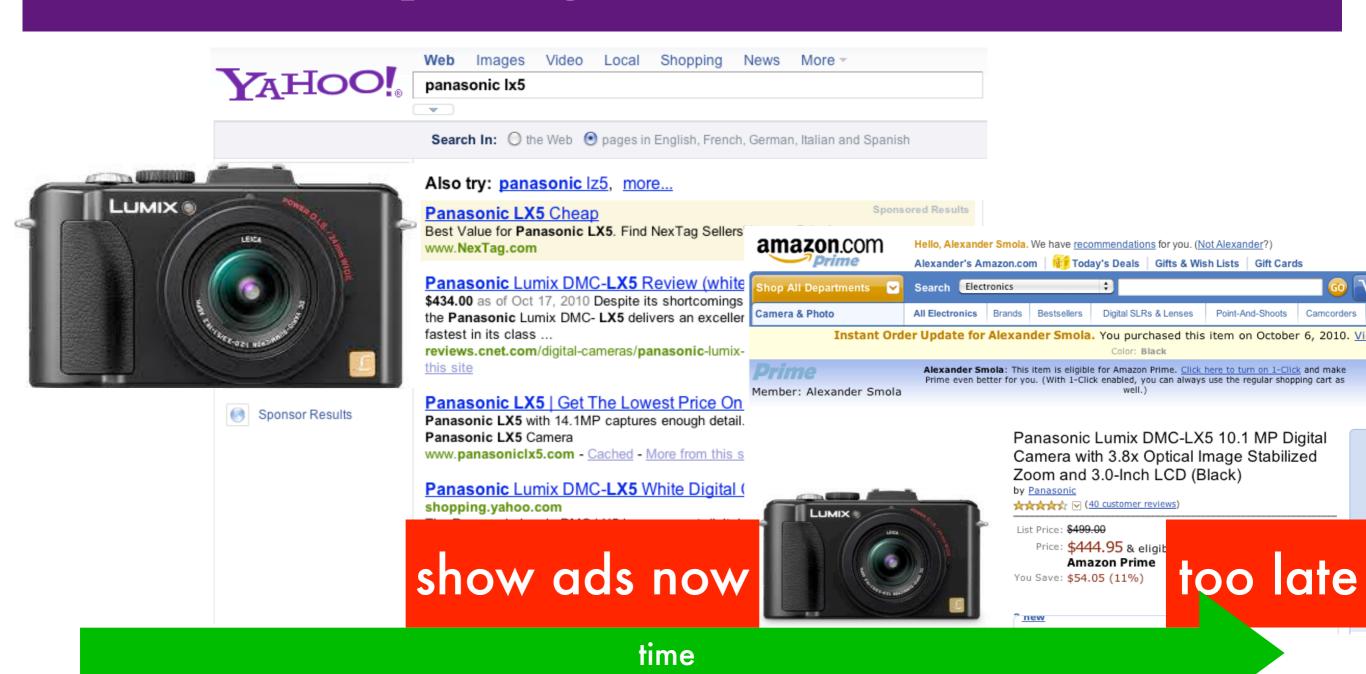
# User modeling





time







Car Deals van



job Hiring diet



1	

	Auto	Movies
Car	Price	Theatre
Deals	Used	Art
van	inspection	gallery

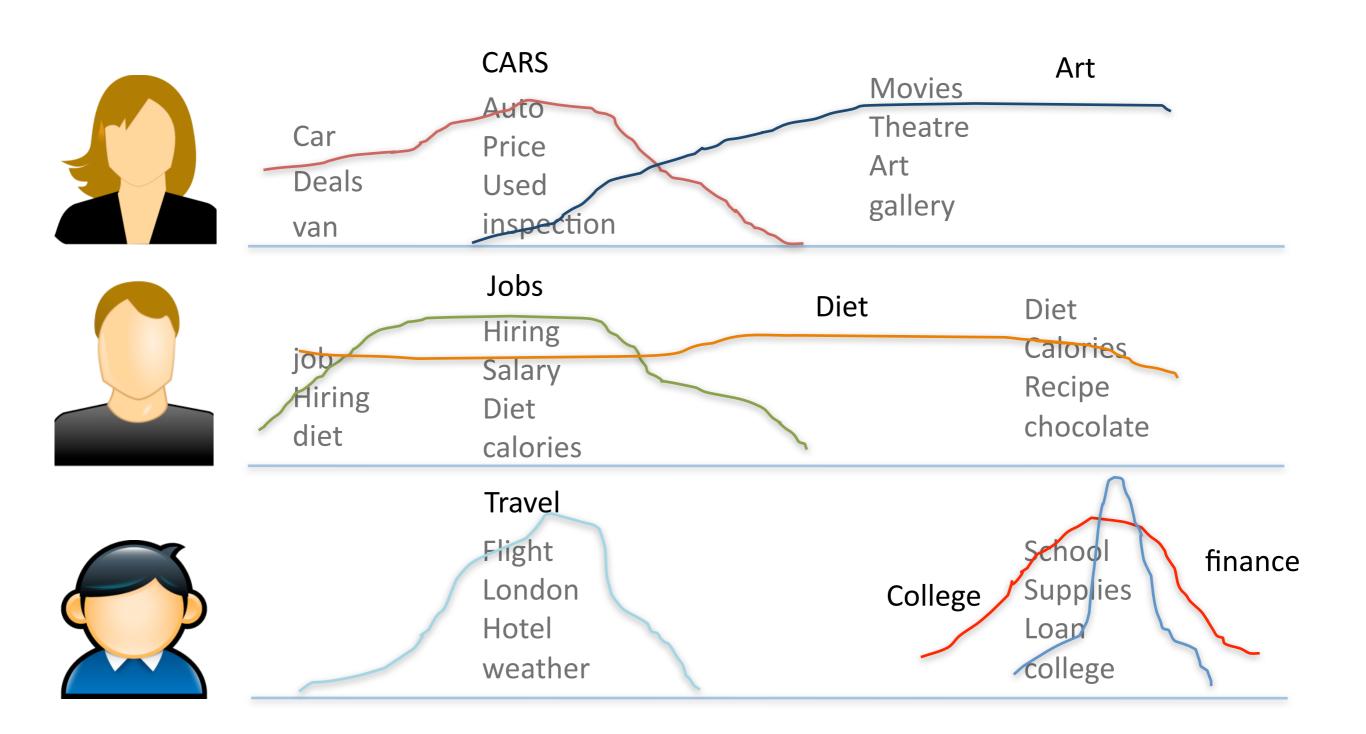


	l liuin a	DIEL
job	Hiring	Calories
	Salary	Recipe
Hiring	Diet	•
diet		chocolate
uict	calories	



Flight School
London Supplies
Hotel Loan
weather college

Diet



### User modeling

#### Input

- Queries issued by the user or Tags of watched content
- Snippet of page examined by user
- Time stamp of each action (day resolution)

#### Output

- Users' daily distribution over intents
- Dynamic intent representation



Flight London Hotel weather

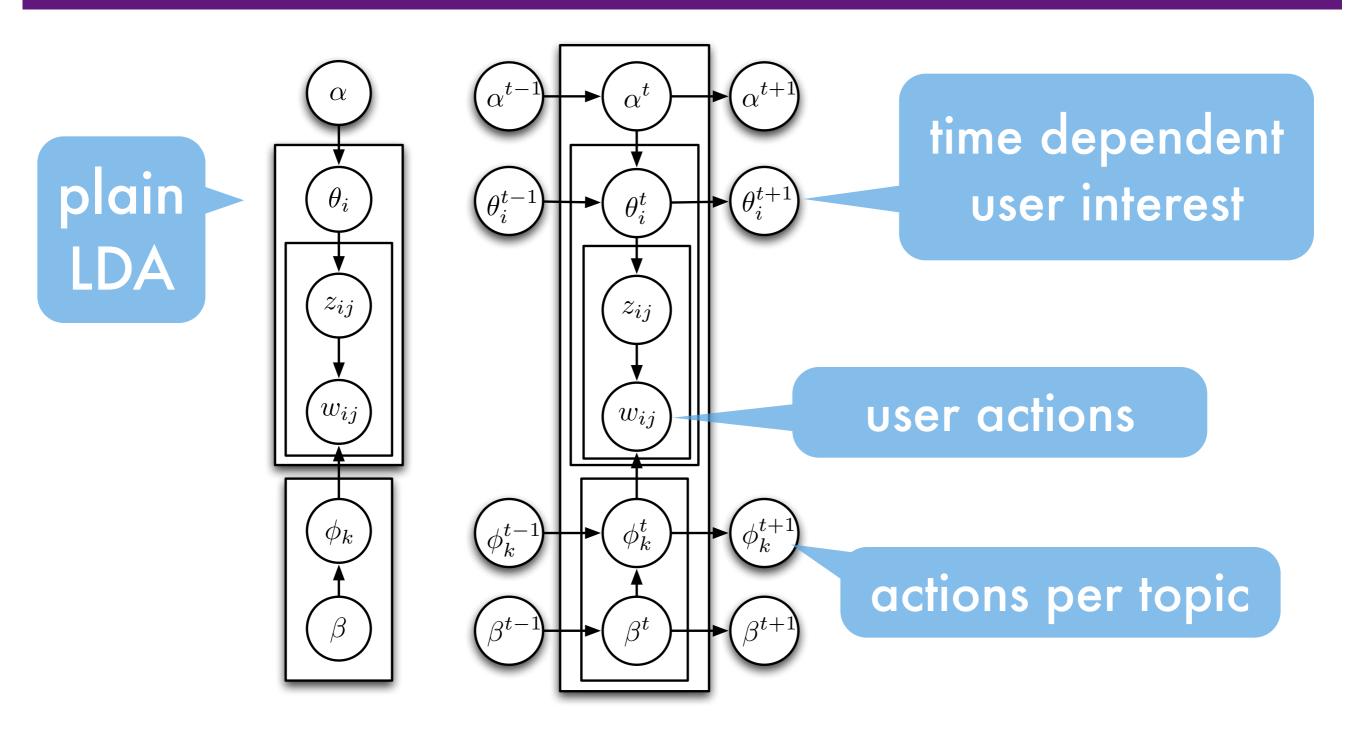
Travel

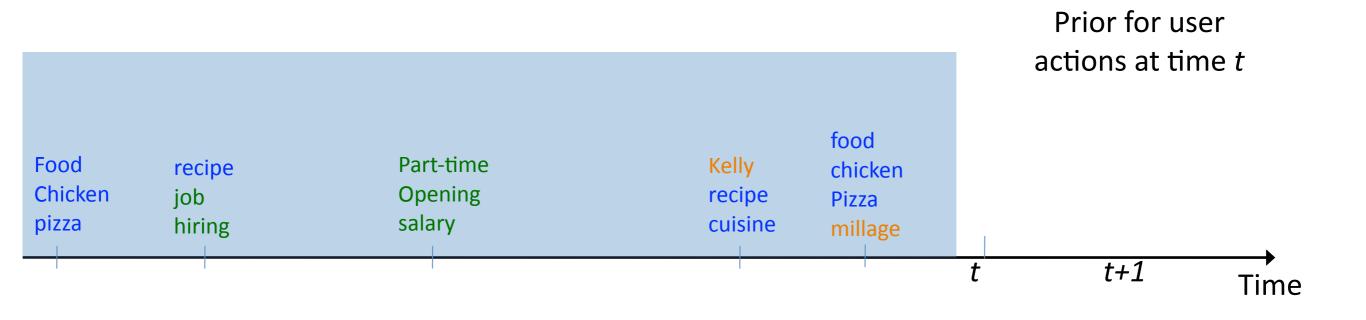
School finance
College Supplies
Loan
college

# Time dependent models

- LDA for topical model of users where
  - User interest distribution changes over time
  - Topics change over time
- This is like a Kalman filter except that
  - Don't know what to track (a priori)
  - Can't afford a Rauch-Tung-Striebel smoother
  - Much more messy than plain LDA

# Graphical Model





Recipe
Chocolate
Pizza
Food
Chicken
Milk
Butter
Powder

Diet

Car Blue Book Kelley Prices Small Speed large

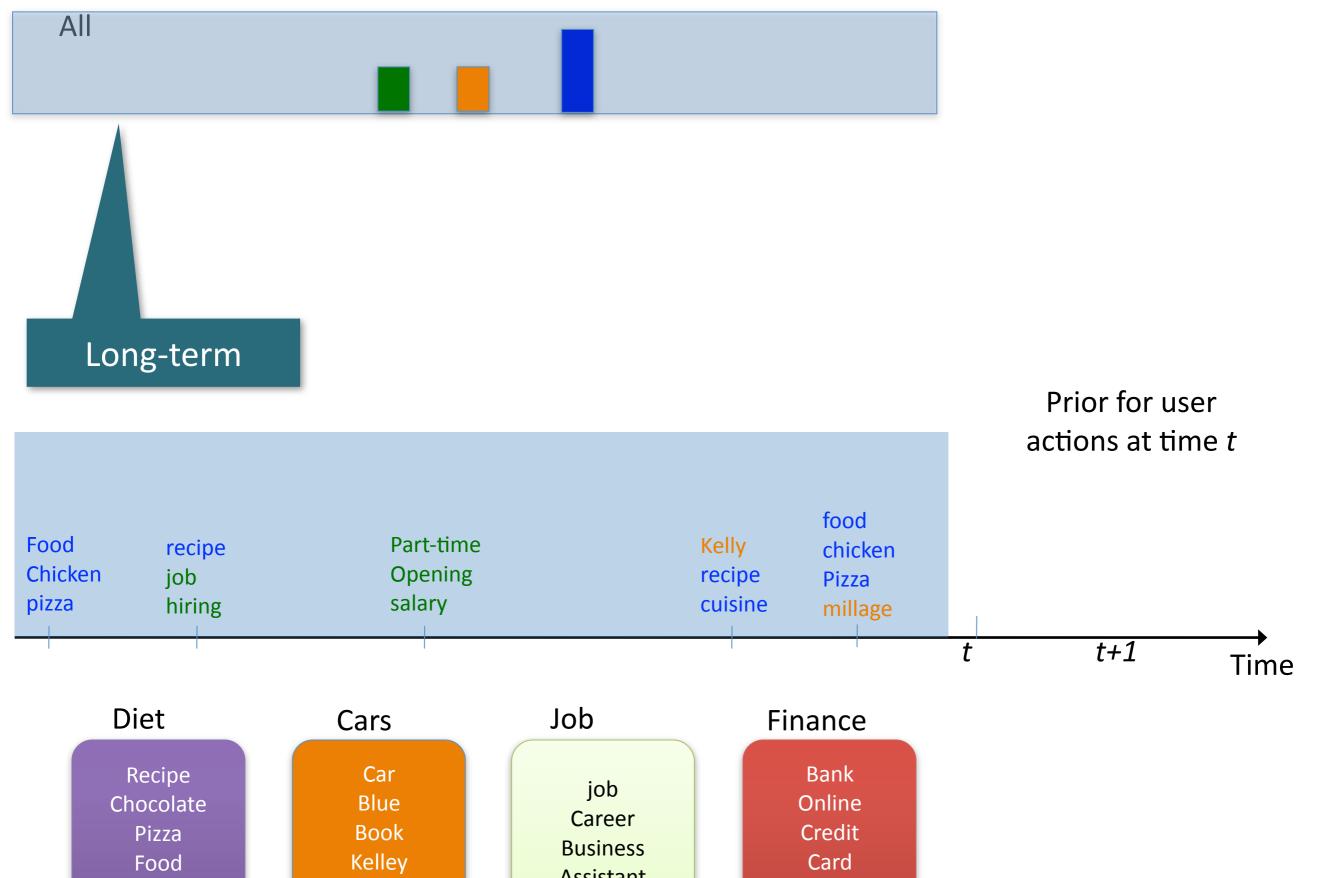
Cars

job
Career
Business
Assistant
Hiring
Part-time
Receptionist

Job

Bank
Online
Credit
Card
debt
portfolio
Finance
Chase

**Finance** 

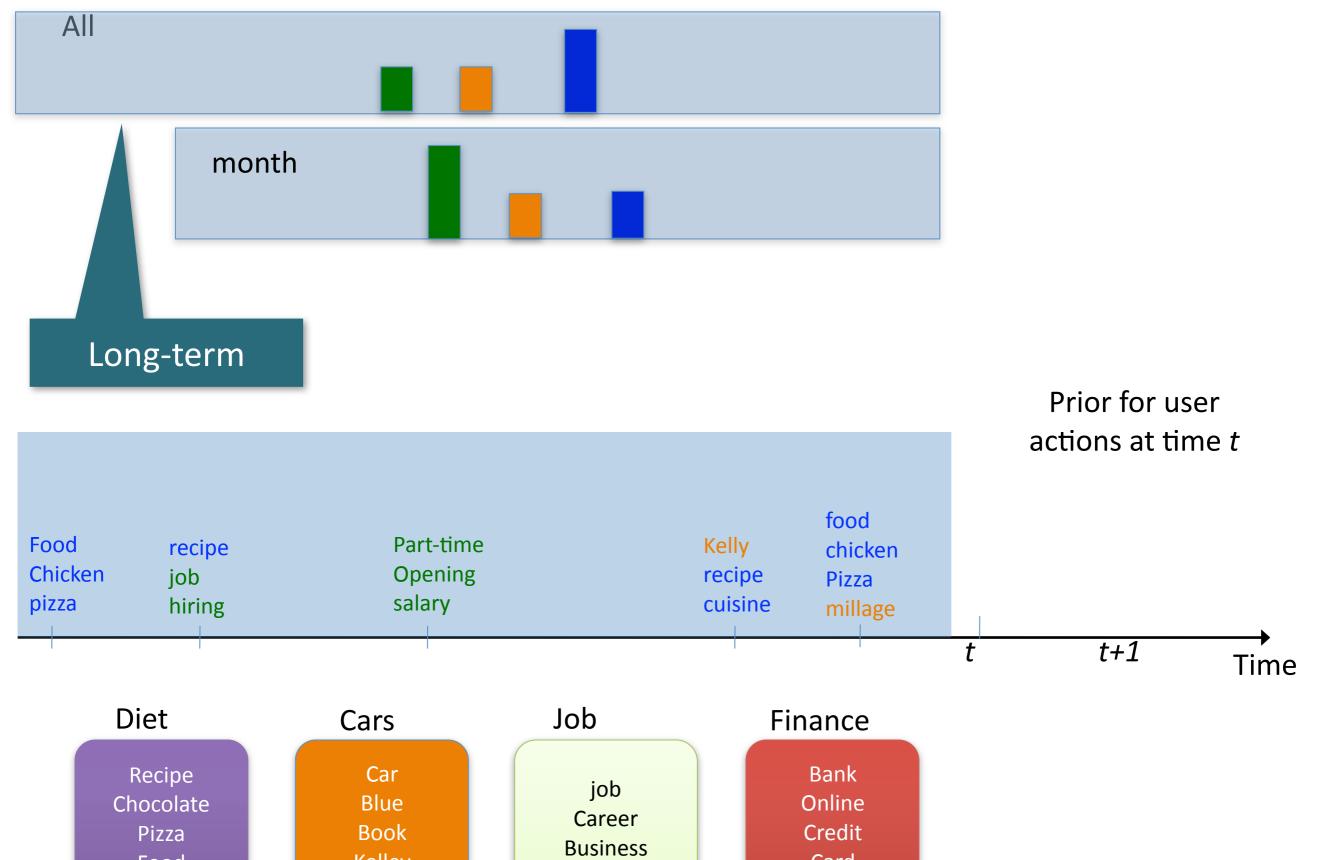


Chicken Milk Butter Powder

Prices Small Speed large

**Assistant** Hiring Part-time Receptionist

debt portfolio Finance Chase

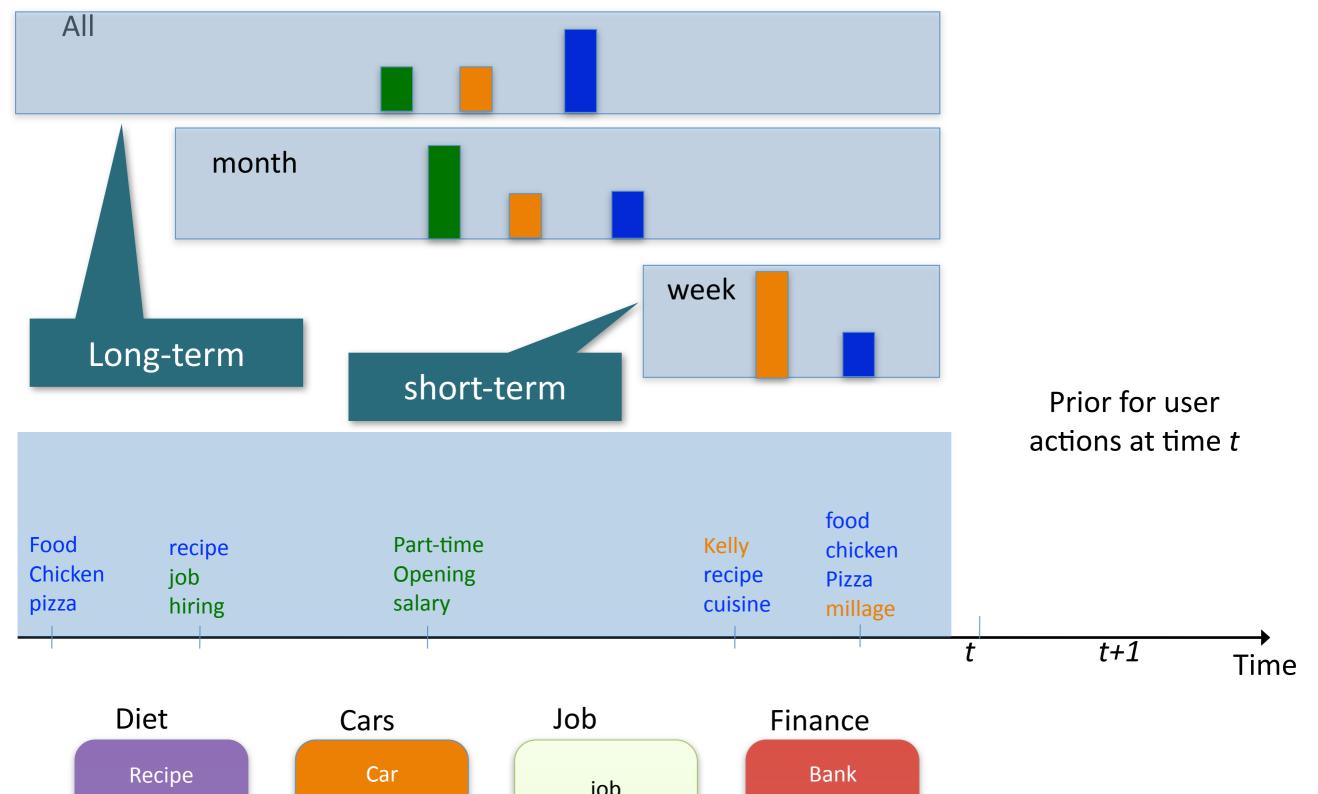


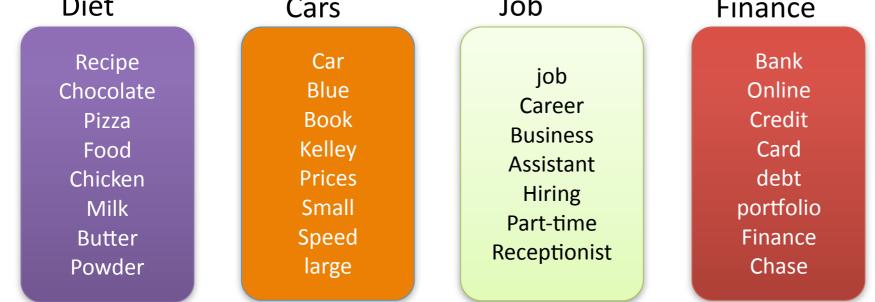
Recipe
Chocolate
Pizza
Food
Chicken
Milk
Butter
Powder

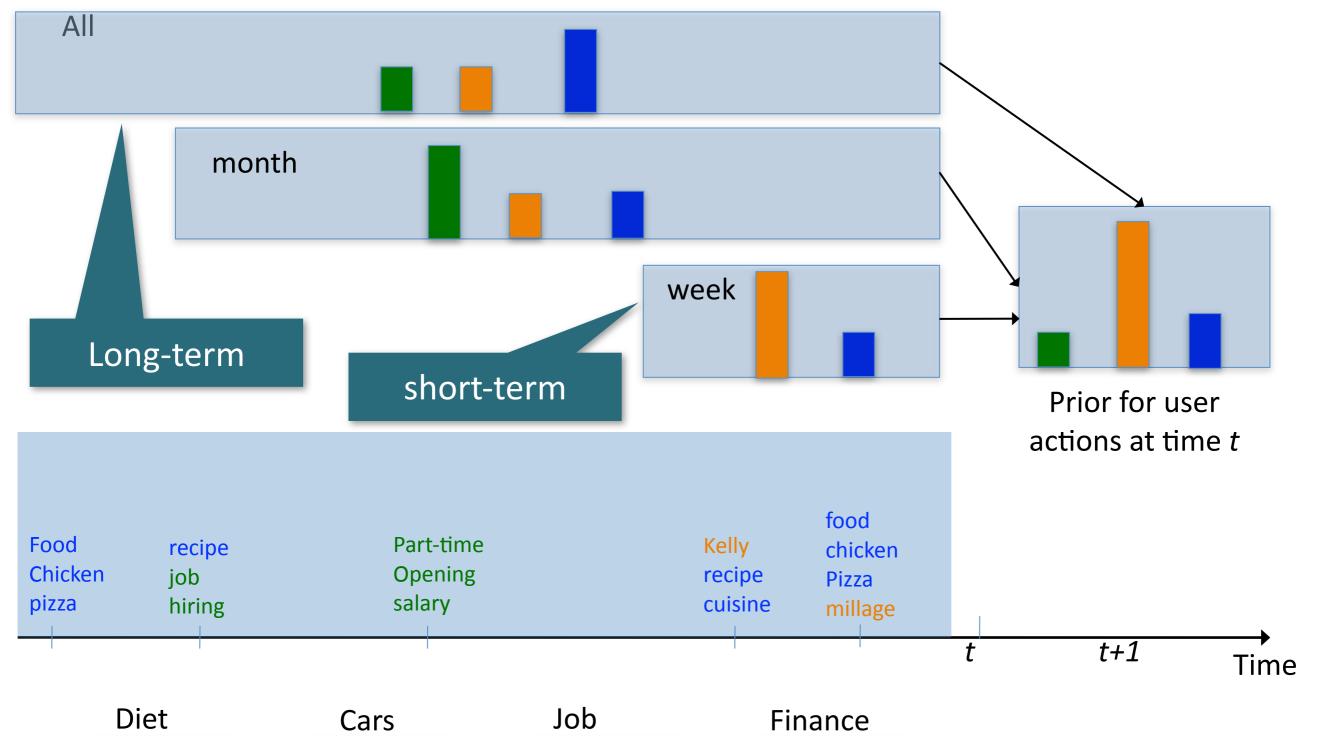
Car Blue Book Kelley Prices Small Speed large

job
Career
Business
Assistant
Hiring
Part-time
Receptionist

Bank
Online
Credit
Card
debt
portfolio
Finance
Chase





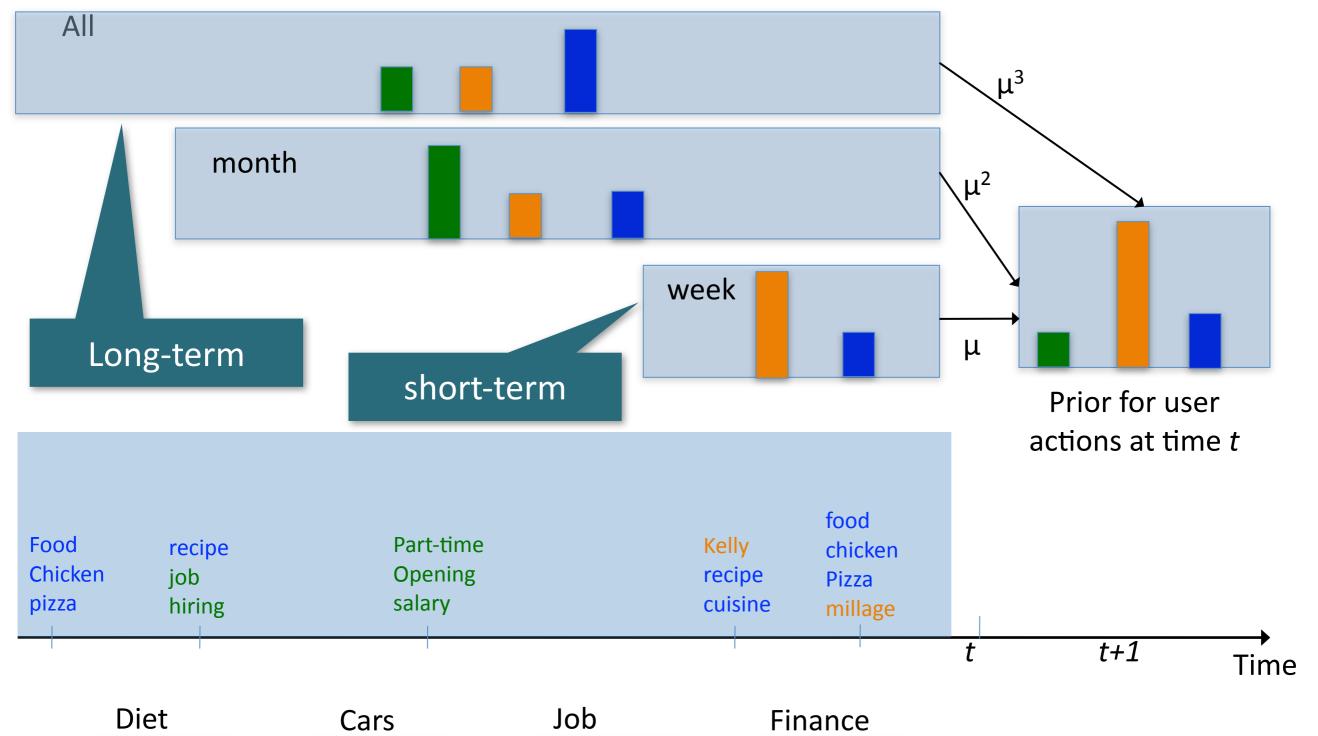




Car Blue Book Kelley Prices Small Speed large

job
Career
Business
Assistant
Hiring
Part-time
Receptionist

Bank
Online
Credit
Card
debt
portfolio
Finance
Chase

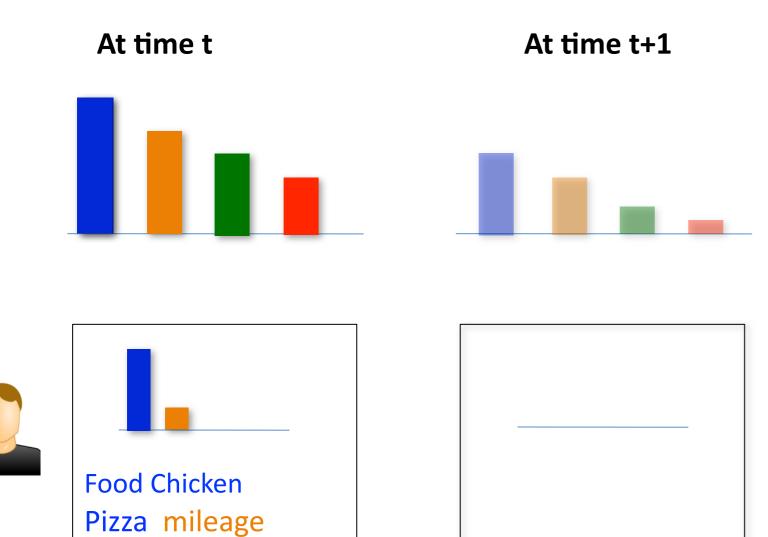




Car Blue Book Kelley Prices Small Speed large

job
Career
Business
Assistant
Hiring
Part-time
Receptionist

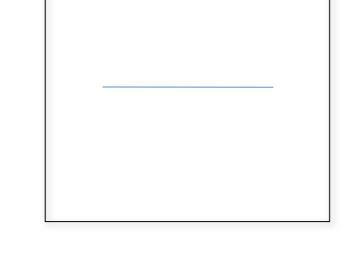
Bank
Online
Credit
Card
debt
portfolio
Finance
Chase





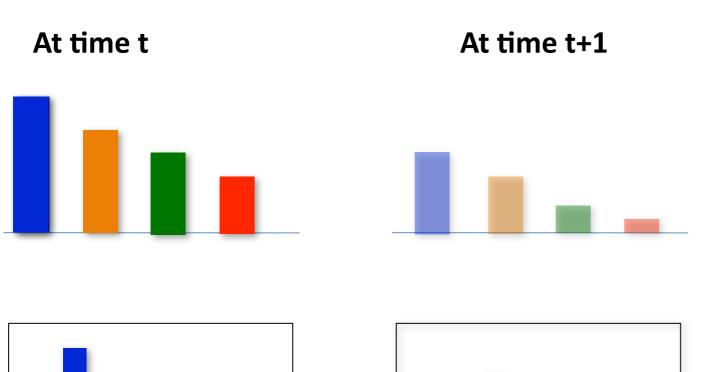
job Bank Career Online Business Credit Assistant Card Hiring debt Part-time portfolio Receptioni Finance Chase st







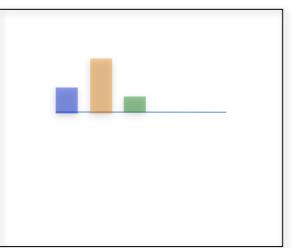






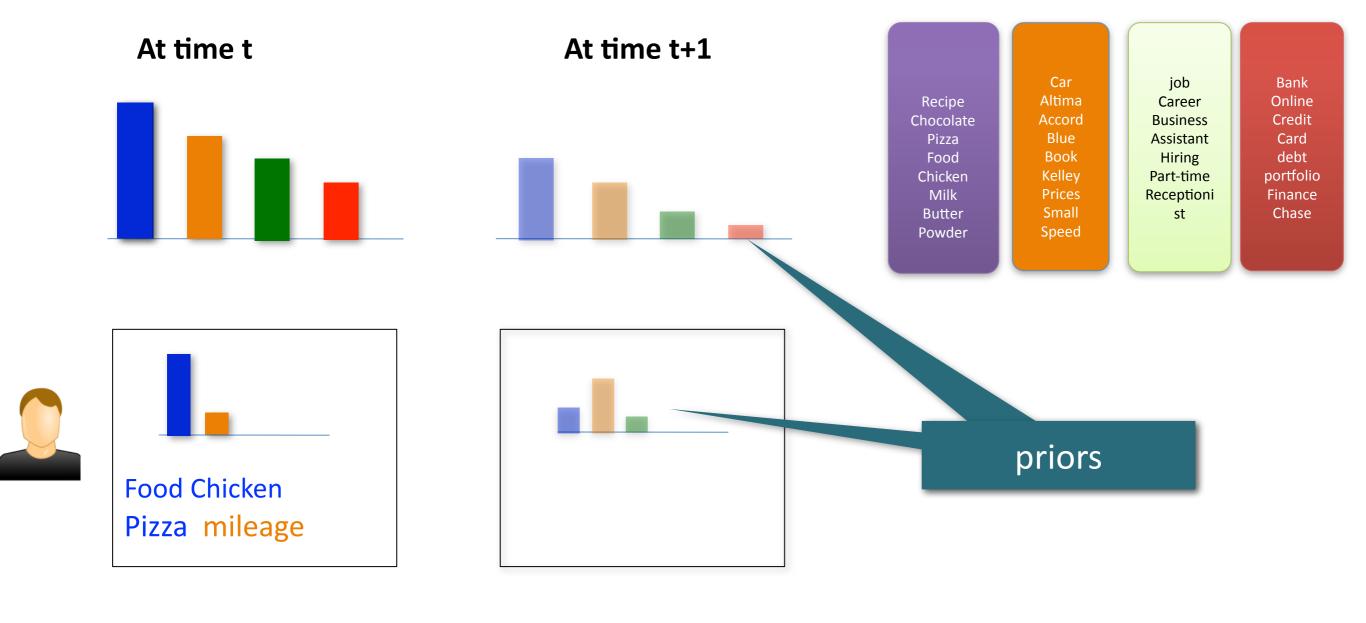






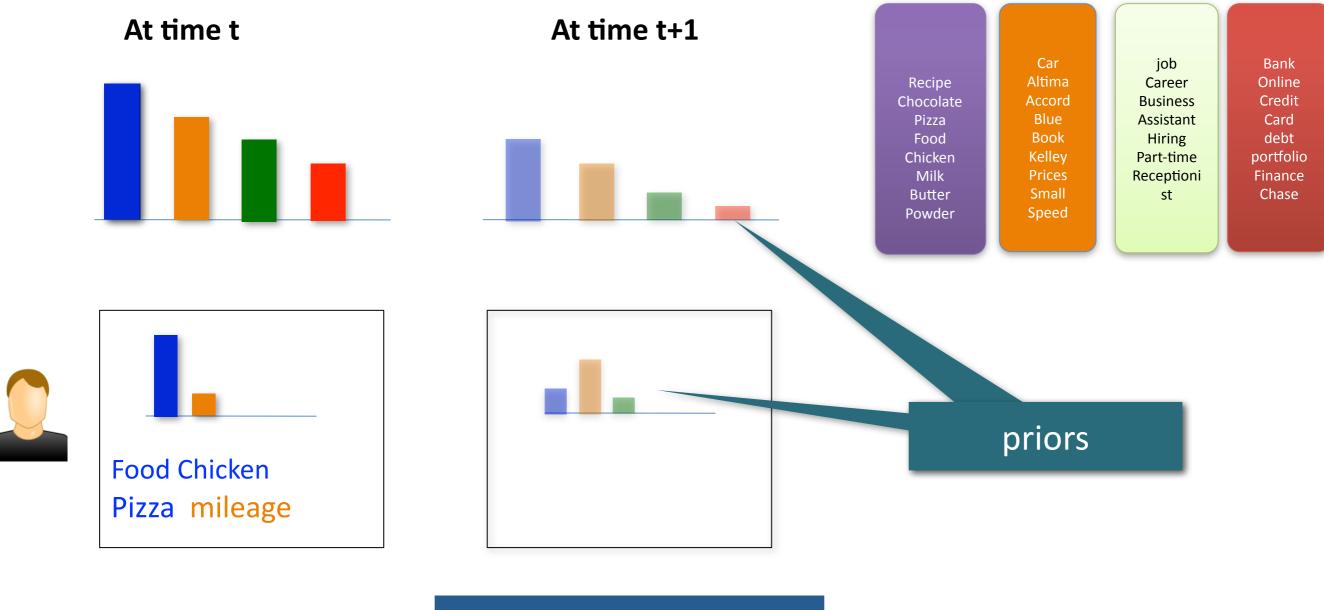










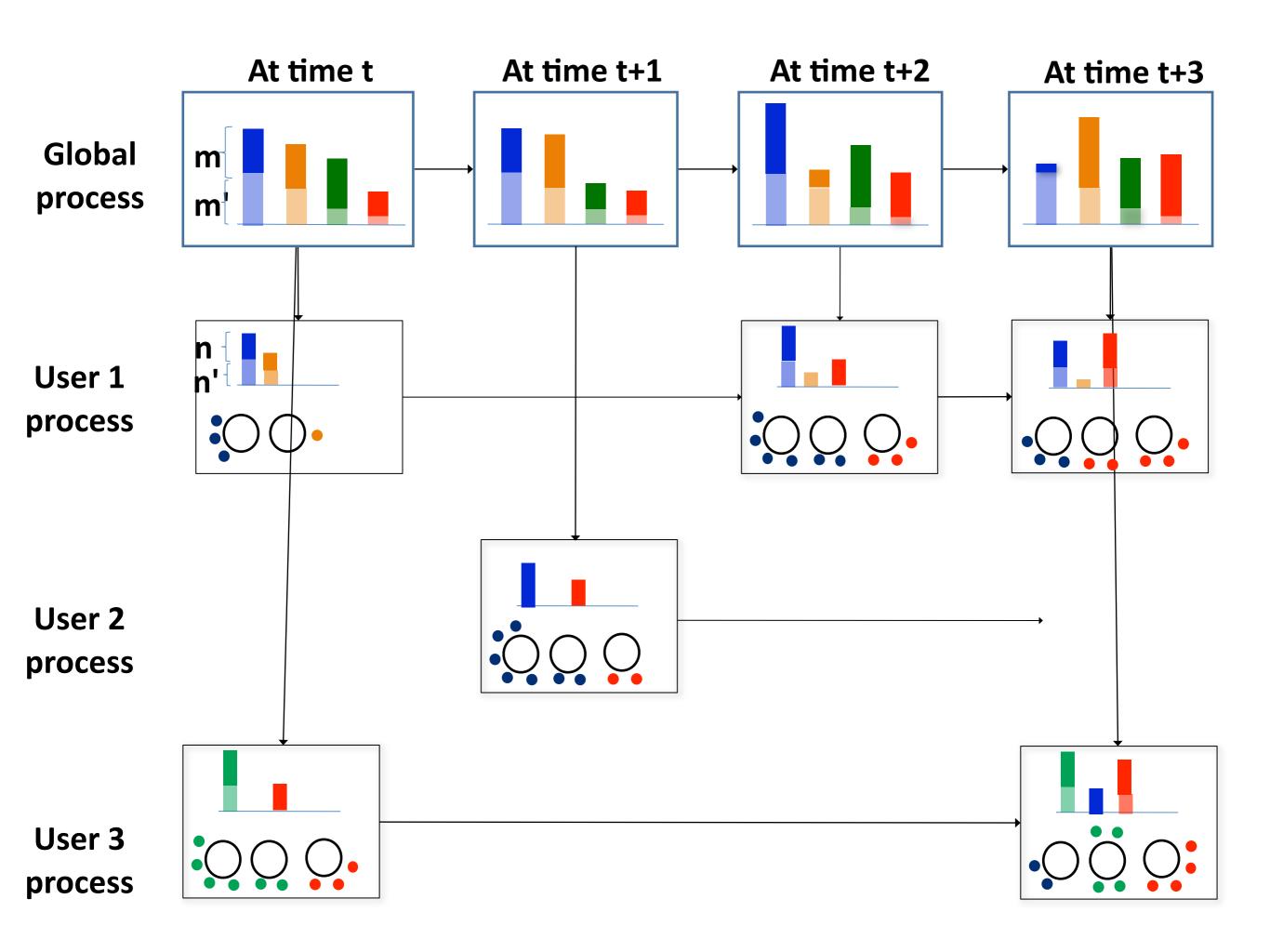




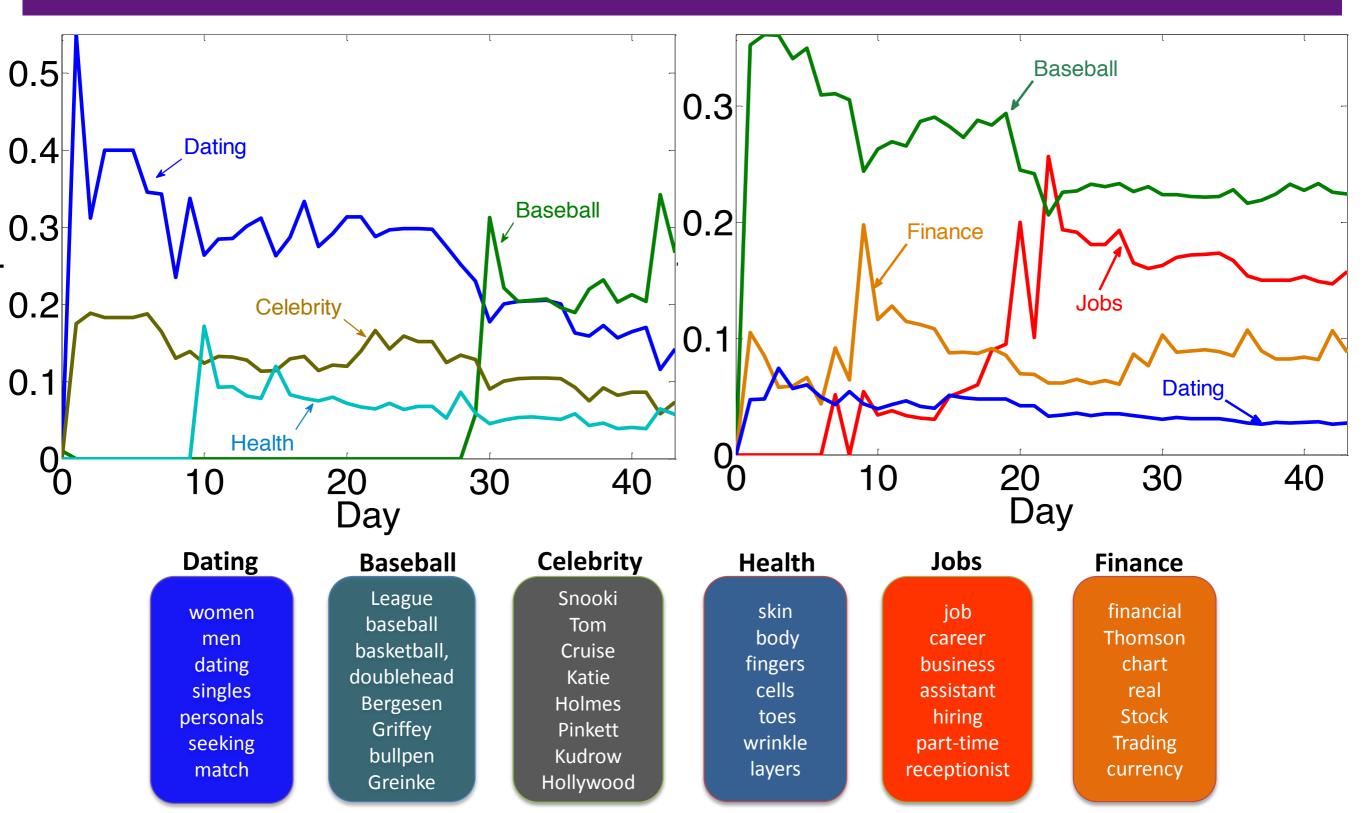


#### **Generative Process**

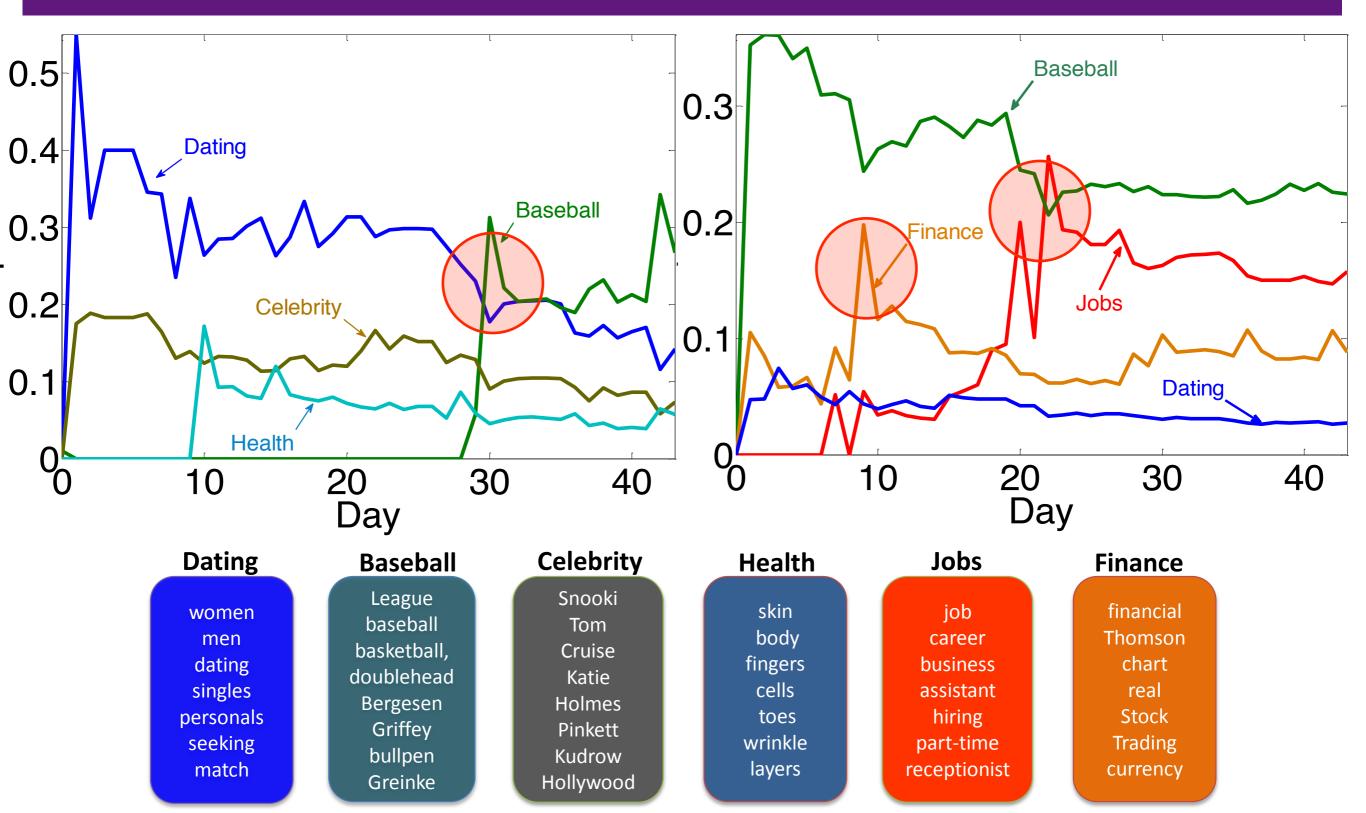
- For each user interaction
  - Choose an intent from local distribution
    - Sample word from the topic's word-distribution
  - •Choose a new intent  $\propto \alpha$ 
    - Sample a new intent from the global distribution
      - Sample word from the new topic word-distribution



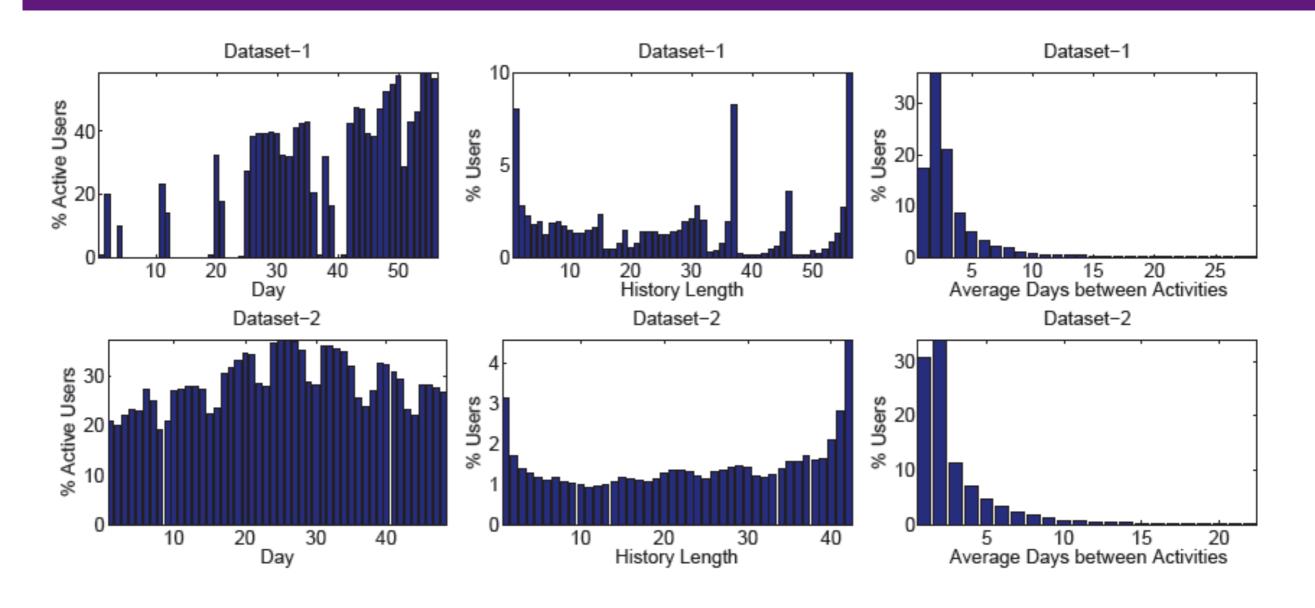
# Sample users



# Sample users

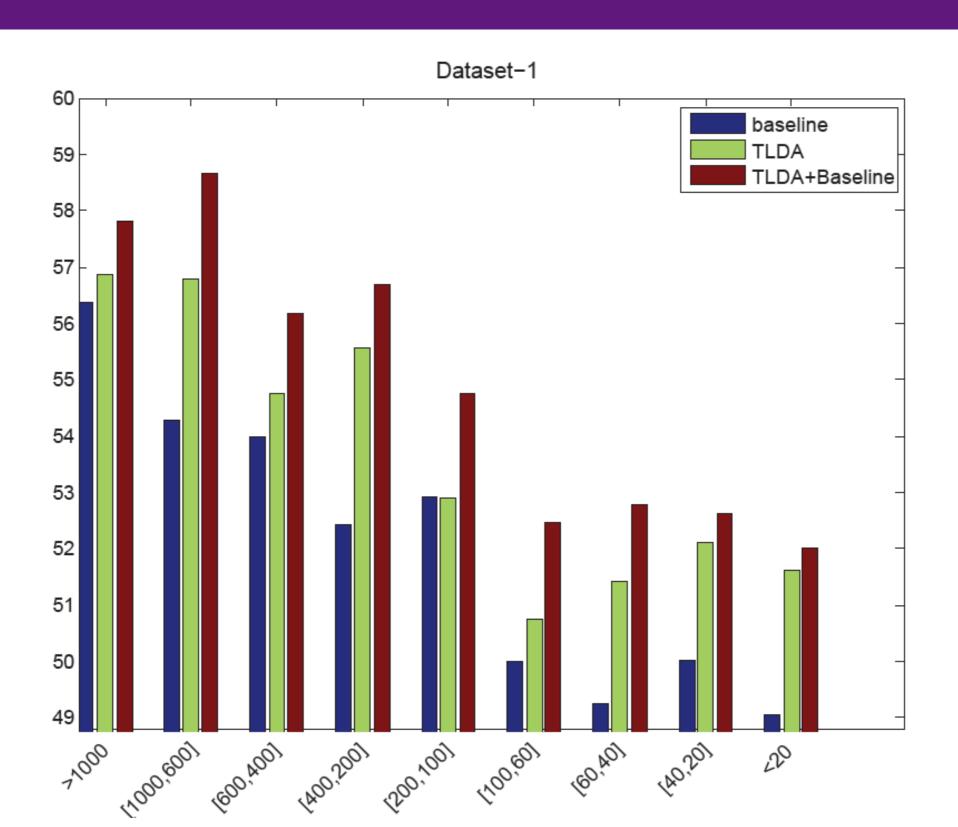


### Data

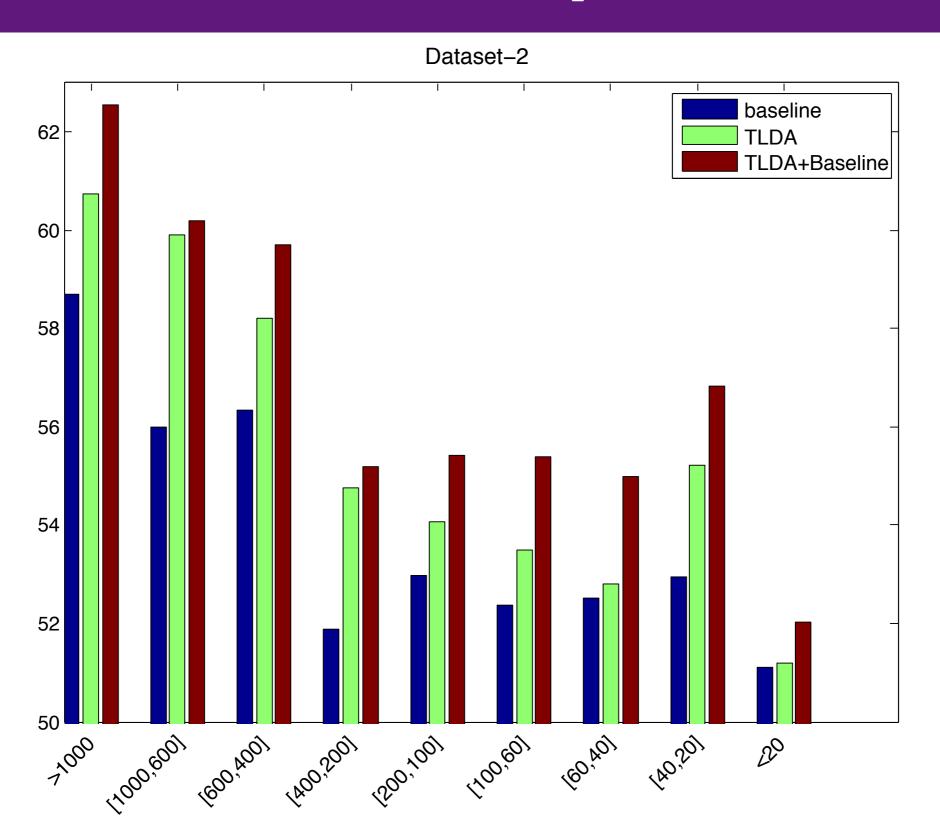


dataset	# days	# users	# campaigns	size	
1	56	13.34M	241	242GB	
2	44	33.5M	216	435GB	

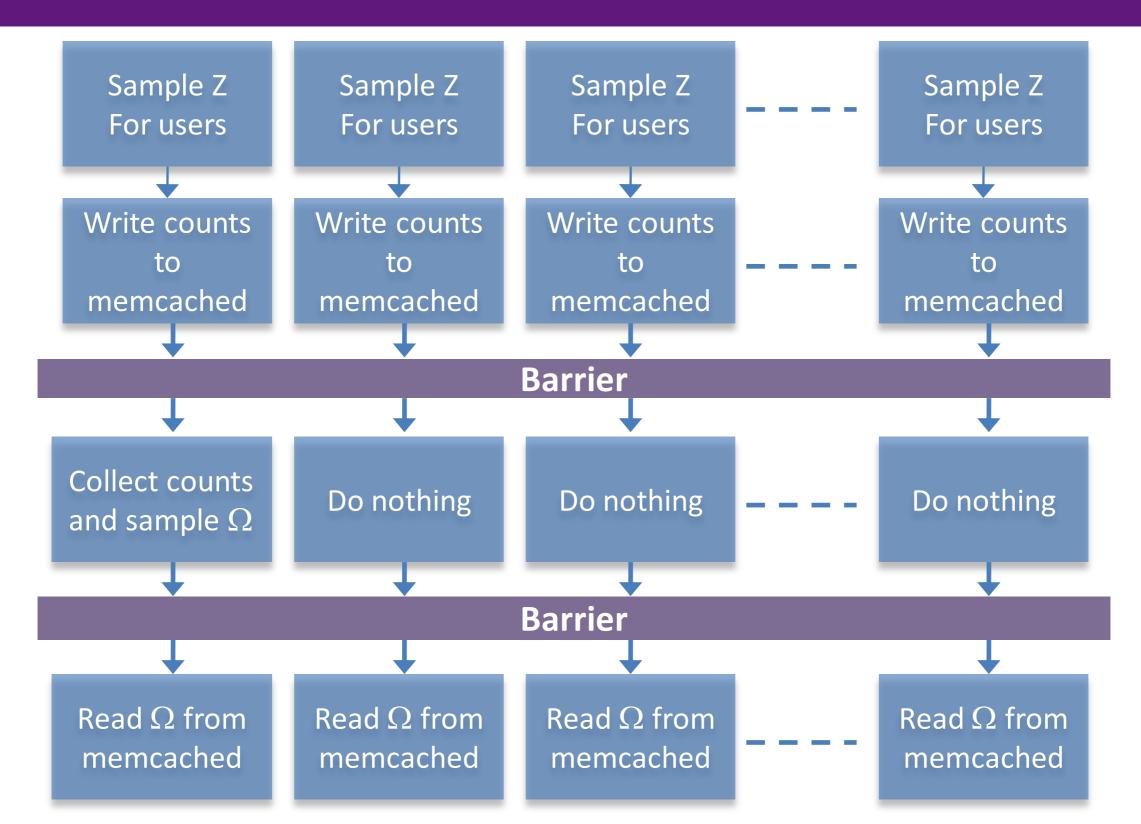
### ROC score improvement



### ROC score improvement



# LDA for user profiling



# News

### News Stream

### News Stream



Add-ons turn tax cut bill into 'Christmas t

AP - 1 hr 32 mins ago

WASHINGTON - In the

China says inflation up 5.1 perce PETER LATTMAN







By CARA ANNA, Associated Press

BEIJING - China's inflation su

Sonja Kohn, an Austrian banker, is accused of Print masterminding a 23-year

conspiracy that played a central role in financing the

Suit to Recover

Madoff's Money

Calls Austrian

an Accomplice

officials said Saturday, despite

er

ovember. ase food

Post a Comment supplies and end diesel shorts

gigantic Ponzi scheme.

The 5.1 percent inflation rate was driven by a 11.7 percent jump in food prices year on year.

The news comes as China's leaders meet for the top economic planning conference of the year and as financial markets watch for a widely anticipated interest rate hike to help bring rapid economic growth to a more sustainable level.

"I think this means that an interest rate hike of 25 basis points is very likely by the end of the year," said CLSA analyst Andy Rothman.

#### BEYOND FOSSIL FUELS

and lawmakers. But Bill Clinton even ba Full Story »

Wideo: Gibbs: I Have

Slideshow: Preside

Related: Tax fight |





Wall Street Video: Charting Consumer Sentiment CNBC



Wall Street Video: Bright Future TheStreet.com

#### **RELATED QUOTES**

^DJI	11,410.32	+40.26
^GSPC	1,240.40	+7.40
^IXIC	2,637.54	+20.87



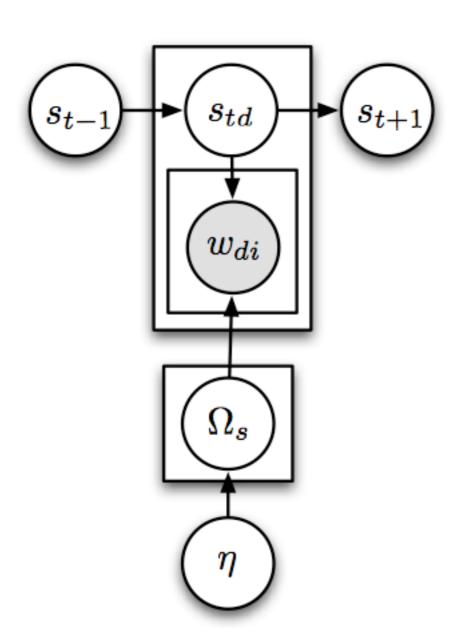
As part of its citywide system, Kristianstad burns wood waste like tree prunings and scraps from flooring factories to power an underground district heating grid.

### News Stream

- Over 1 high quality news article per second
- Multiple sources (Reuters, AP, CNN, ...)
- Same story from multiple sources
- Stories are related

- Goals
  - Aggregate articles into a storyline
  - Analyze the storyline (topics, entities)

# Clustering / RCRP



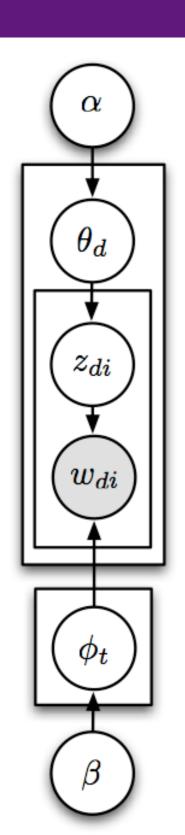
- Assume active story distribution at time t
- Draw story indicator
- Draw words from story distribution
- Down-weight story counts for next day

Ahmed & Xing, 2008

# Clustering / RCRP

- Pro
  - Nonparametric model of story generation (no need to model frequency of stories)
  - No fixed number of stories
  - Efficient inference via collapsed sampler
- Con
  - We learn nothing!
  - No content analysis

#### Latent Dirichlet Allocation



- Generate topic distribution per article
- Draw topics per word from topic distribution
- Draw words from topic specific word distribution

Blei, Ng, Jordan, 2003

#### Latent Dirichlet Allocation

- Pro
  - Topical analysis of stories
  - Topical analysis of words (meaning, saliency)
  - More documents improve estimates
- Con
  - No clustering

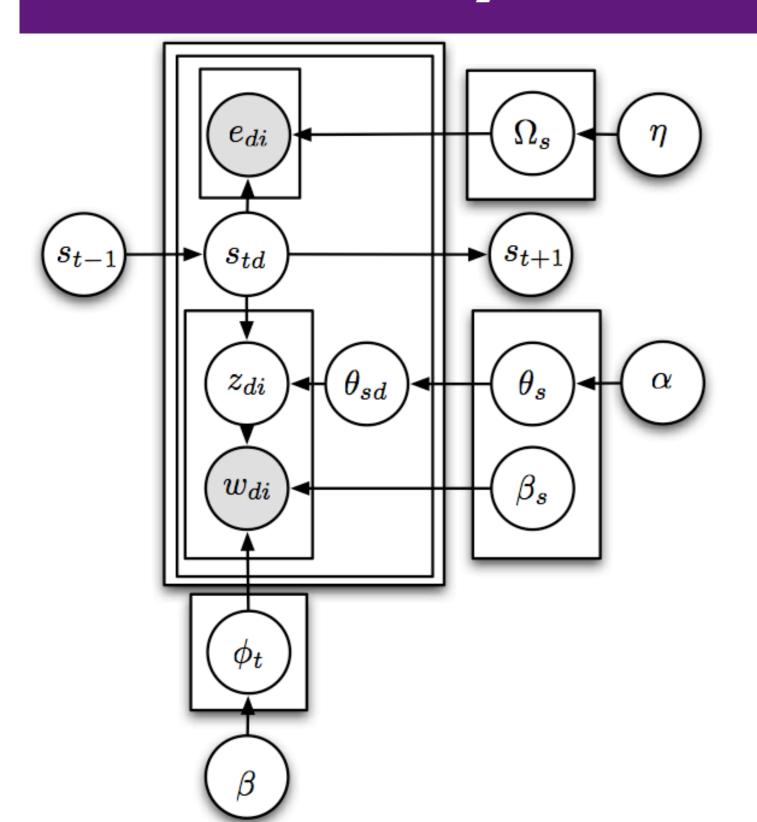
### More Issues



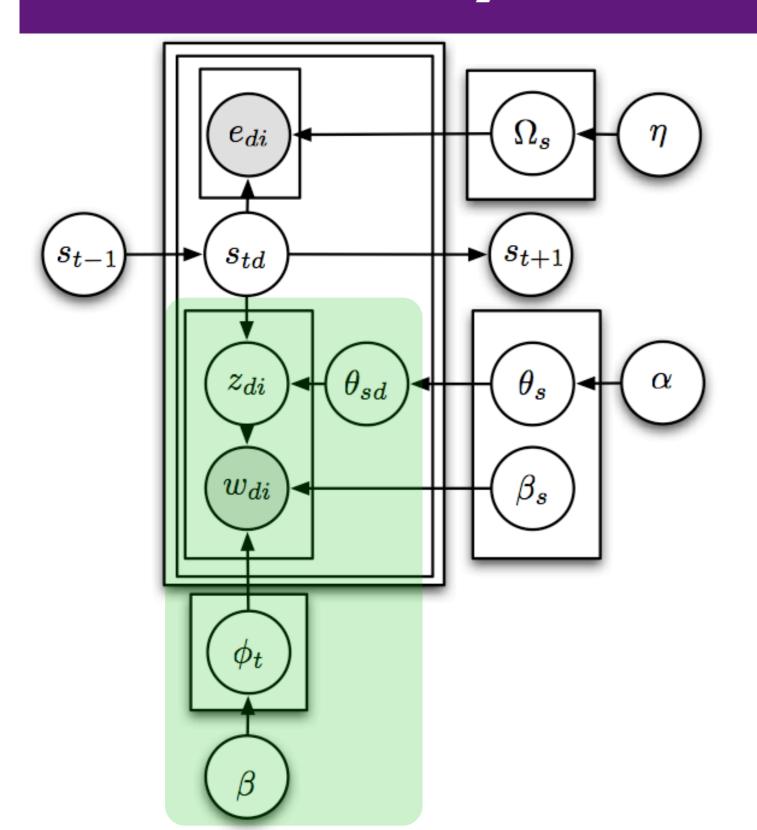
#### More Issues

- Named entities are special, topics less (e.g. Tiger Woods and his mistresses)
- Some stories are strange (topical mixture is not enough - dirty models)
- Articles deviate from general story (Hierarchical DP)

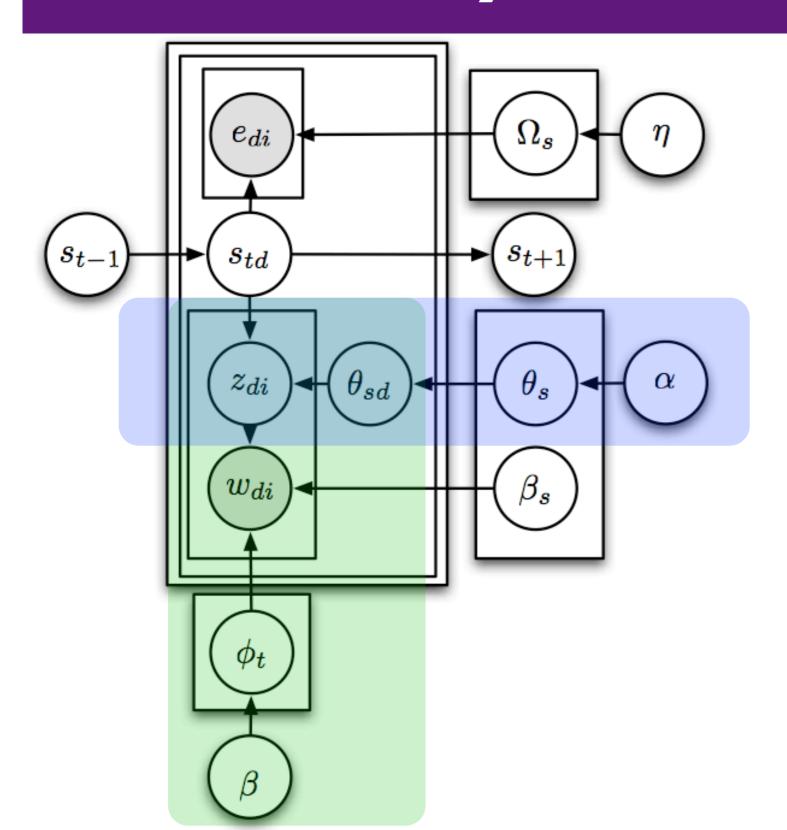
# Storylines



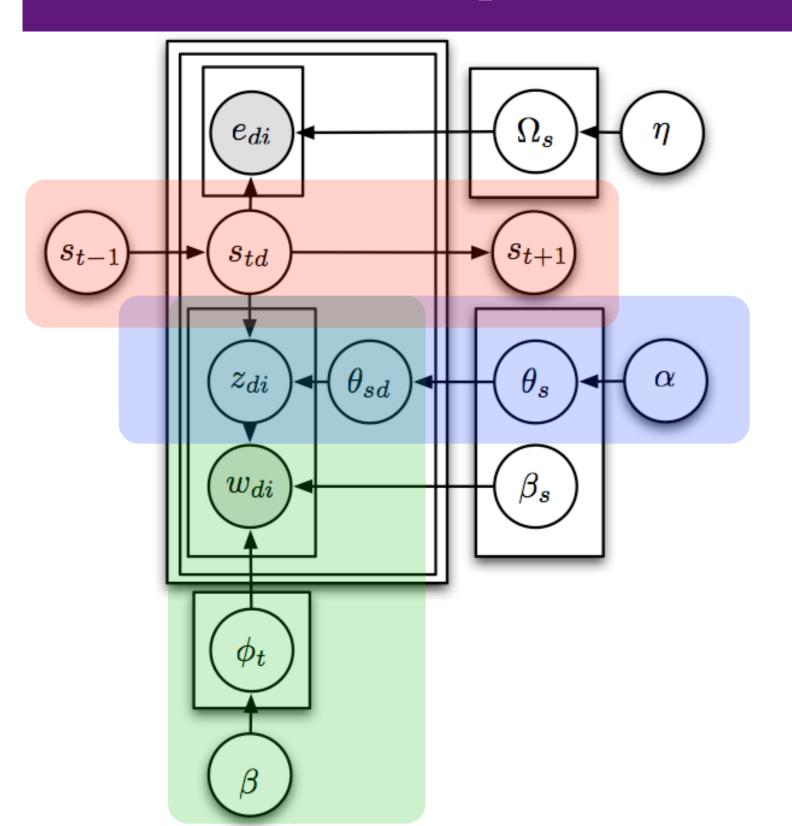
- Topic model
- Topics per cluster
- RCRP for cluster
- Hierarchical DP for article
- Separate model for named entities
- Story specific correction



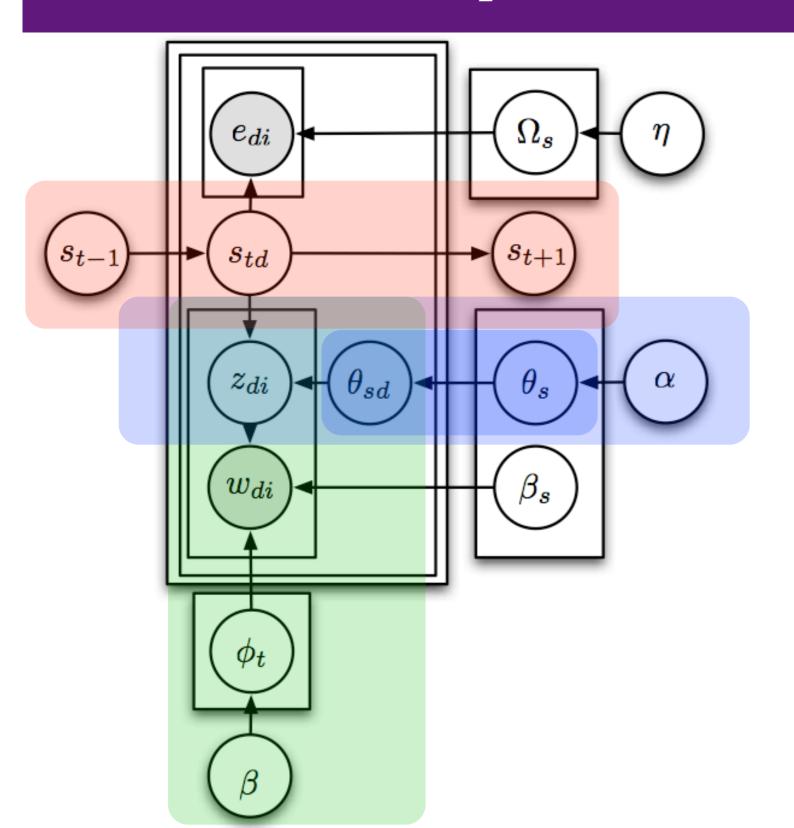
- Topic model
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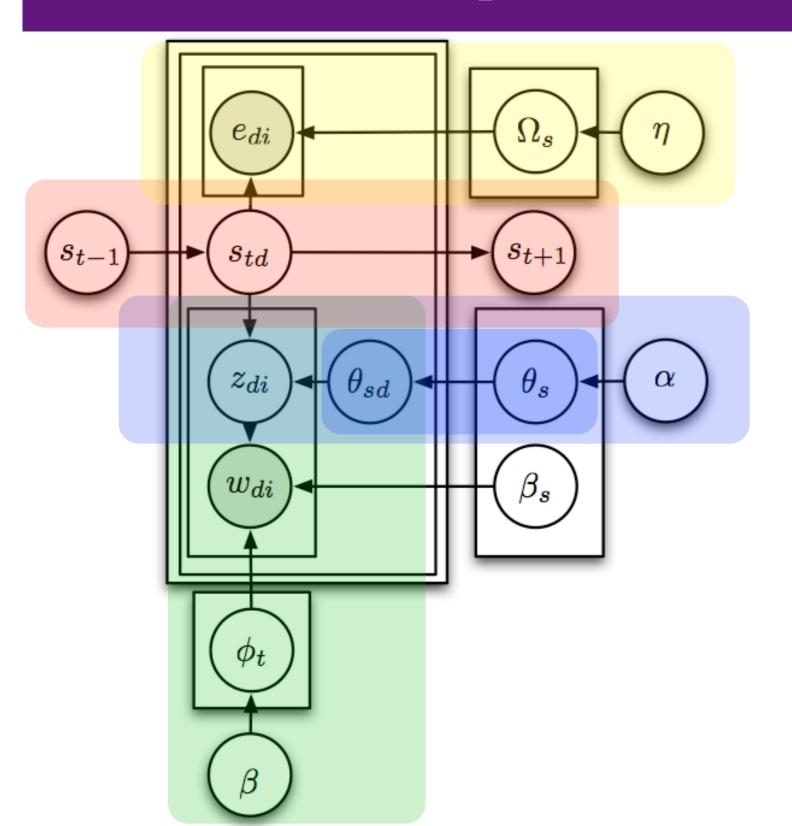
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- Story specific correction



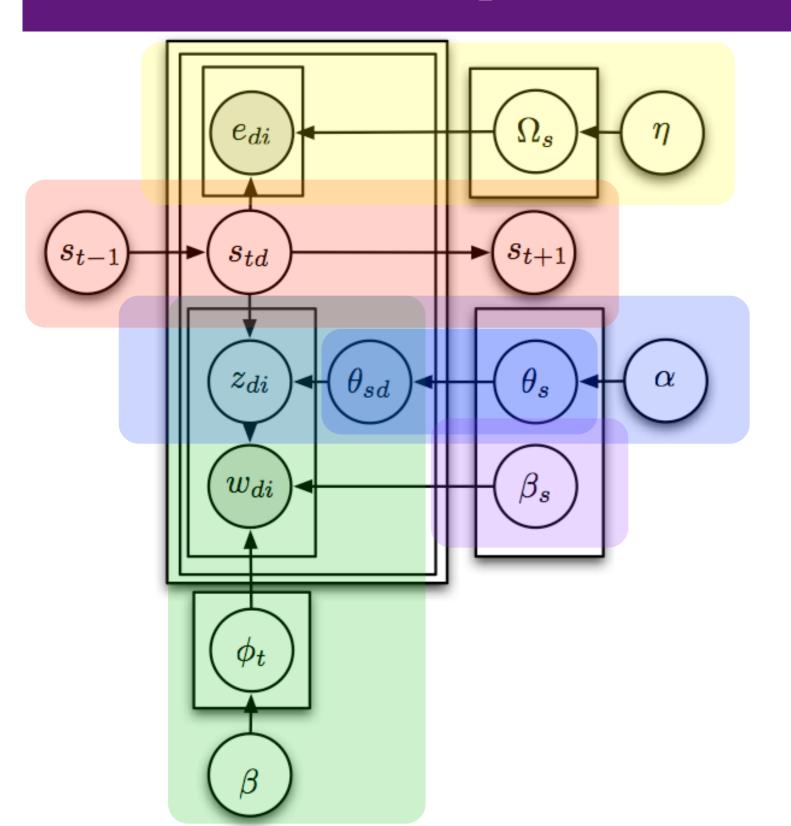
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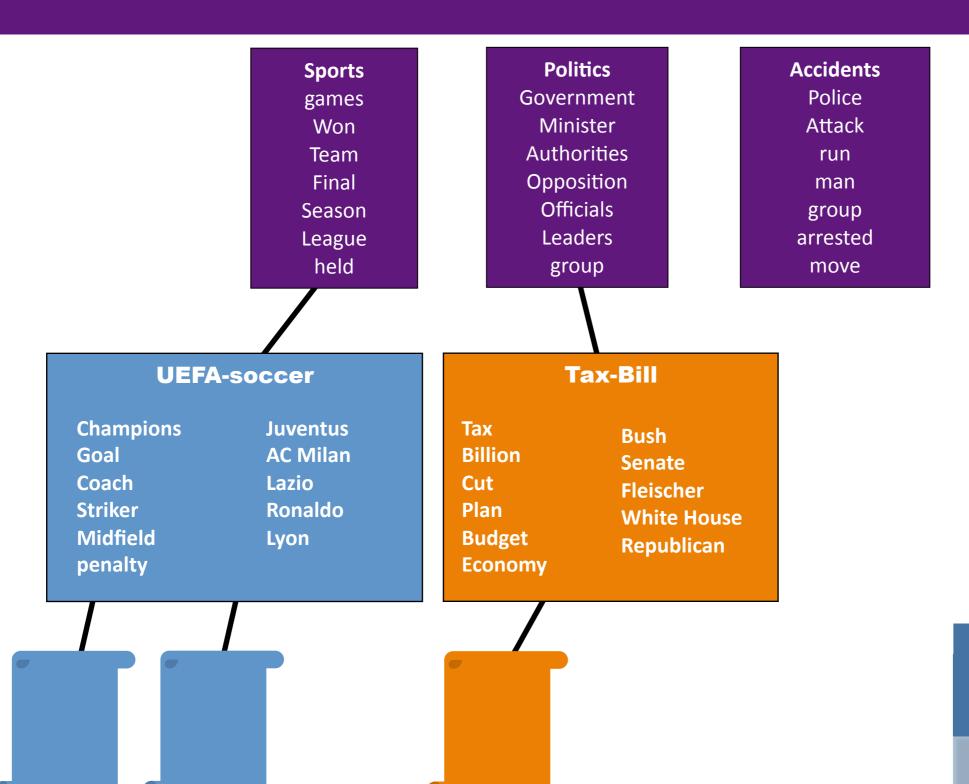


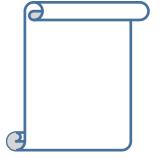
- Topic model
- Topics per cluster
- RCRP for cluster
- Hierarchical DP for article
- Separate model for named entities
- Story specific correction



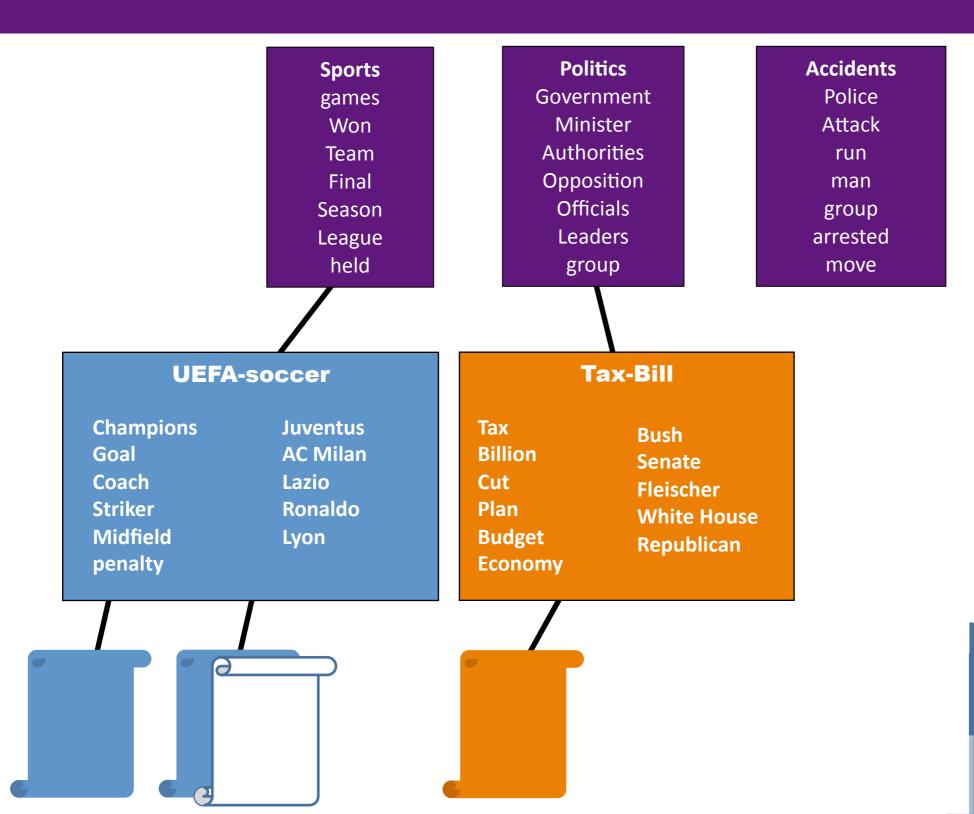
- Topic model
- Topics per cluster
- RCRP for cluster
- Hierarchical DP for article
- Separate model for named entities
- Story specific correction

### Dynamic Cluster-Topic Hybrid

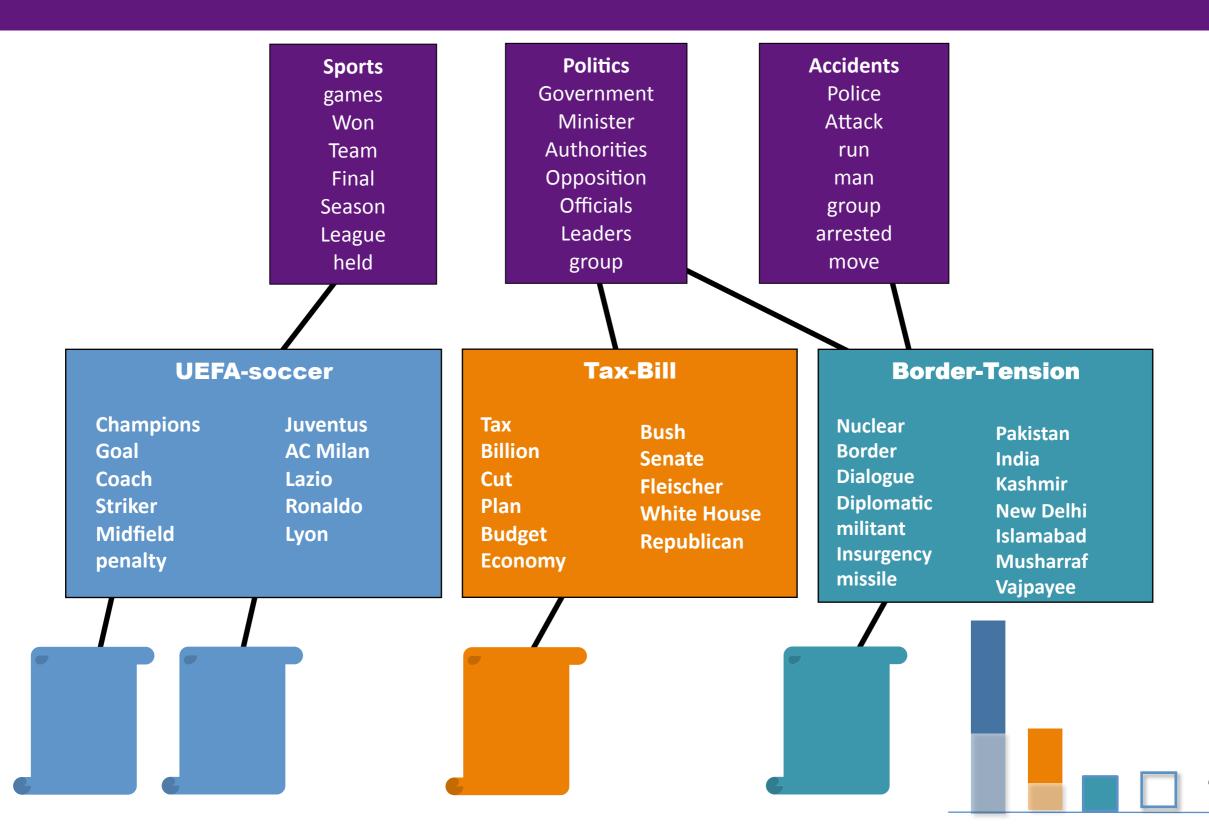




### Dynamic Cluster-Topic Hybrid



### Dynamic Cluster-Topic Hybrid



### Inference

- We receive articles as a stream
   Want topics & stories now
- Variational inference infeasible (RCRP, sparse to dense, vocabulary size)
- We have a 'tracking problem'
  - Sequential Monte Carlo
  - Use sampled variables of surviving particle
  - Use ideas from Cannini et al. 2009

• Proposal distribution - draw stories s, topics z

$$p(s_{t+1}, z_{t+1}|x_{1...t+1}, s_{1...t}, z_{1...t})$$

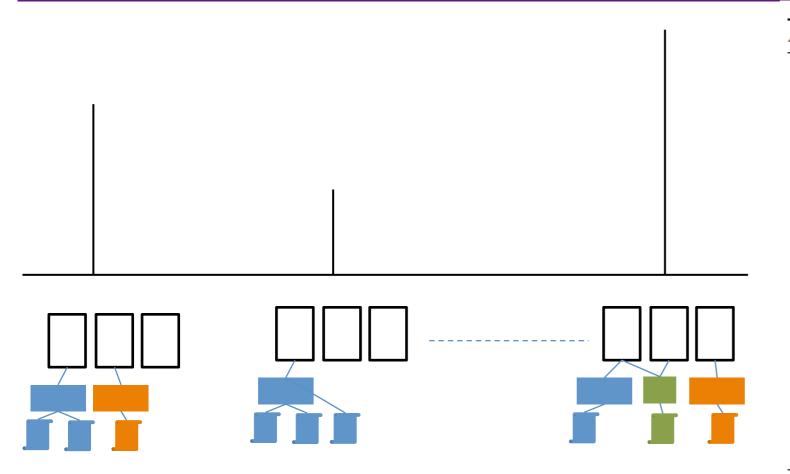
using Gibbs Sampling for each particle

Reweight particle via

past state

new data 
$$p(x_{t+1}|x_{1...t},s_{1...t},z_{1...t})$$

- Resample particles if l<sub>2</sub> norm too large (resample some assignments for diversity, too)
- Compare to multiplicative updates algorithm In our case predictive likelihood yields weights



```
Algorithm 1 A Particle Filter Algorithm

Initialize \omega_1^f to \frac{1}{F} for all f \in \{1, \dots F\}

for each document d with time stamp t do

for f \in \{1, \dots F\} do

Sample s_{td}^f, \mathbf{z}_{td}^f using MCMC

\omega^f \leftarrow \omega^f P(\mathbf{x}_{td} | \mathbf{z}_{td}^f, \mathbf{s}_{td}^f, \mathbf{x}_{1:t,d-1})

end for

Normalize particle weights

if \|\omega_t\|_2^{-2} < threshold then

resample particles

for f \in \{1, \dots F\} do

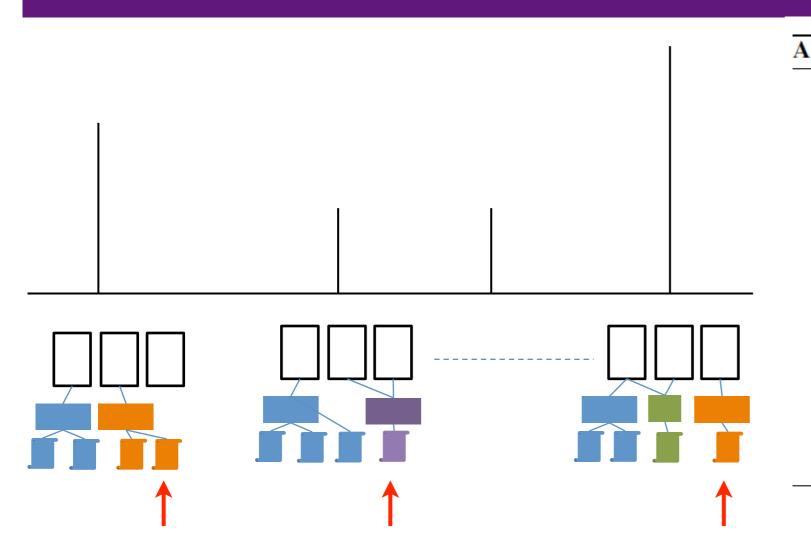
MCMC pass over 10 random past documents

end for

end if

end for
```

- •s and z are tightly coupled
- Alternative to MCMC
  - •Sample **s** then sample **z** (high variance)
  - •Sample **z** then sample **s** (doesn't make sense)
- Idea (following a similar trick by Jain and Neal)
  - •Run a few iterations of MCMC over s and z
  - Take last sample as the proposed value



```
Algorithm 1 A Particle Filter Algorithm

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end for

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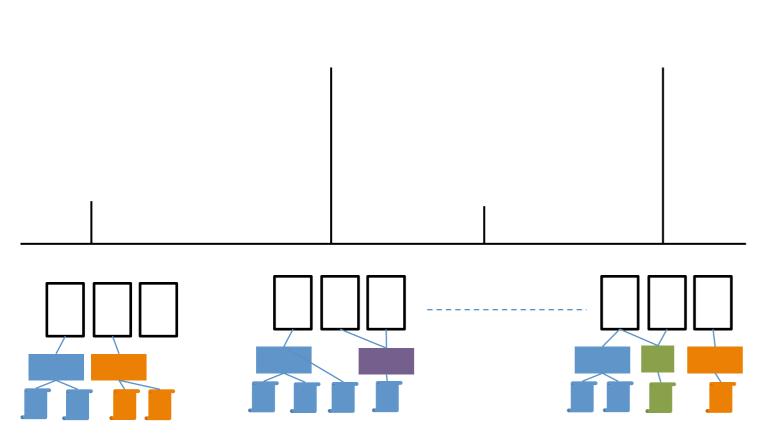
for f \in \{1, \dots F\} do

MCMC pass over 10 random past documents

end for

end if
```

end for



#### Algorithm 1 A Particle Filter Algorithm

```
Initialize \omega_1^f to \frac{1}{F} for all f \in \{1, \dots F\} for each document d with time stamp t do for f \in \{1, \dots F\} do

Sample s_{td}^f, \mathbf{z}_{td}^f using MCMC

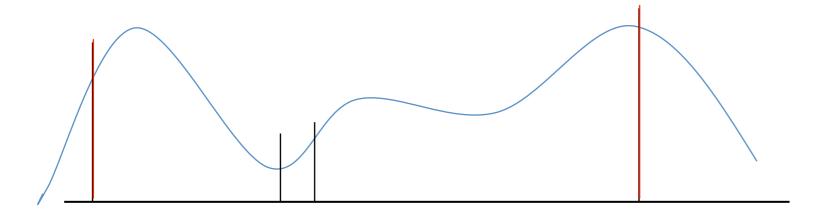
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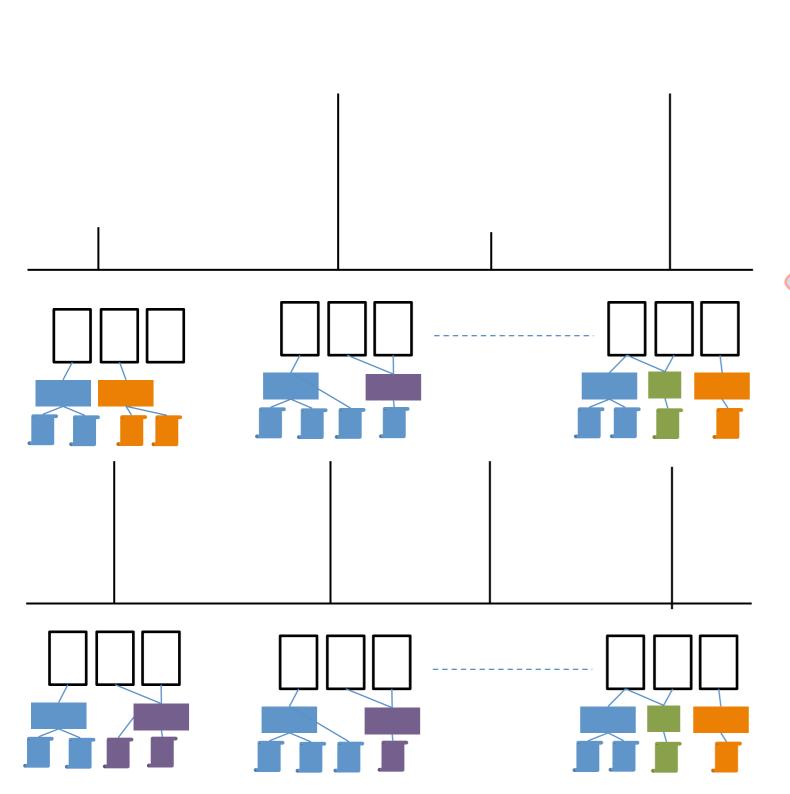
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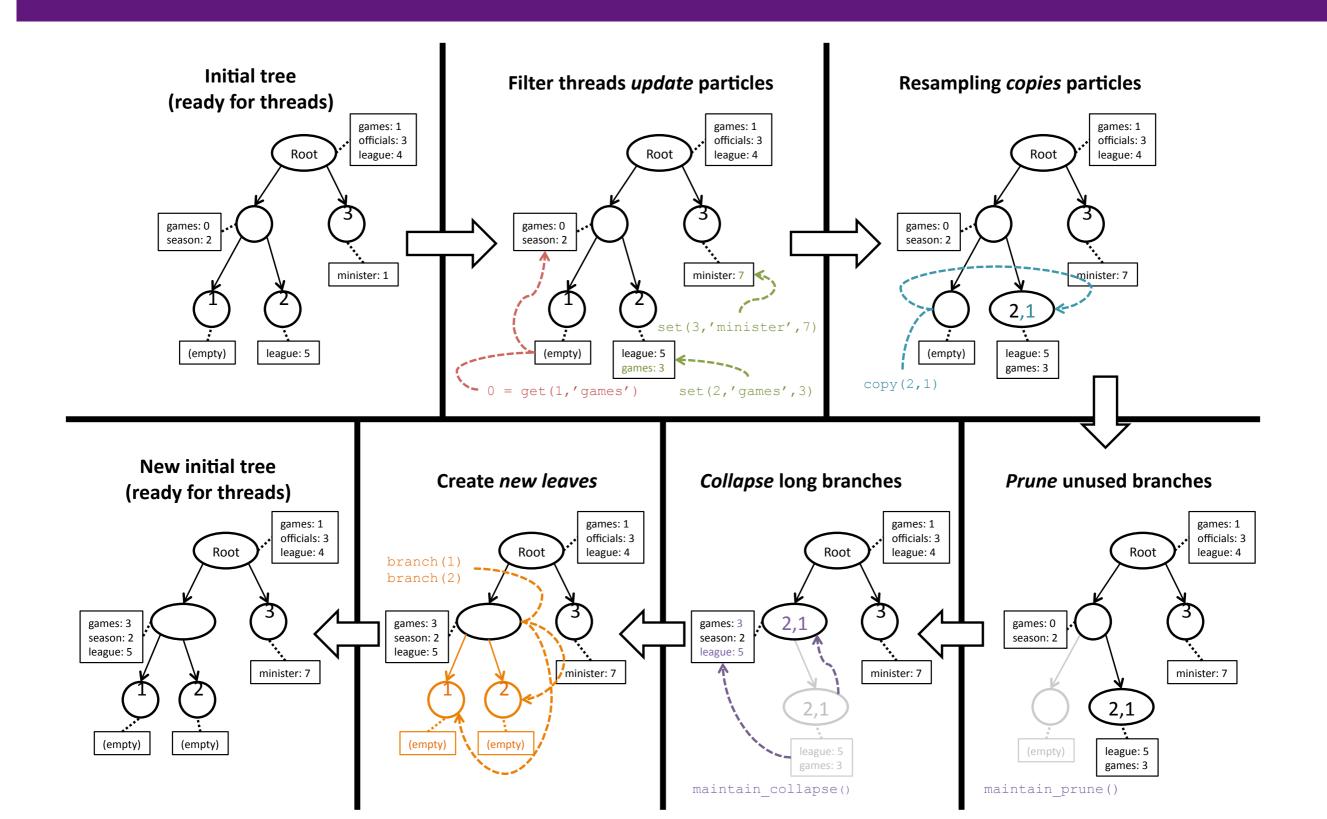
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if \|\omega_t\|_2^{-2} < threshold then resample particles

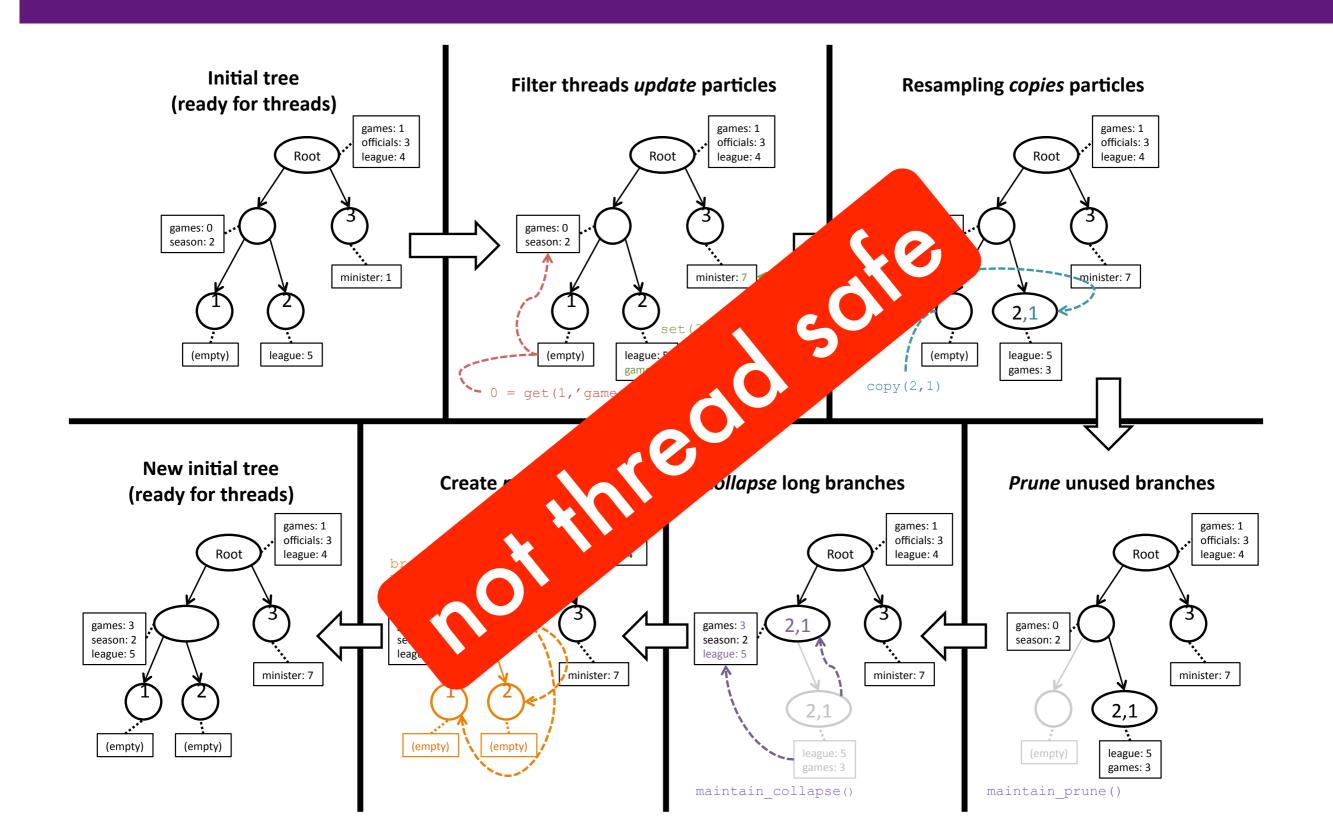
for f \in \{1, \dots F\} do

MCMC pass over 10 random past documents end for end if end for
```

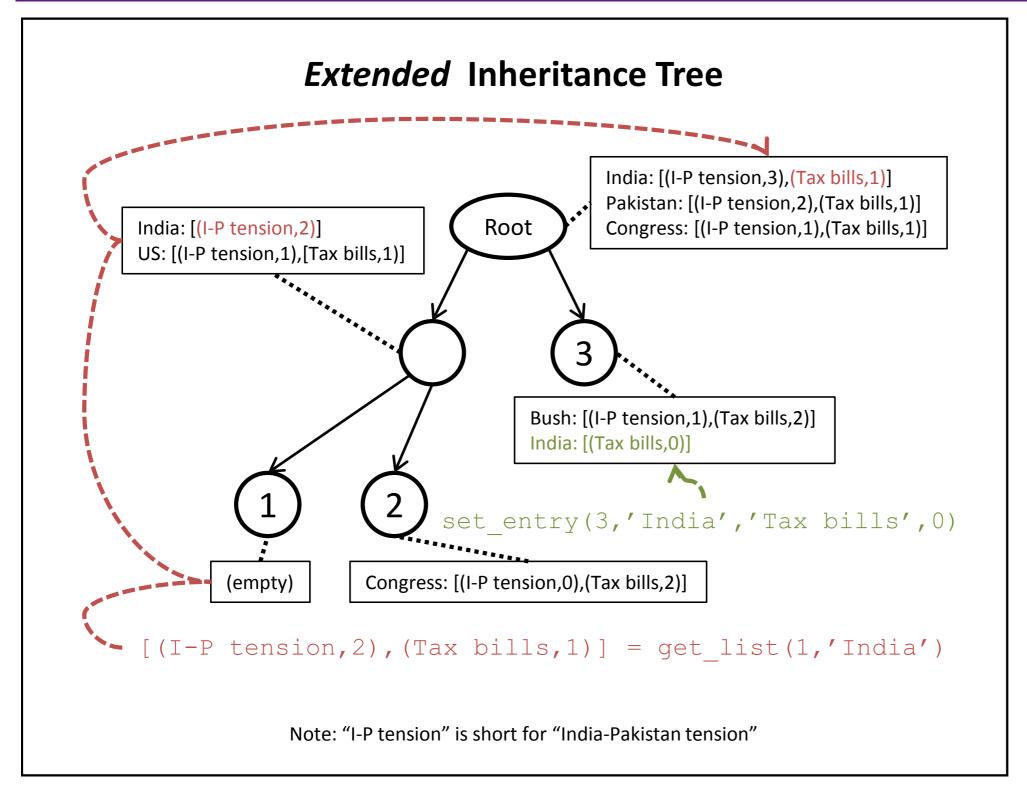
### Inheritance Tree



### Inheritance Tree



#### Extended Inheritance Tree



write only in the leaves (per thread)

# Results

### Ablation studies

TDT5 (Topic Detection and Tracking)
 macro-averaged minimum detection cost: 0.714

Removing features

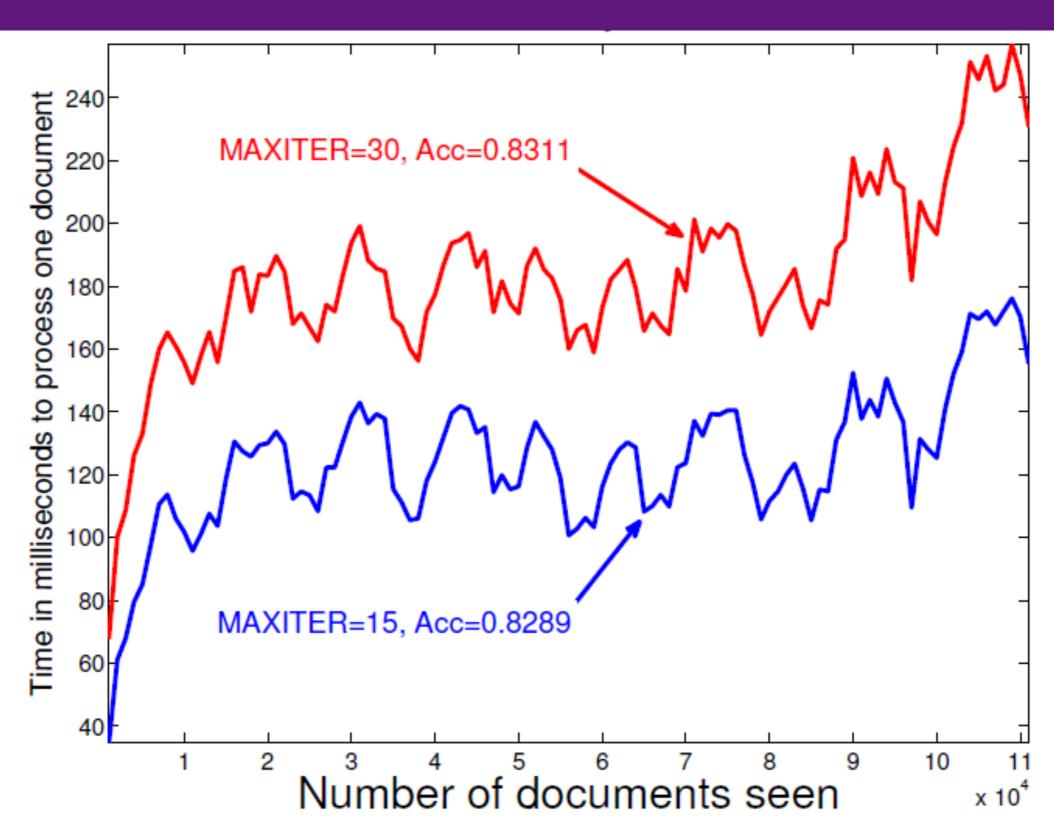
time	entities	topics	story words
0.84	0.90	0.86	0.75

# Comparison

Sample	Sample	Num	Num	Story	LSHC
No.	size	Words	Entities	Acc.	Acc.
1	111,732	19,218	12,475	0.8289	0.738
2	274,969	29,604	21,797	0.8388	0.791
3	547,057	40,576	32,637	0.8395	0.800

Hashing & correlation clustering

# Time-Accuracy trade off



### Stories

# TOPICS

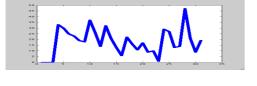
# STORYLINES

#### **Sports**

games won team final season league held

#### UEFA-soccer

champions Juventus
goal AC Milan
leg Real Madrid
coach Milan
striker Lazio
midfield Ronaldo
penalty Lyon



#### **Politics**

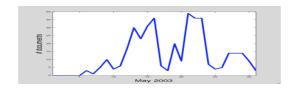
government minister authorities opposition officials leaders group

#### **Unrest**

police attack run man group arrested move

#### Tax bills

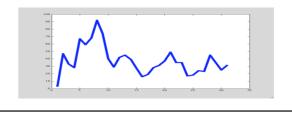
tax Bush
billion Senate
cut US
plan Congress
budget Fleischer
economy White House
lawmakers Republican



#### **India-Pakistan tension**

nuclear
border
dialogue
diplomatic
militant
insurgency
missile

Pakistan
India
Kashmir
New Delhi
Islamabad
Islamabad
Musharraf
Vajpayee

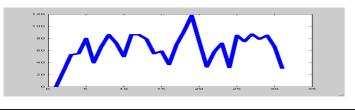


### Related Stories

"Show similar stories by topic"

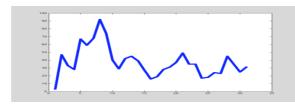
#### Middle-east conflict

Peace Israel
Roadmap Palestinian
Suicide West bank
Violence Sharon
Settlements Hamas
bombing Arafat

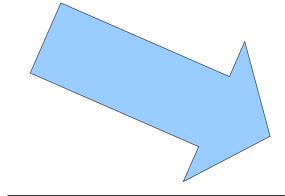


#### **India-Pakistan tension**

nuclearPakistanborderIndiadialogueKashmirdiplomaticNew DelhimilitantIslamabadinsurgencyMusharrafmissileVajpayee



"Show similar stories, require the word nuclear"

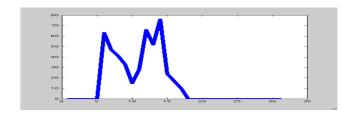


#### **North Korea nuclear**

nuclear summit warning policy missile program

North Korea South Korea U.S Bush

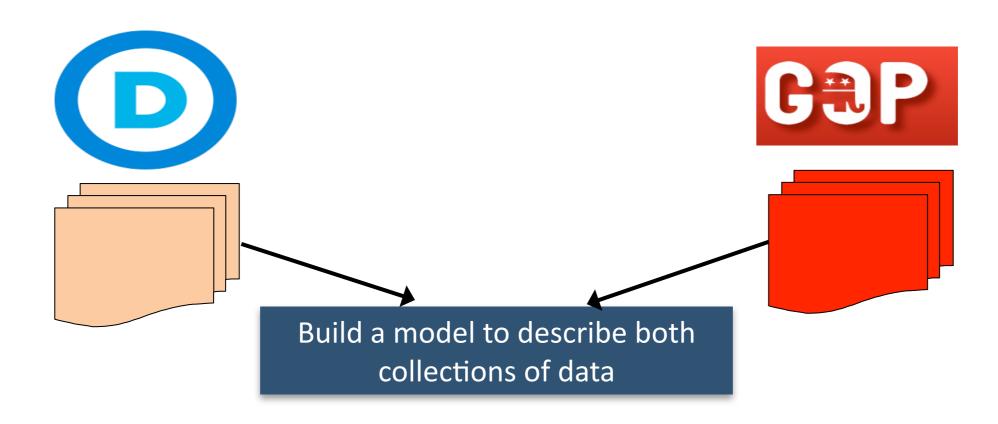
Pyongyang



### Detecting Ideologies

Ahmed and Xing, 2010

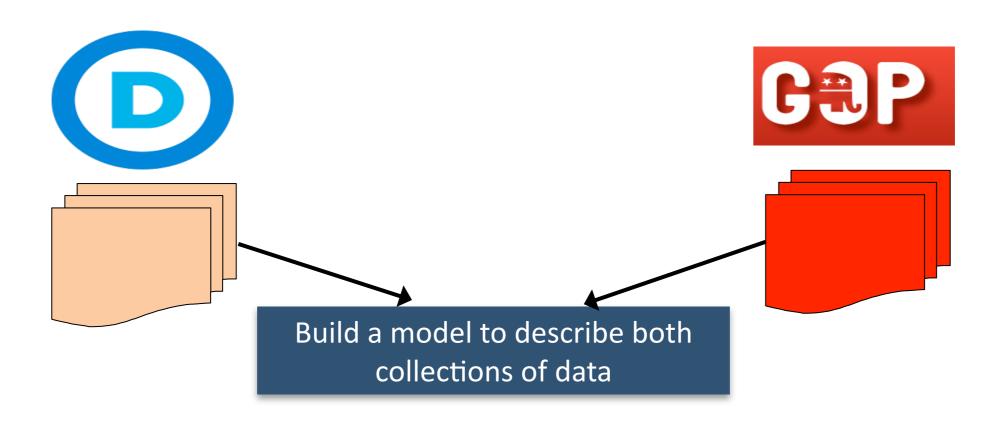
# Ideologies



#### Visualization

- How does each ideology view mainstream events?
- On which topics do they differ?
- On which topics do they agree?

# Ideologies

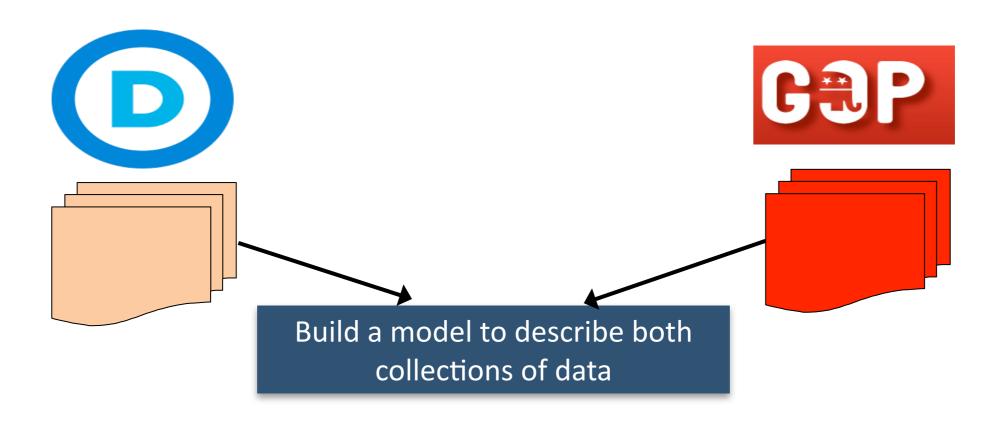


#### Visualization

#### Classification

- •Given a new news article or a blog post, the system should infer
  - From which side it was written
  - Justify its answer on a topical level (view on abortion, taxes, health care)

# Ideologies

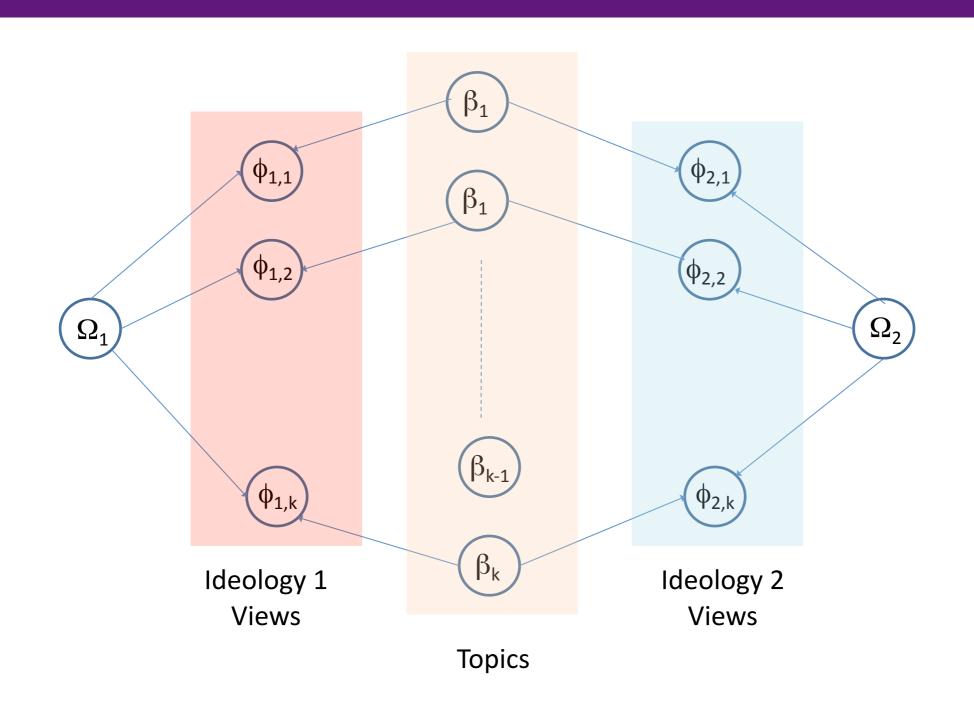


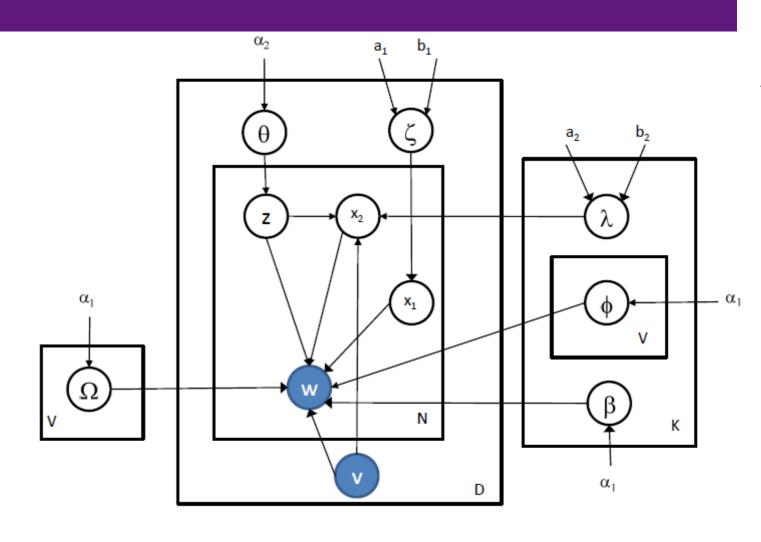
Visualization

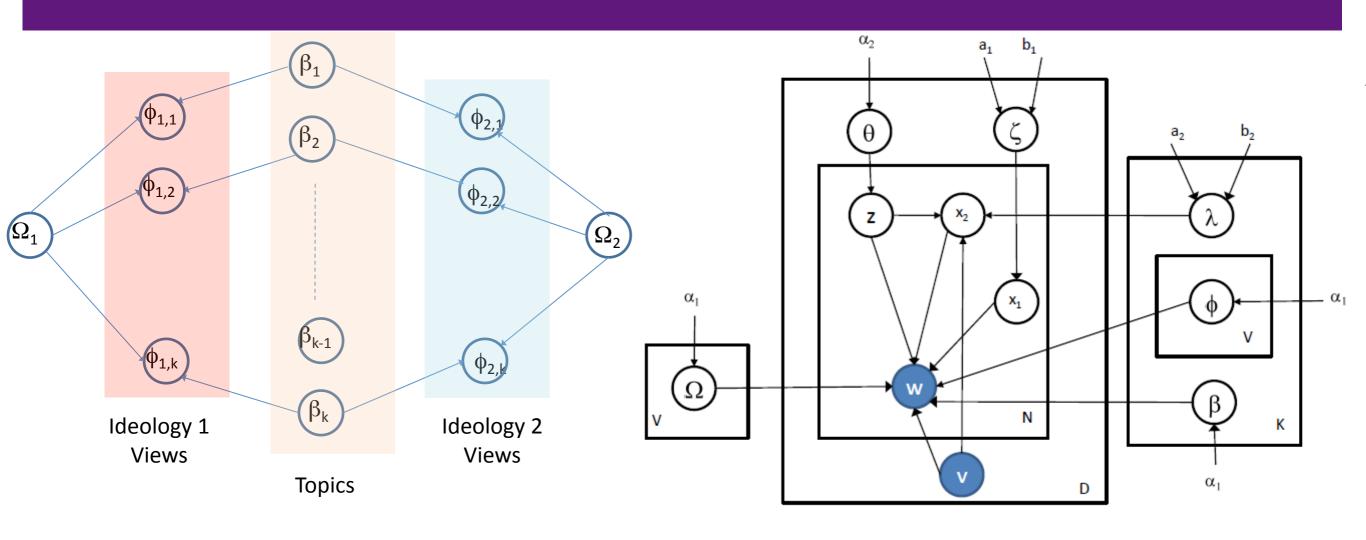
Classification

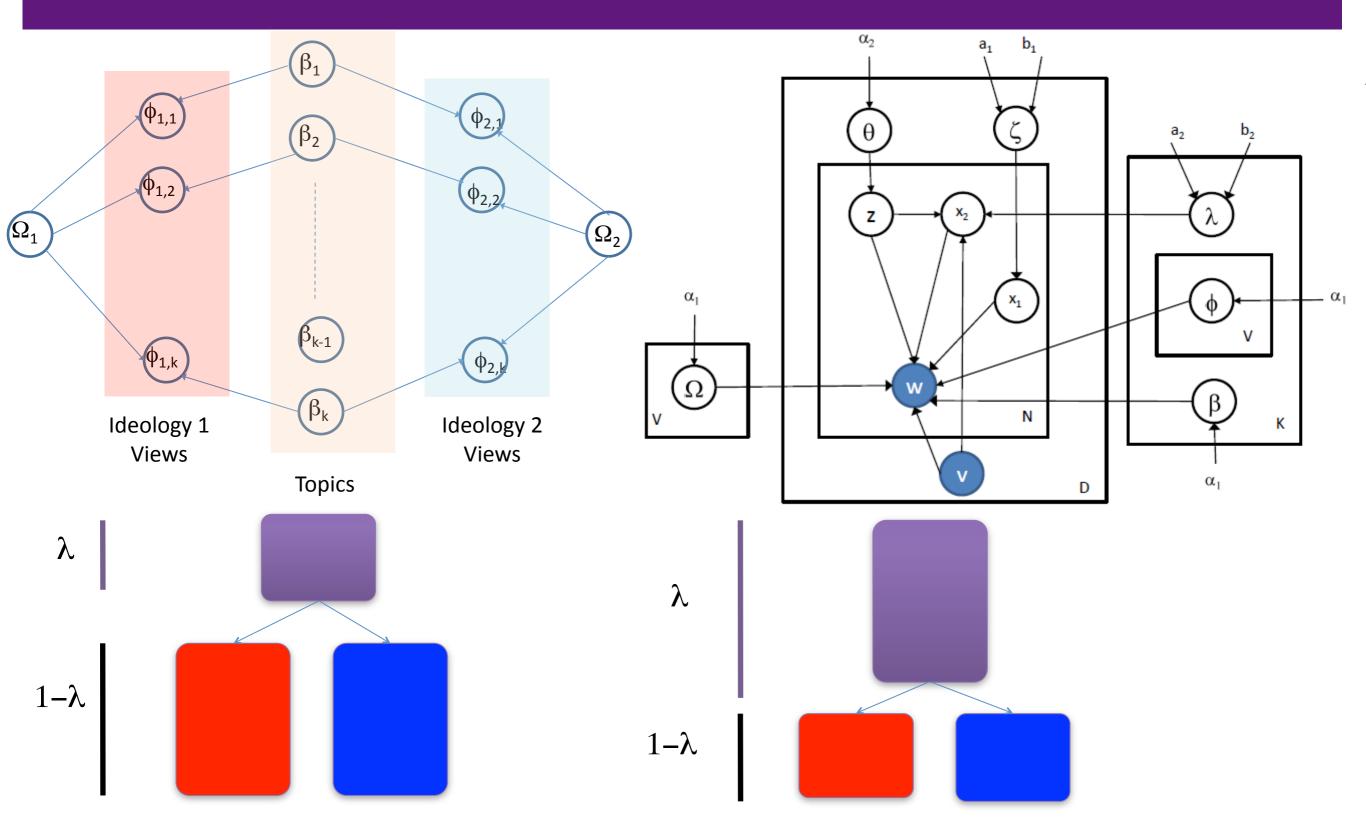
Structured browsing

- •Given a **new** news article or a blog post, the user can ask for :
  - •Examples of other articles from the same ideology about the same topic
  - Documents that could exemplify alternative views from other ideologies









#### Data

#### Bitterlemons:

- Middle-east conflict, document written by Israeli and Palestinian authors.
- ~300 documents form each view with average length 740
- Multi author collection
- 80-20 split for test and train

#### Political Blog-1:

- American political blogs (Democrat and Republican)
- 2040 posts with average post length = 100 words
- Follow test and train split as in (Yano et al., 2009)
- Political Blog-2 (test generalization to a new writing style)
  - Same as 1 but 6 blogs, 3 from each side
  - ~14k posts with ~200 words per post
  - 4 blogs for training and 2 blogs for test

### Bitterlemons dataset

#### US role

Israeli View arafat state leader roadmap election month iraq yasir senior involvement clinton terrorism

bush US president american sharon administration prime pressure policy washington

Roadmap process

powell minister colin visit internal policy statement express pro previous package work transfer european

Palestinian View

palestinian israeli peace

year political process state end right government need conflict way security

process force terrorism unit provide confidence element interim discussion union succee point build positive recognize present timetable

roadmap phase security ceasefire state plan international step authority

end settlement
implementation obligation
stop expansion commitment
fulfill unit illegal present
previous assassination meet
forward

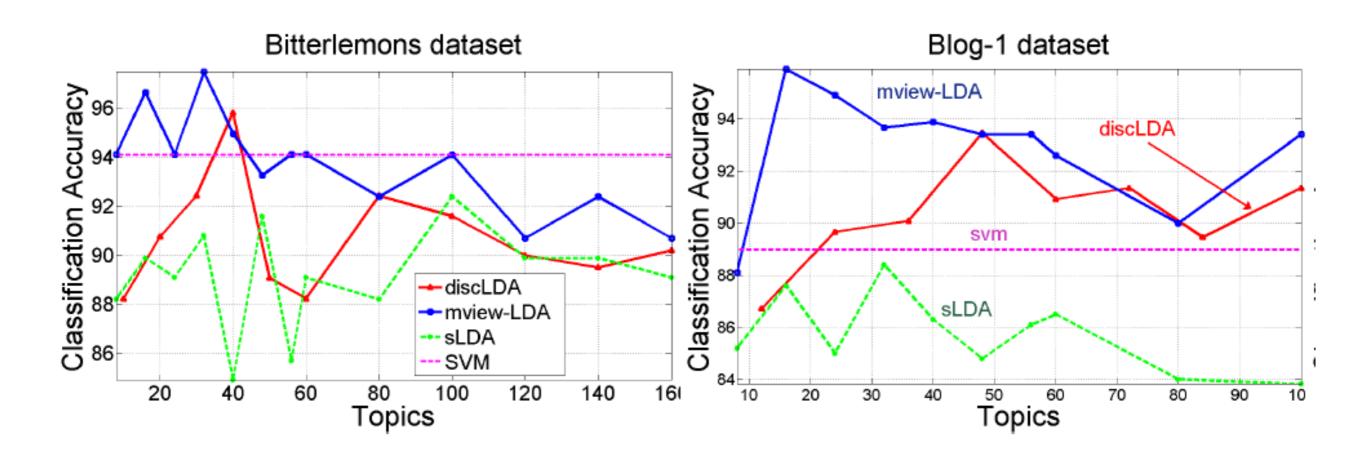
#### **Arab Involvement**

peace strategic plo hizballah islamic neighbor territorial radical iran relation think obviou countri mandate greater conventional intifada affect jihad time

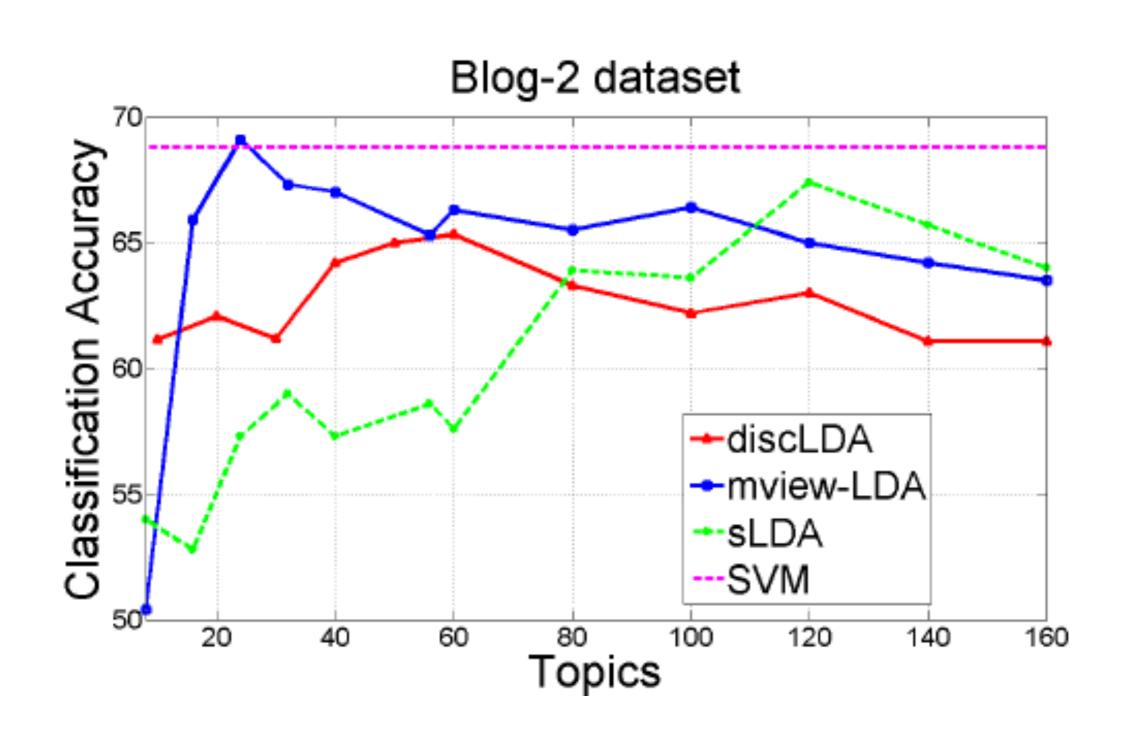
syria syrian negotiate lebanon deal conference concession asad agreement regional october initiative relationship track negotiation official leadership position withdrawal time victory present second stand circumstance represent sense talk strategy issue participant parti negotiator

palestinian
israeli
Peace
political
occupation
process
end security
conflict
way
government
people
time year
force
negotiation

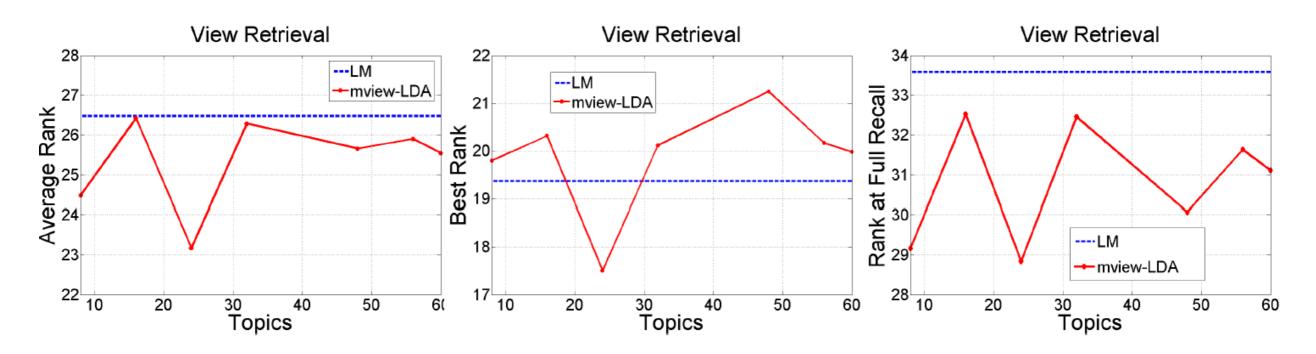
### Classification accuracy



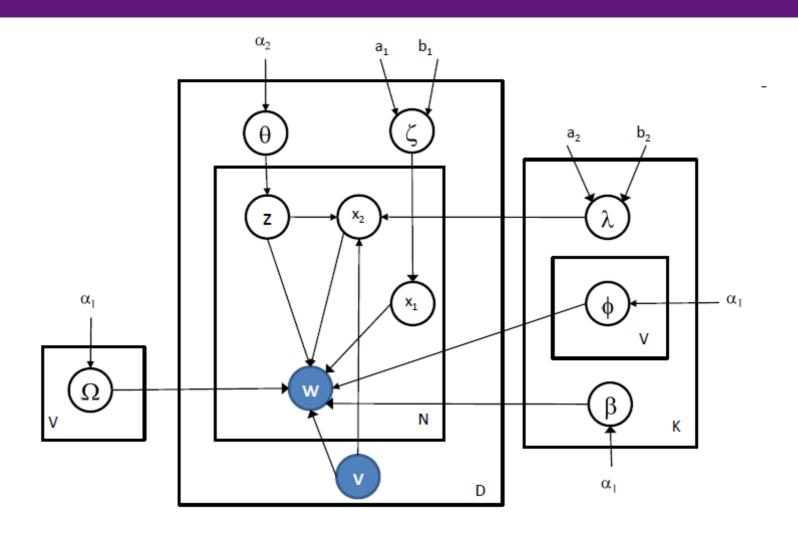
### Generalization to new blogs

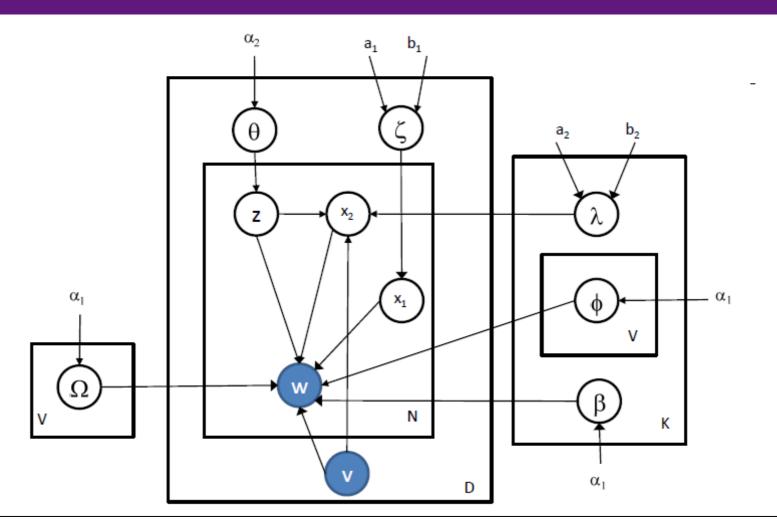


### Finding alternate views

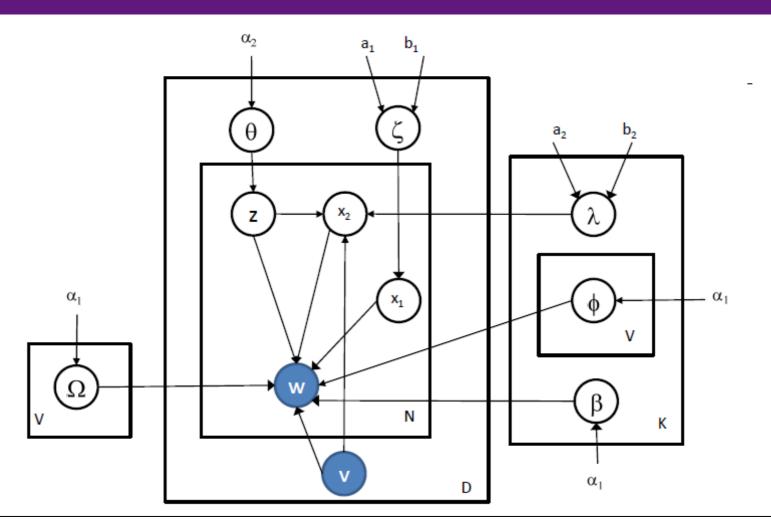


- Given a document written in one ideology, retrieve the equivalent
- Baseline: SVM + cosine similarity

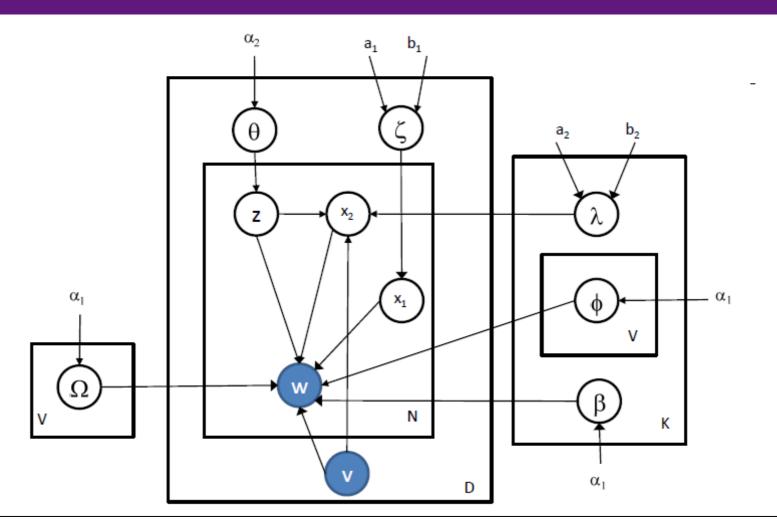




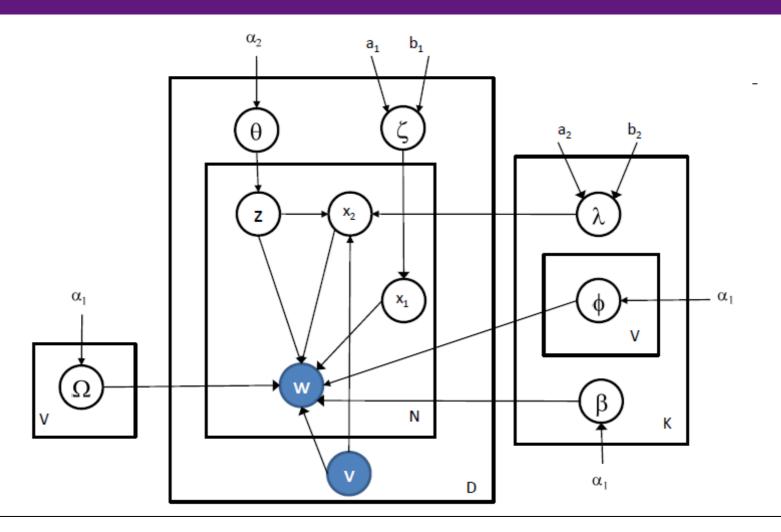
- In theory this is simple
  - Add a step that samples the document view (v)
  - •Doesn't mix in practice because tight coupling between v and  $(x_1,x_2,z)$



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  - Add a step that samples the document view (v)
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- Solution



- In theory this is simple
  - Add a step that samples the document view (v)
  - •Doesn't mix in practice because tight coupling between v and  $(x_1,x_2,z)$
- Solution
  - •Sample v and  $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z})$  as a block using a Metropolis-Hasting step



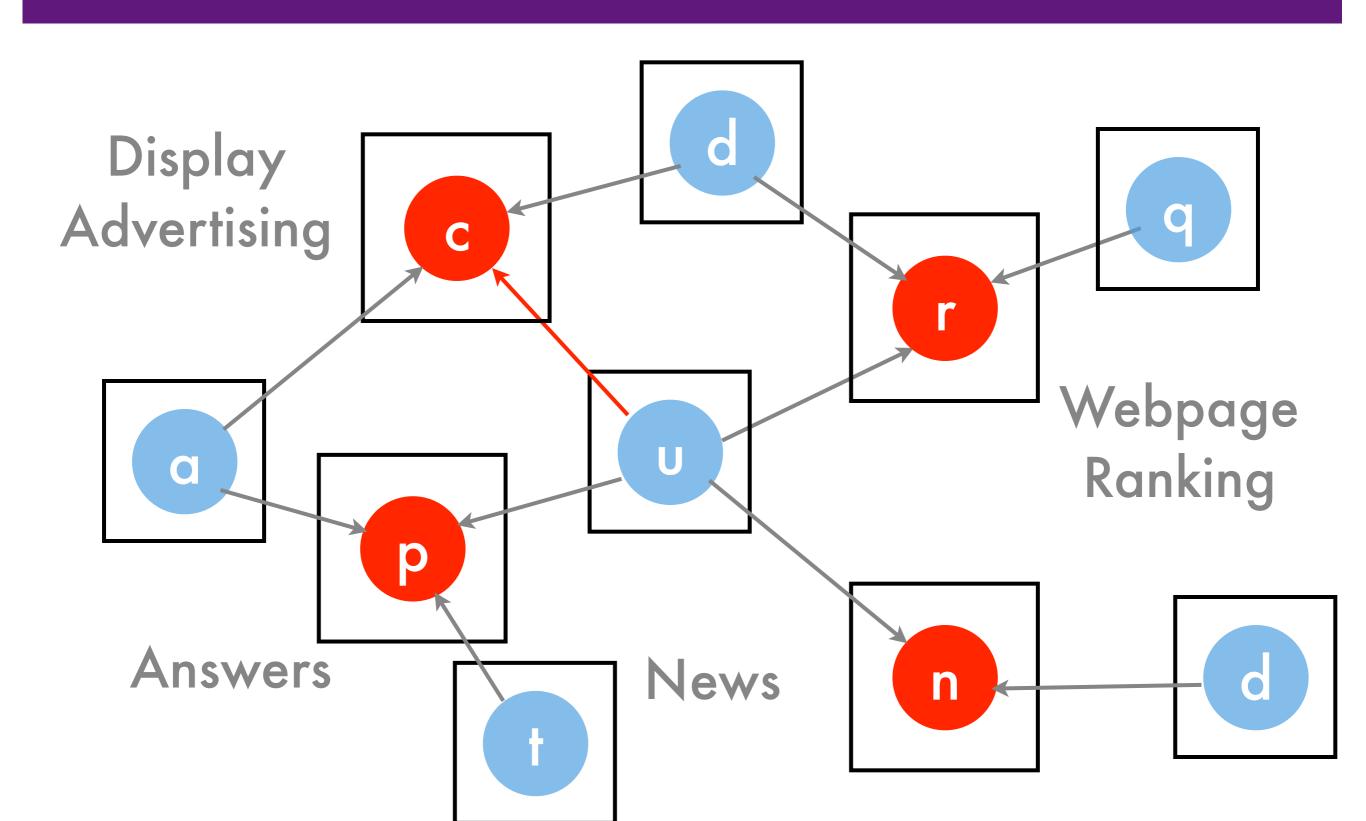
- In theory this is simple
  - Add a step that samples the document view (v)
  - •Doesn't mix in practice because tight coupling between v and  $(x_1,x_2,z)$
- Solution
  - •Sample v and  $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z})$  as a block using a Metropolis-Hasting step
  - This is a huge proposal!

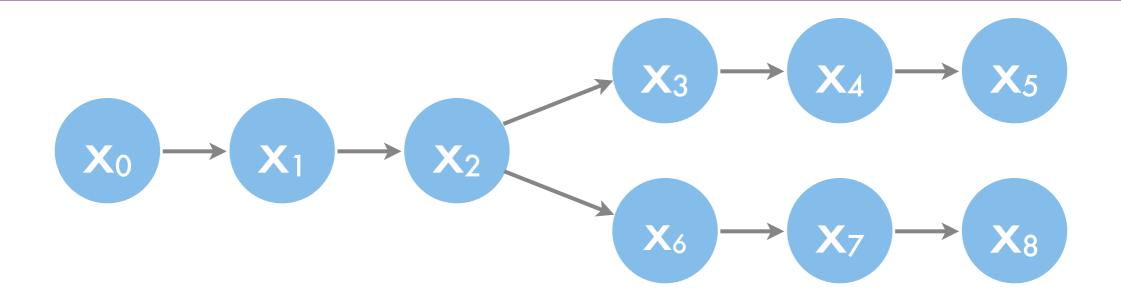
### Part 7 - Undirected Graphical Models

### Review

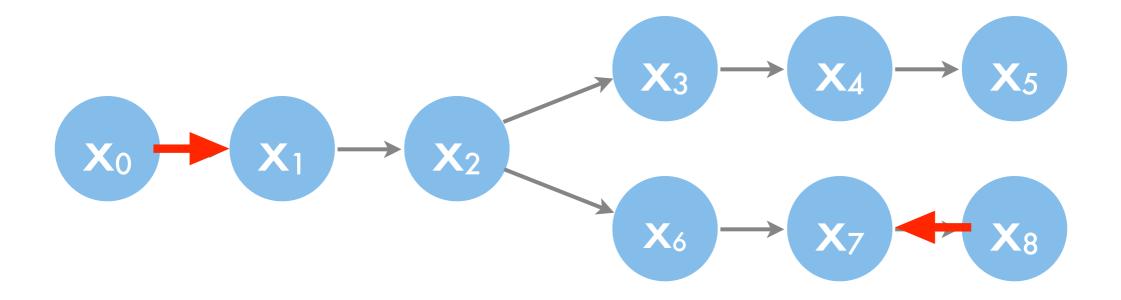


### Data Integration

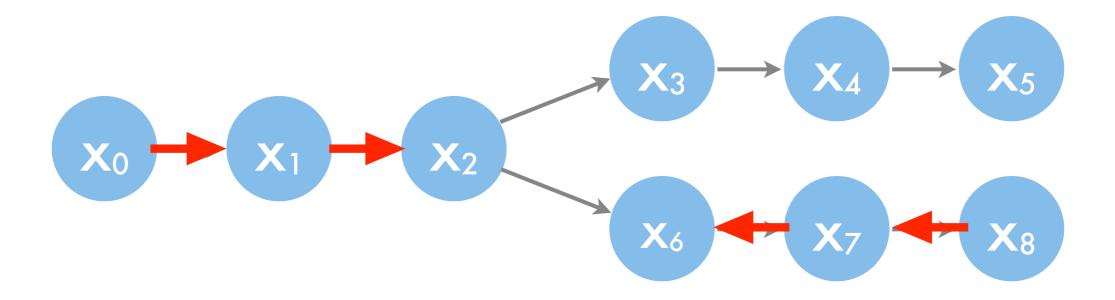




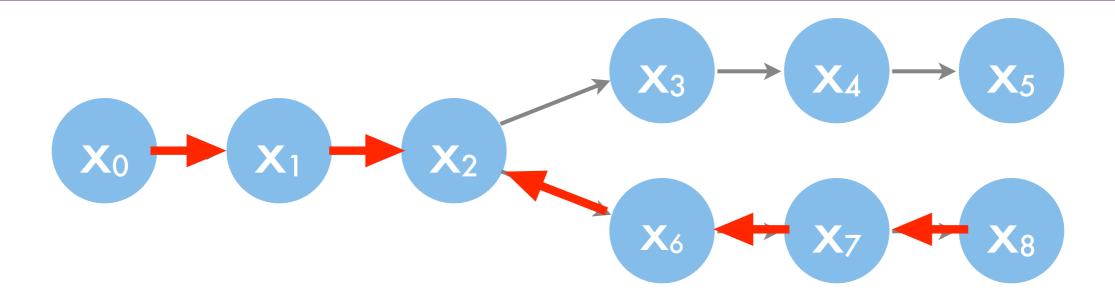
- Forward/Backward messages as normal for chain
- When we have more edges for a vertex use
  - For each outgoing message, send it once you have all other incoming messages
  - PRINCIPLED HACK
     If no message received yet, set it to 1 altogether



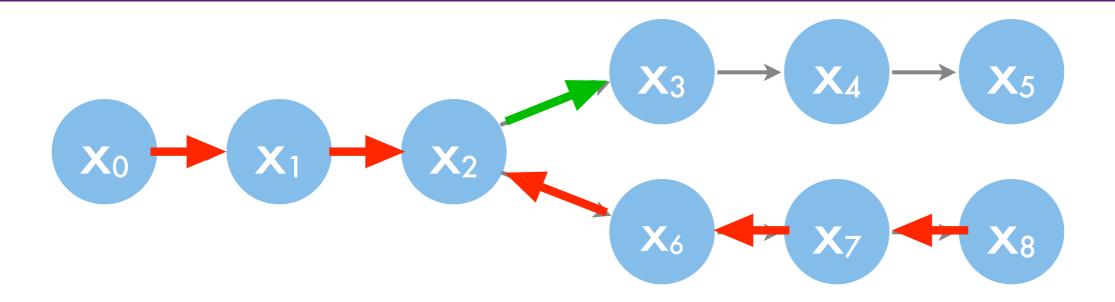
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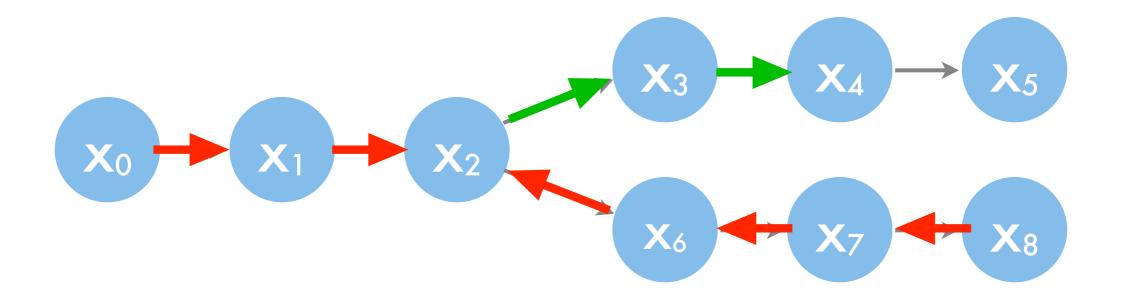
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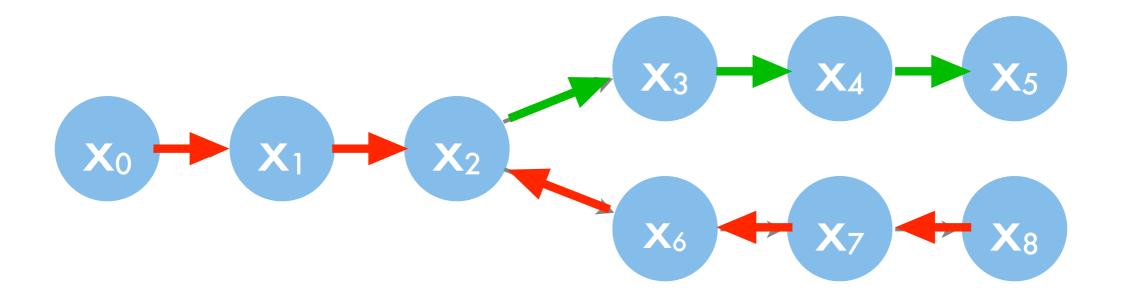
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  - PRINCIPLED HACK
     If no message received yet, set it to 1 altogether

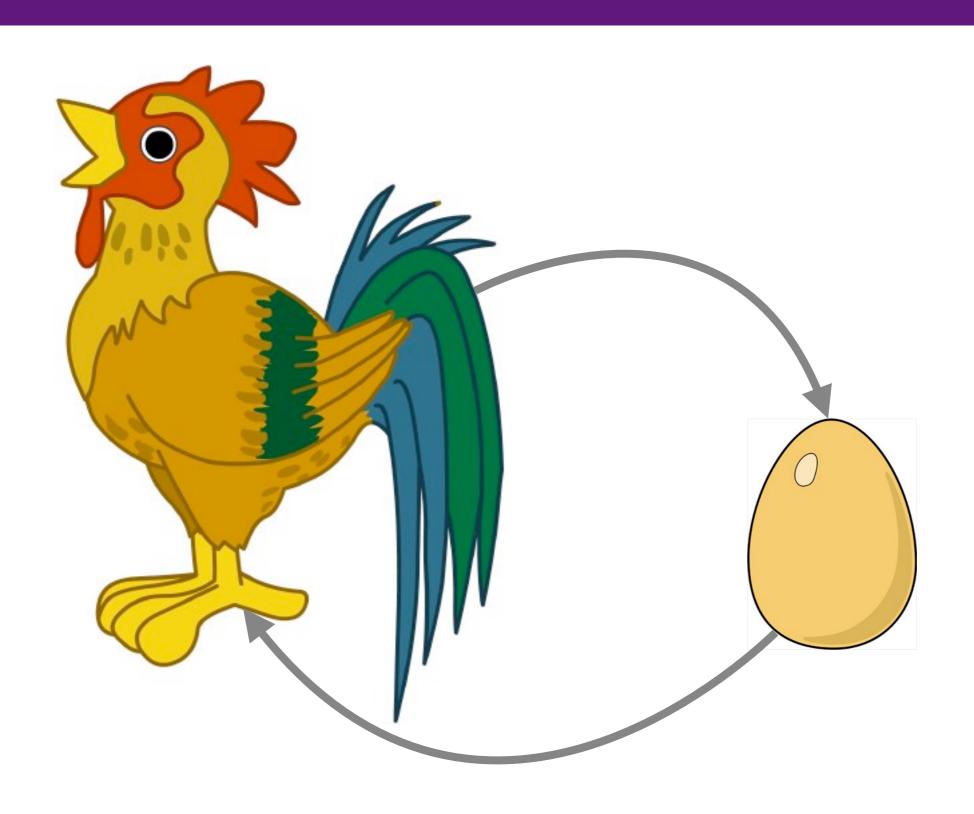


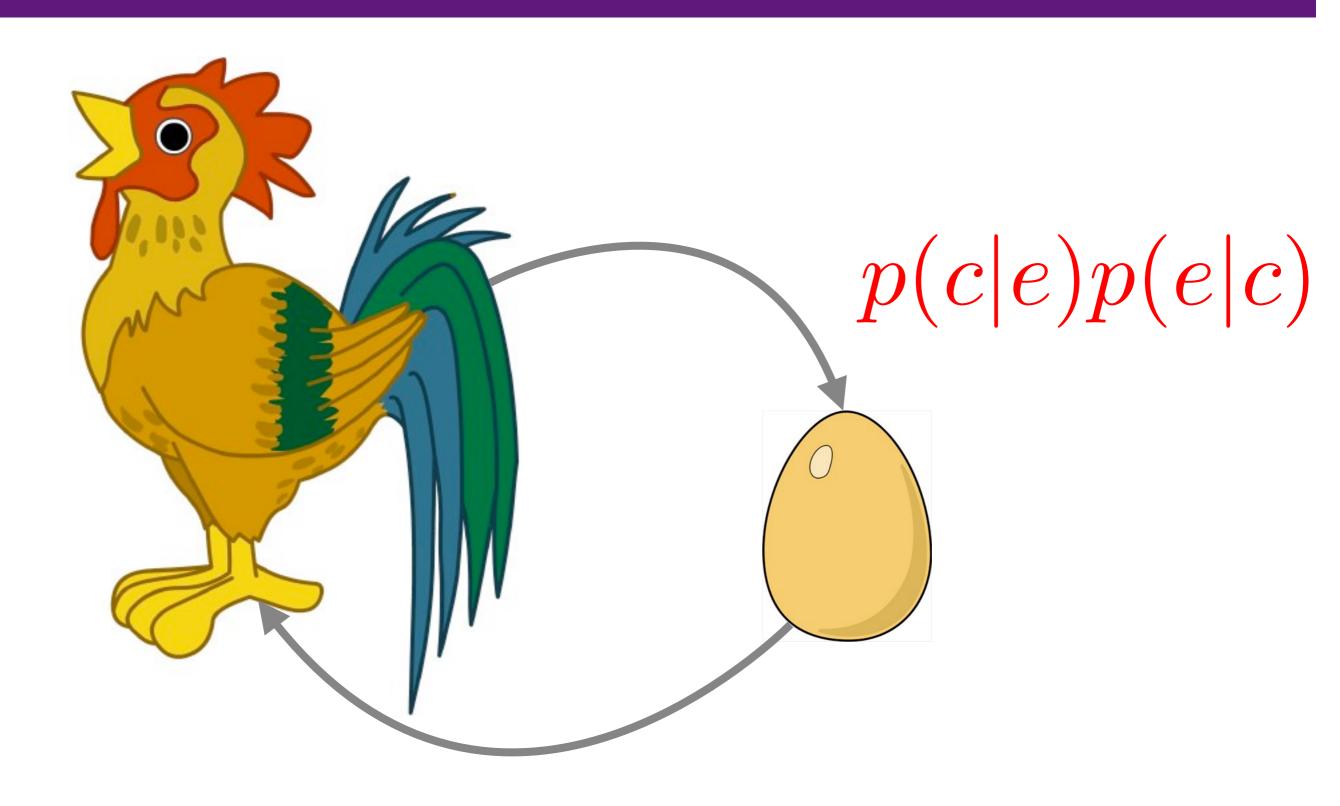
- Forward/Backward messages as normal for chain
- When we have more edges for a vertex use
  - For each outgoing message, send it once you have all other incoming messages
  - PRINCIPLED HACK
     If no message received yet, set it to 1 altogether

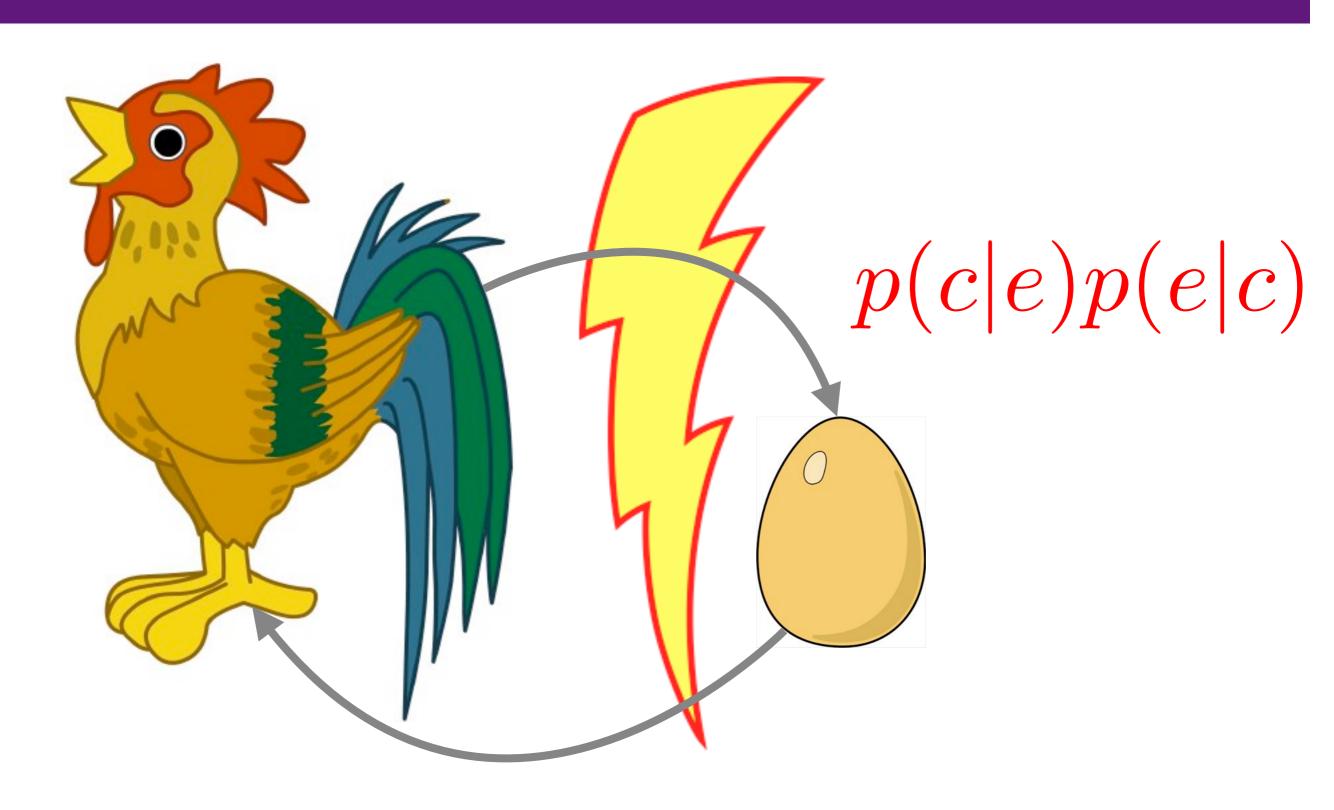


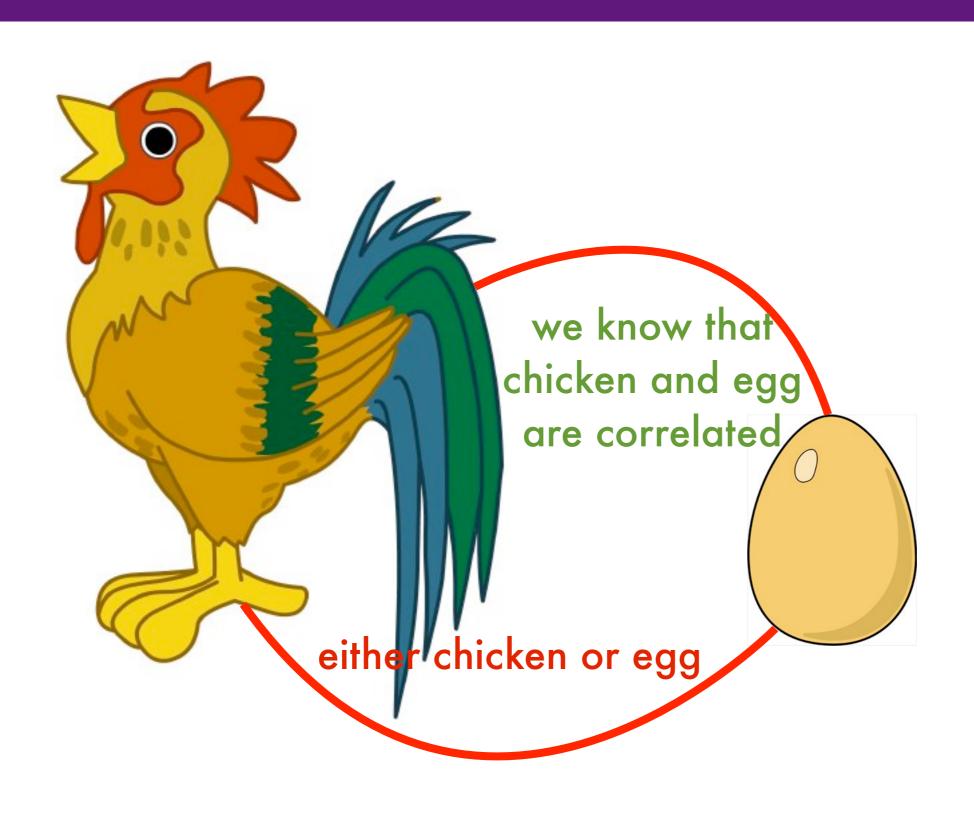
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     If no message received yet, set it to 1 altogether

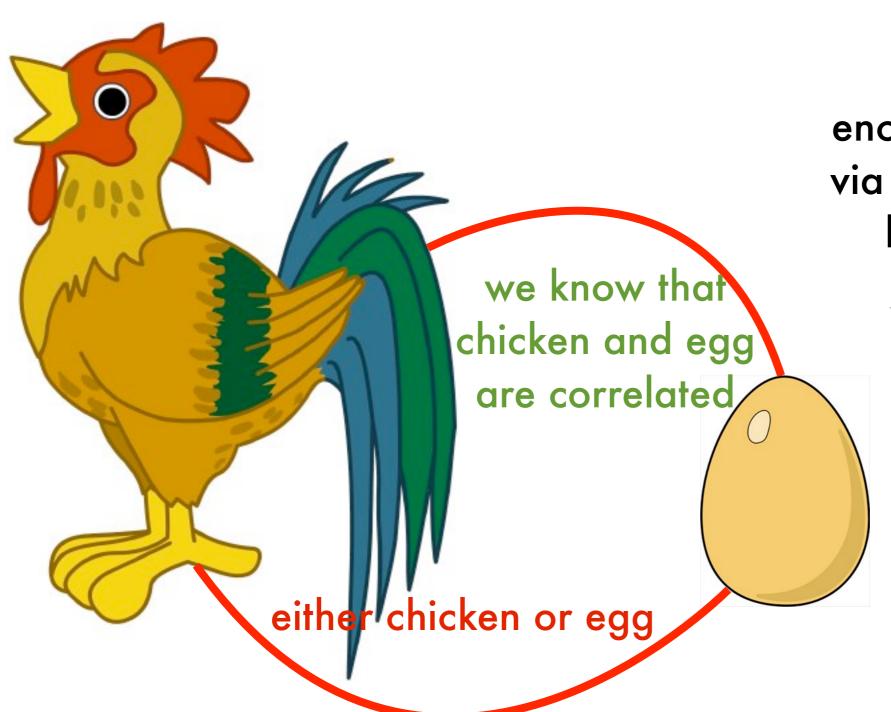
# Blunting the arrows ...





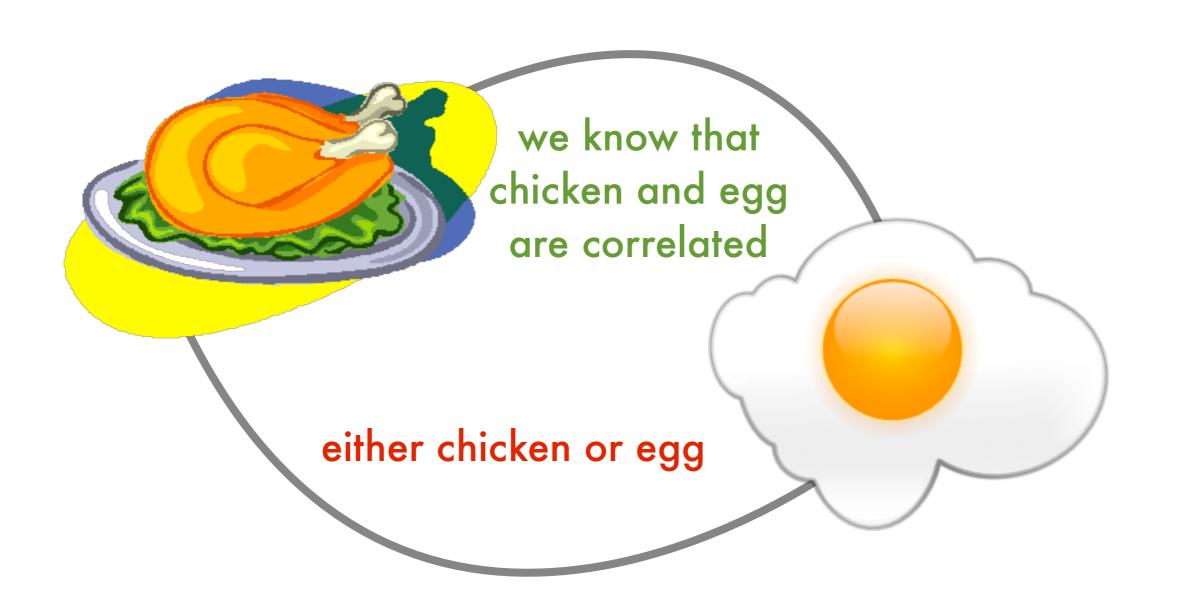






encode the correlation via the clique potential between c and e

 $p(c,e) \propto \exp \psi(c,e)$ 



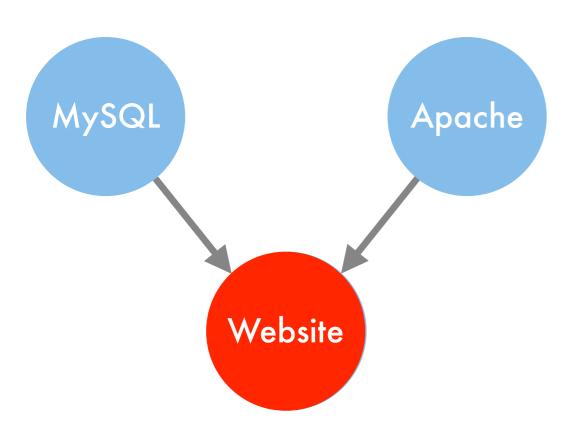
$$p(c, e) = \frac{\exp \psi(c, e)}{\sum_{c', e'} \exp \psi(c', e')}$$
$$= \exp \left[\psi(c, e) - g(\psi)\right] \text{ where } g(\psi) = \log \sum_{c, e} \exp \psi(c, e)$$



we know that chicken and egg are correlated

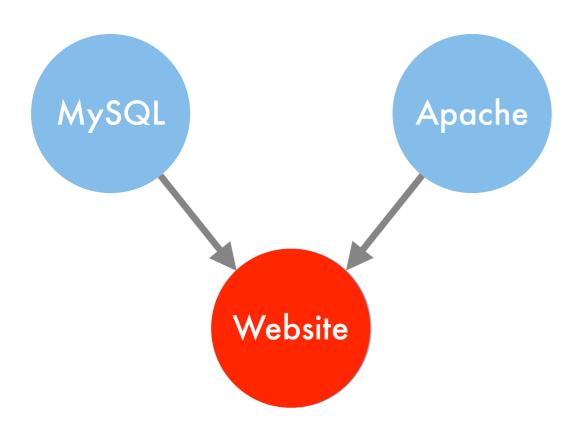
either chicken or egg

### ... some Yahoo service

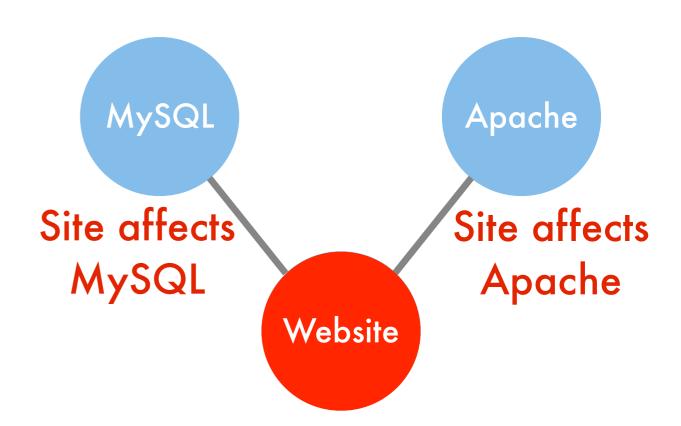


$$p(w|m,a)p(m)p(a)$$
 $m \not\perp \!\!\!\perp a|w$ 

### ... some Yahoo service

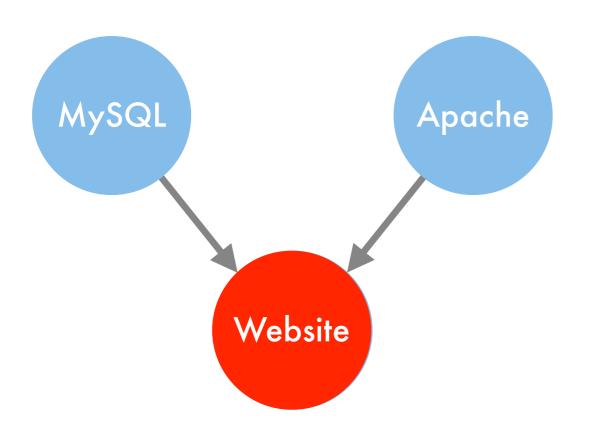


$$p(w|m,a)p(m)p(a)$$
 $m \not\perp \!\!\!\perp a|w$ 

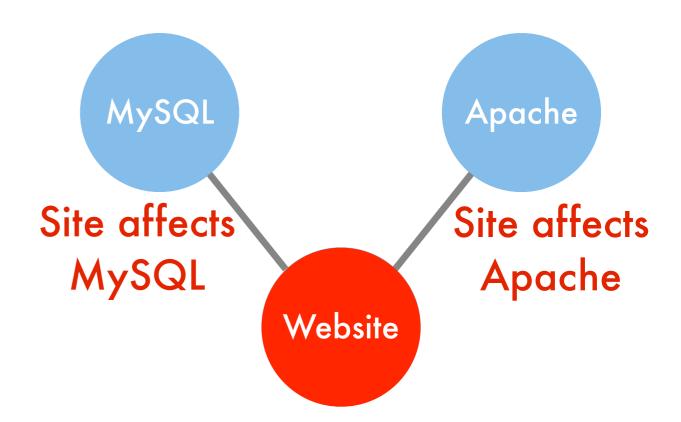


$$p(m, w, a) \propto \phi(m, w)\phi(w, a)$$
  
 $m \perp \!\!\!\perp a|w$ 

### ... some Yahoo service

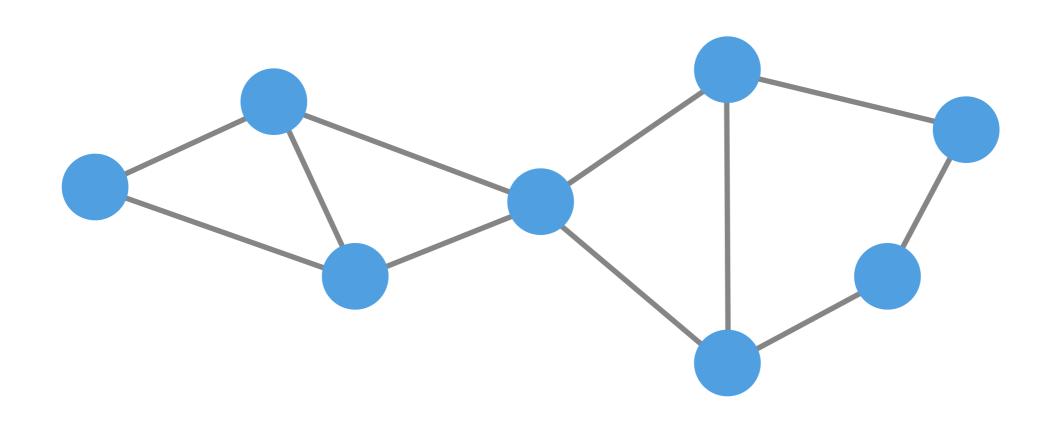


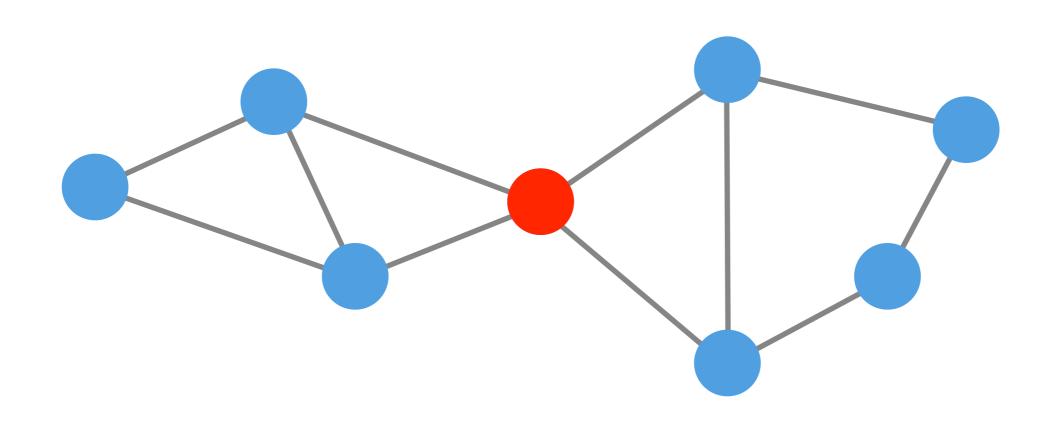
$$p(w|m,a)p(m)p(a)$$
 $m \not\perp \!\!\! \perp a|w$ 
easier
"debugging"

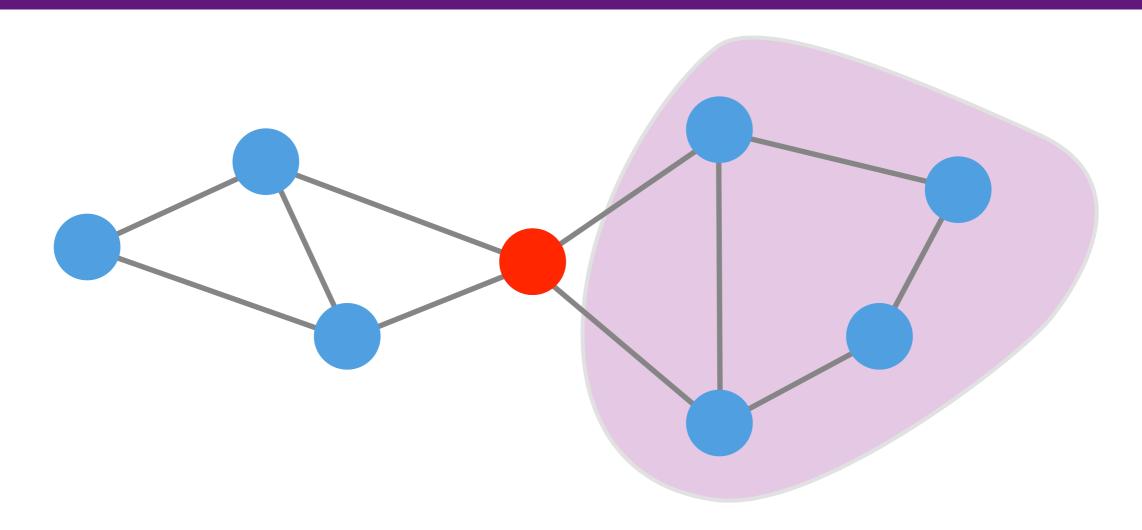


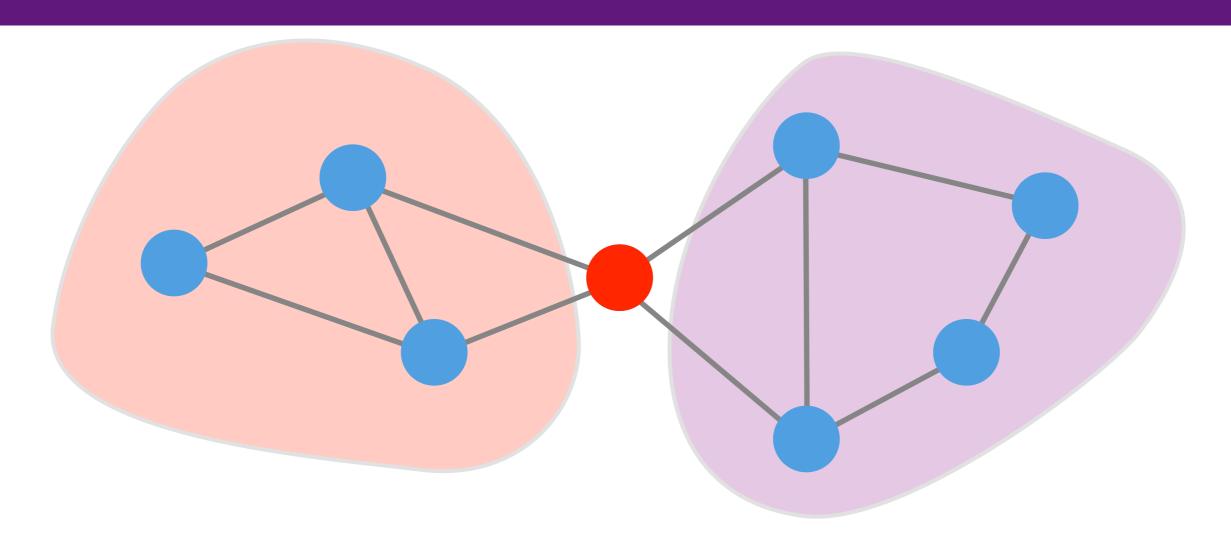
$$p(m, w, a) \propto \phi(m, w)\phi(w, a)$$
  
 $m \perp \!\!\! \perp a|w$ 

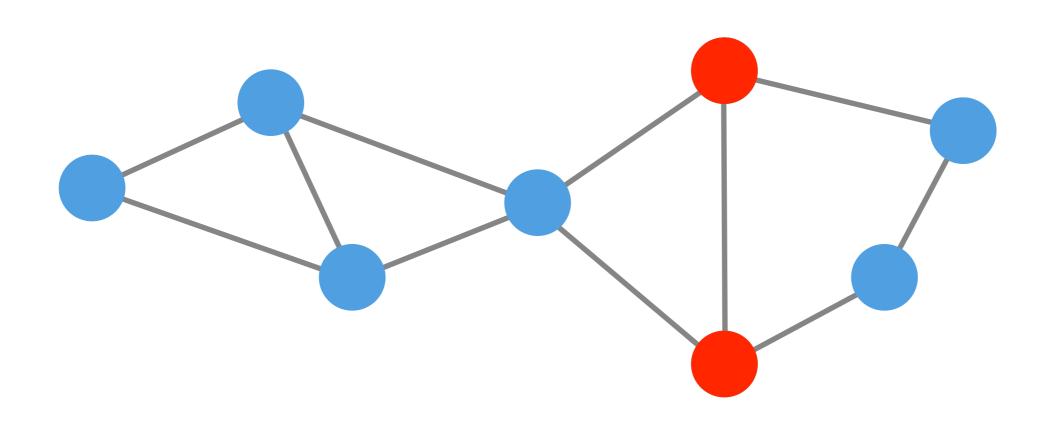
easier
"modeling"

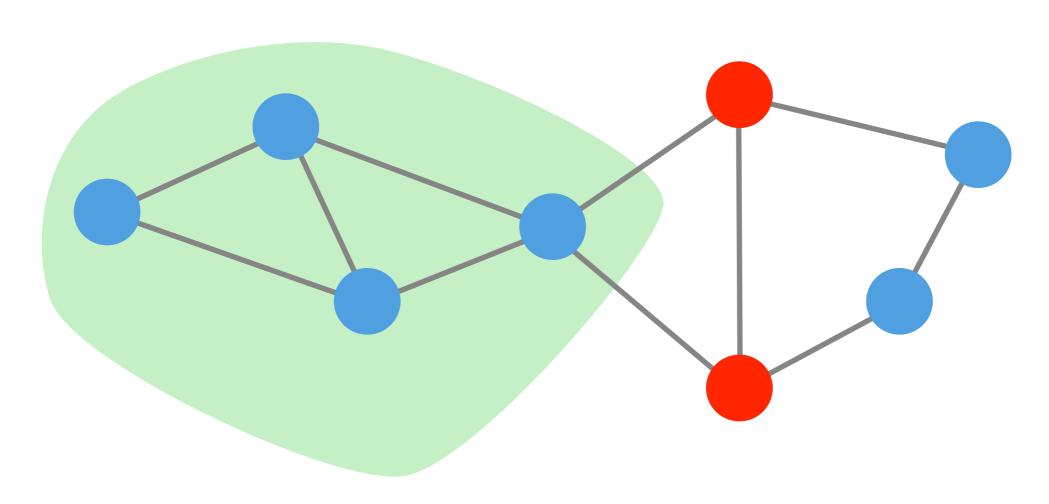


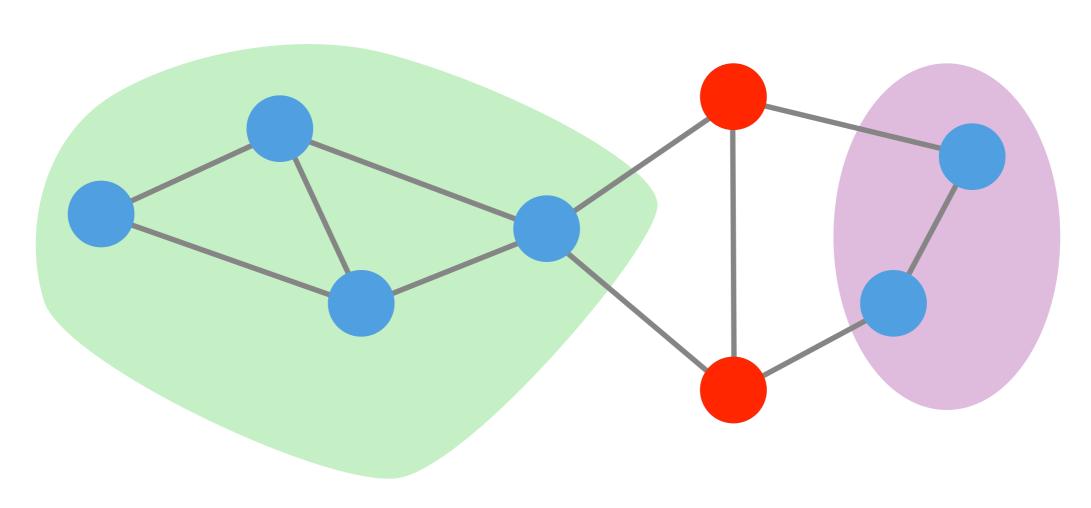


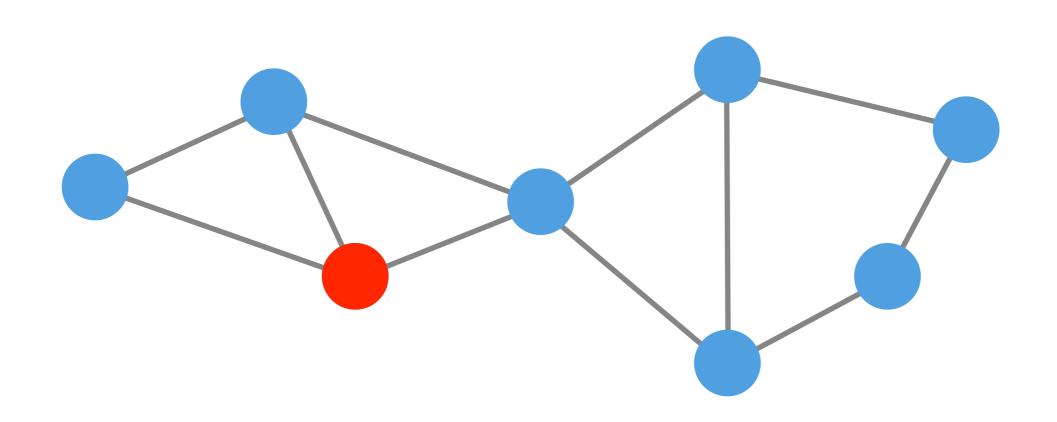


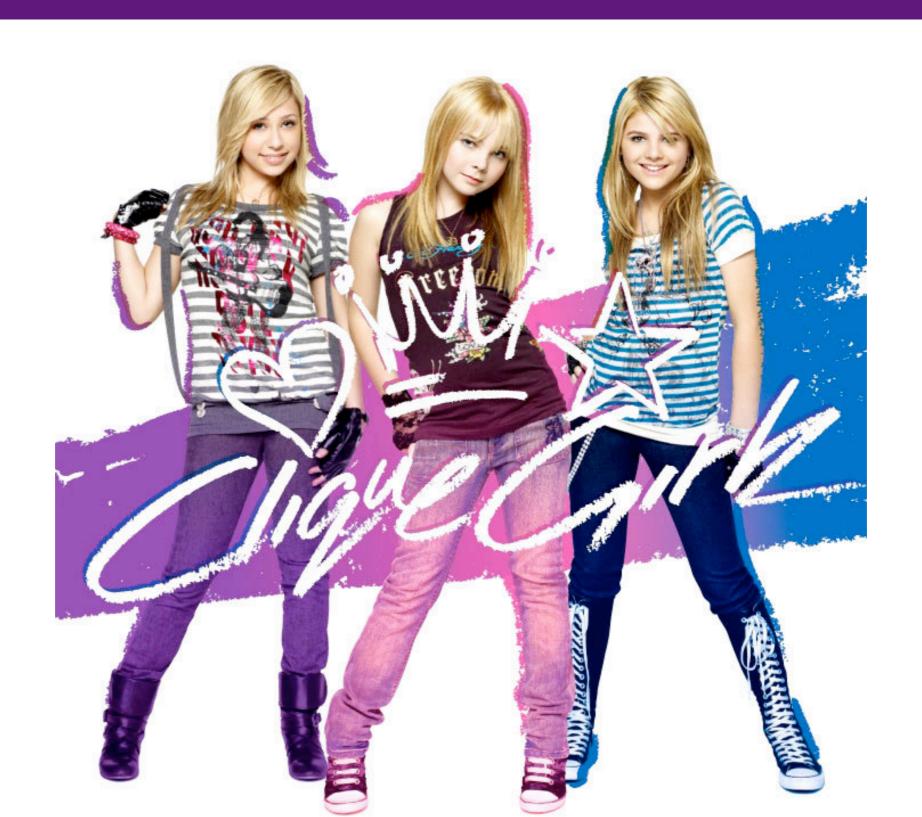




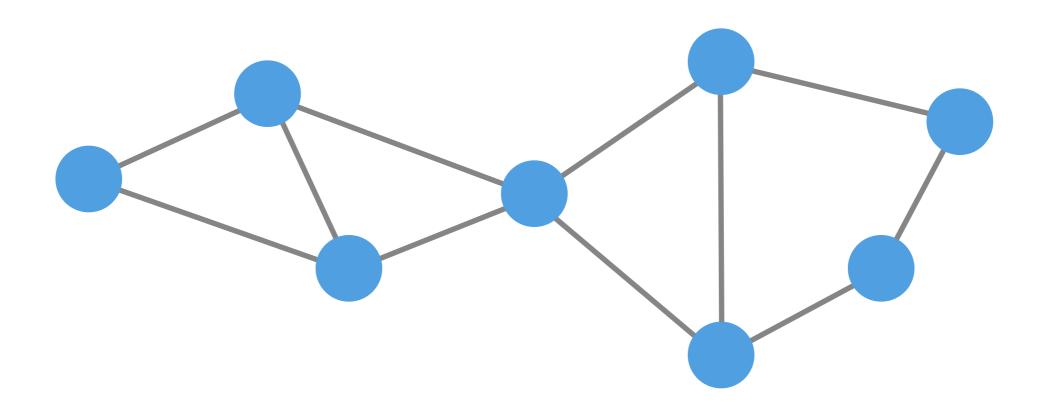




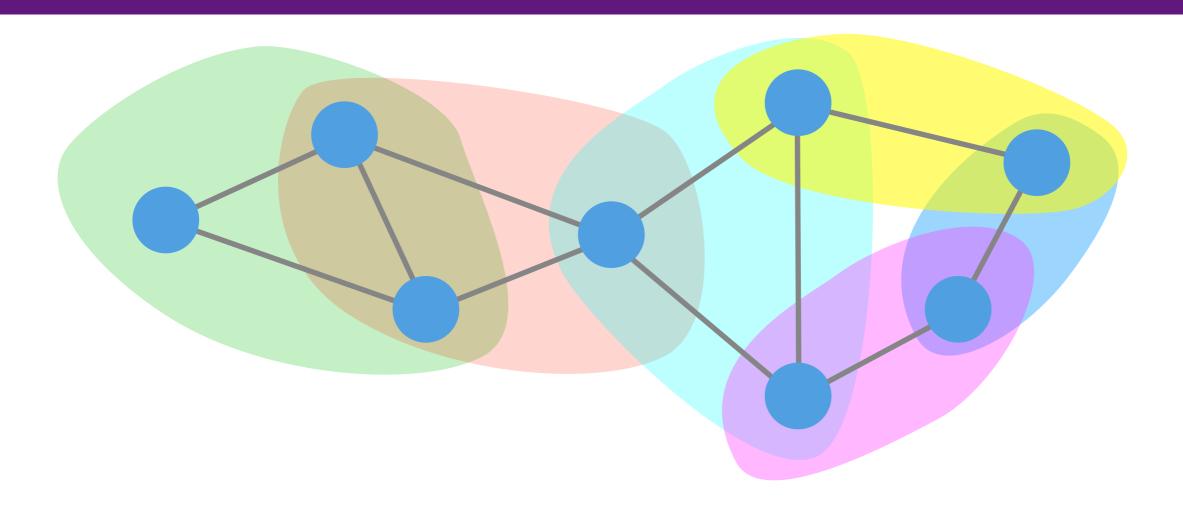






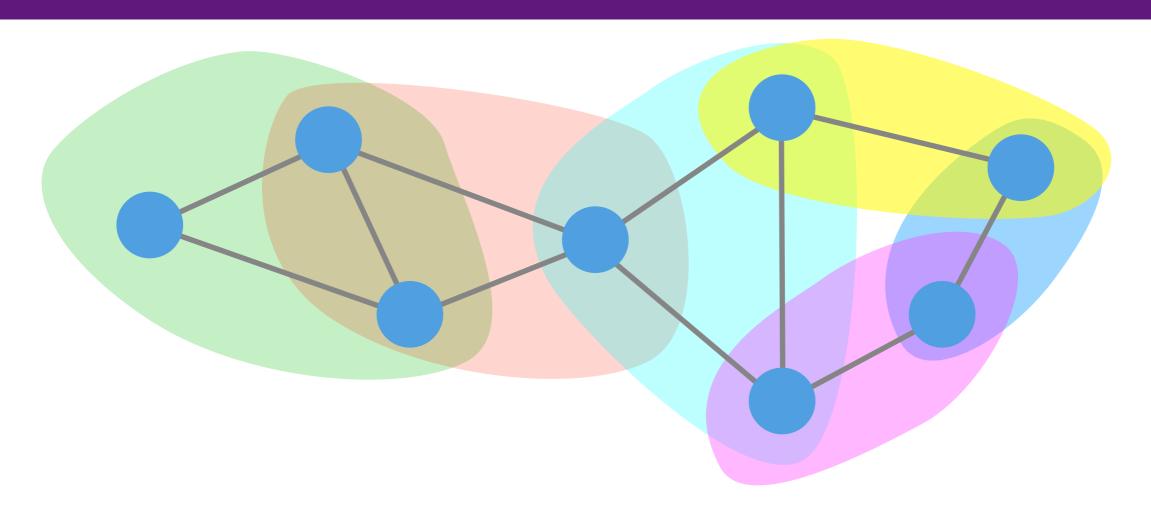


maximal fully connected subgraph



maximal fully connected subgraph

## Hammersley Clifford Theorem



If density has full support then it decomposes into products of clique potentials

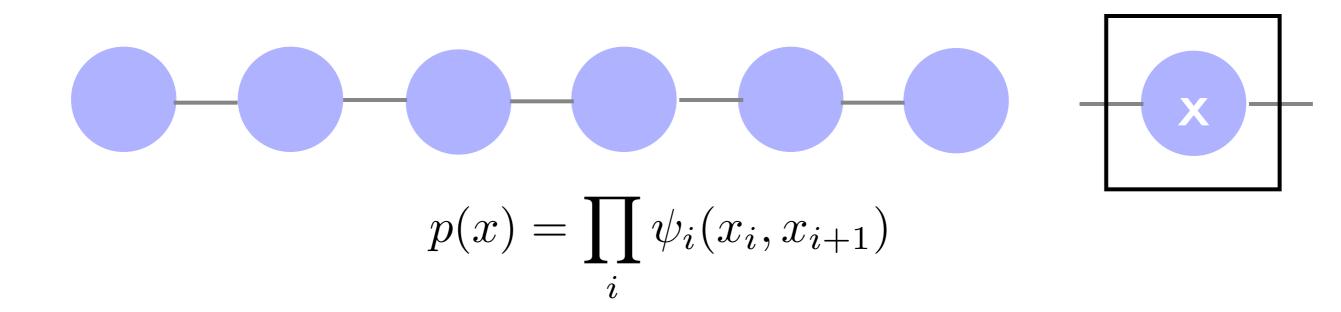
$$p(x) = \prod_{c} \psi_c(x_c)$$

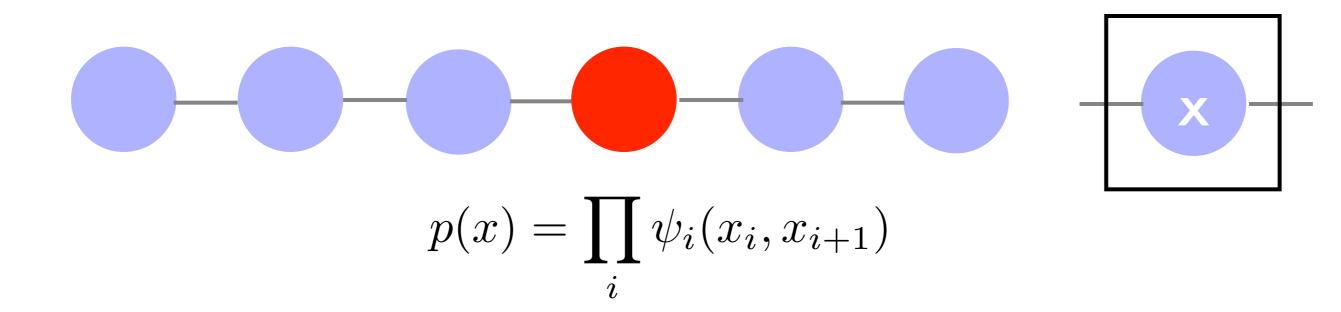
#### Directed vs. Undirected

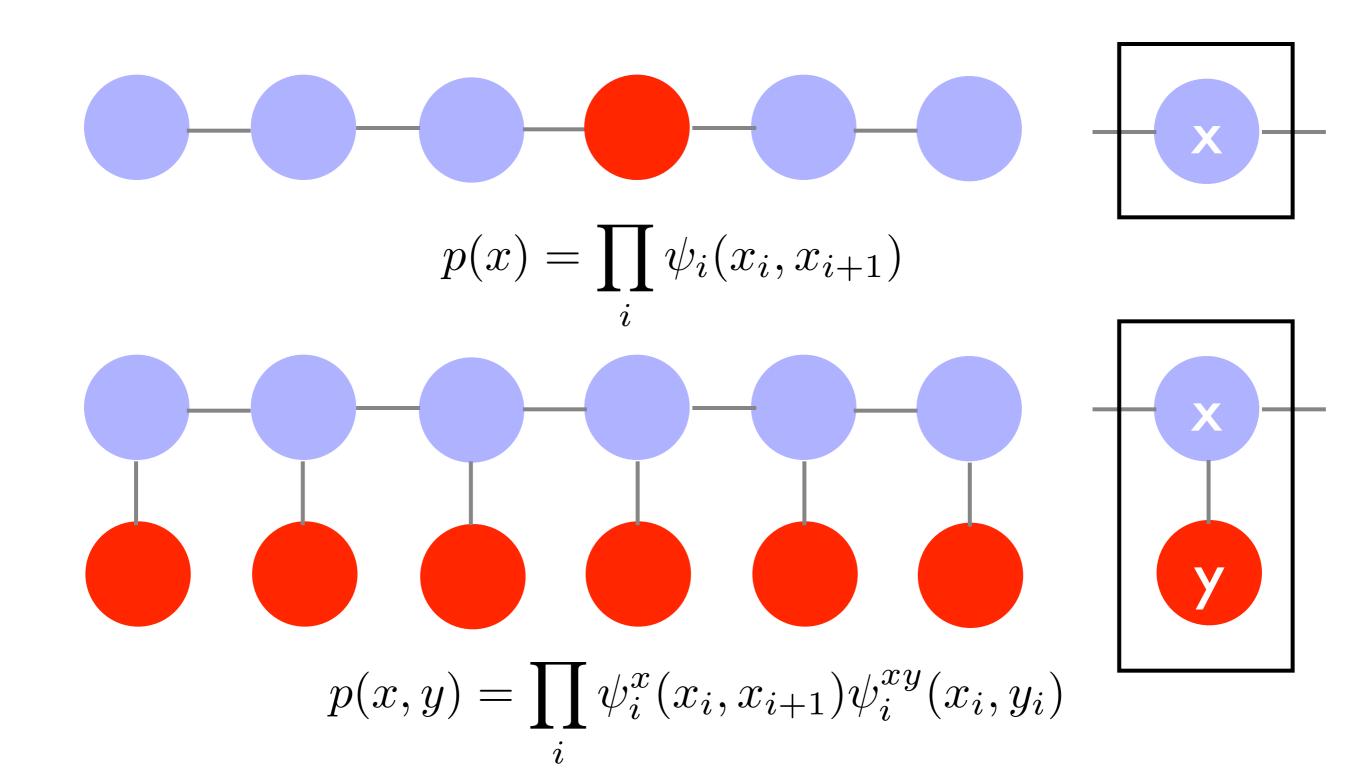
- Causal description
- Normalization automatic
- Intuitive
- Requires knowledge of dependencies
- Conditional independence tricky (Bayes Ball algorithm)

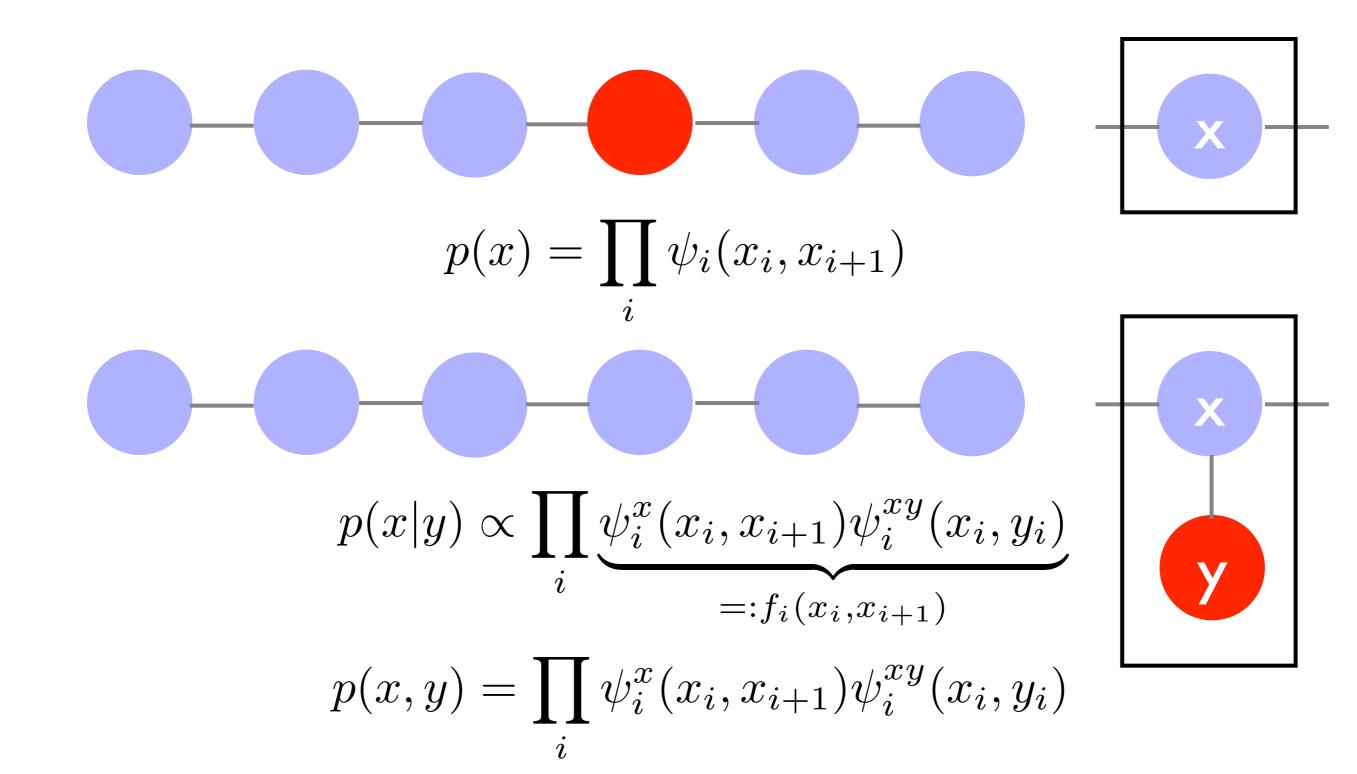
- Noncausal description (correlation only)
- Intuitive
- Easy modeling
- Normalization difficult
- Conditional independence easy to read off (graph connectivity)

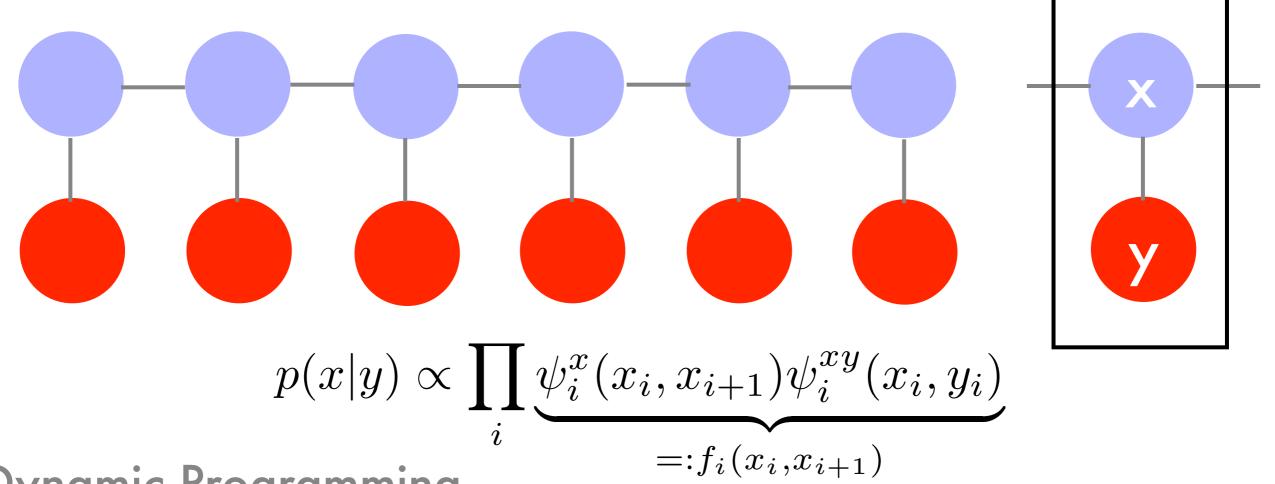
# Examples









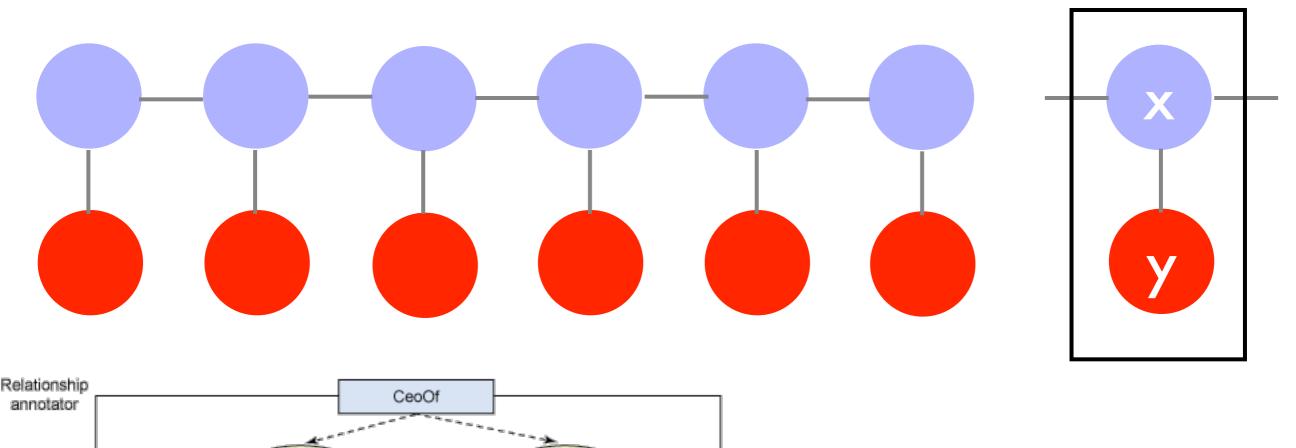


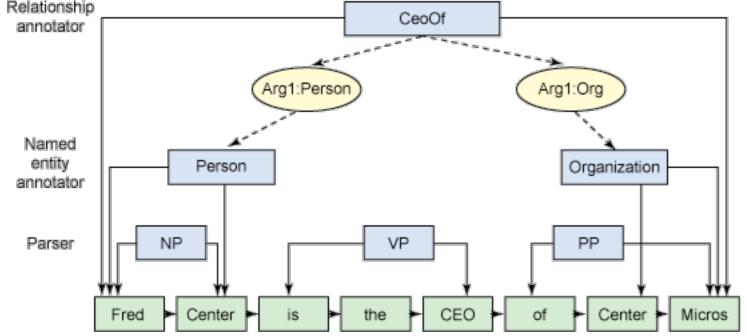
#### **Dynamic Programming**

$$l_1(x_1) = 1$$
 and  $l_{i+1}(x_{i+1}) = \sum_{x_i} l_i(x_i) f_i(x_i, x_{i+1})$ 

$$r_n(x_n) = 1$$
 and  $r_i(x_i) = \sum_{x_{i+1}} r_{i+1}(x_{i+1}) f_i(x_i, x_{i+1})$ 

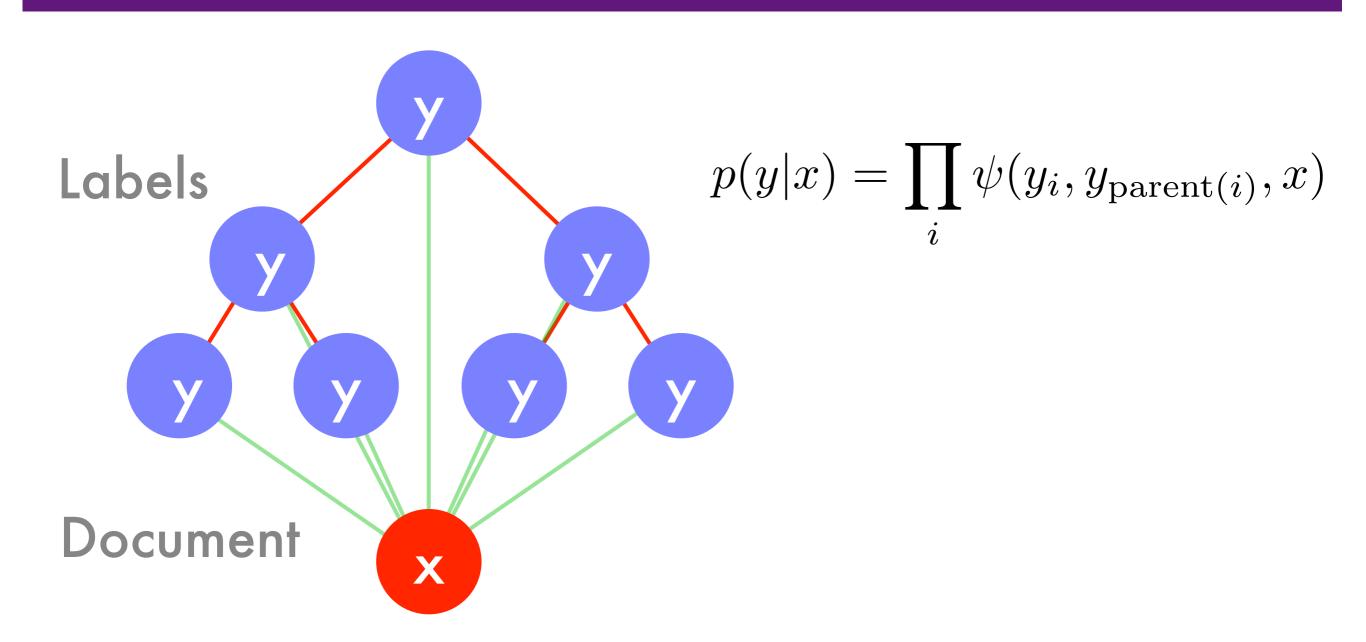
# Named Entity Tagging





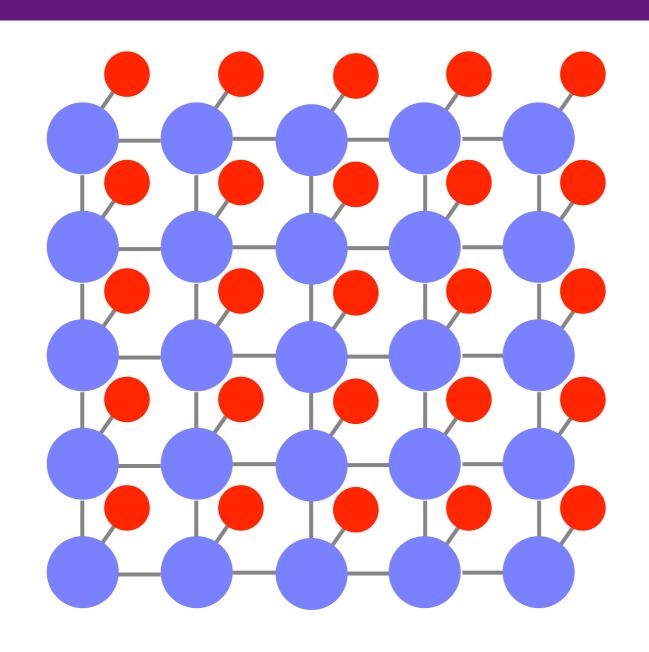
$$p(x|y) \propto \prod_{i} \underbrace{\psi_{i}^{x}(x_{i}, x_{i+1})\psi_{i}^{xy}(x_{i}, y_{i})}_{=:f_{i}(x_{i}, x_{i+1})}$$

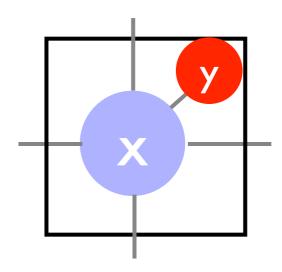
# Trees + Ontologies



Ontology classification (e.g. YDir, DMOZ)

# Spin Glasses + Images

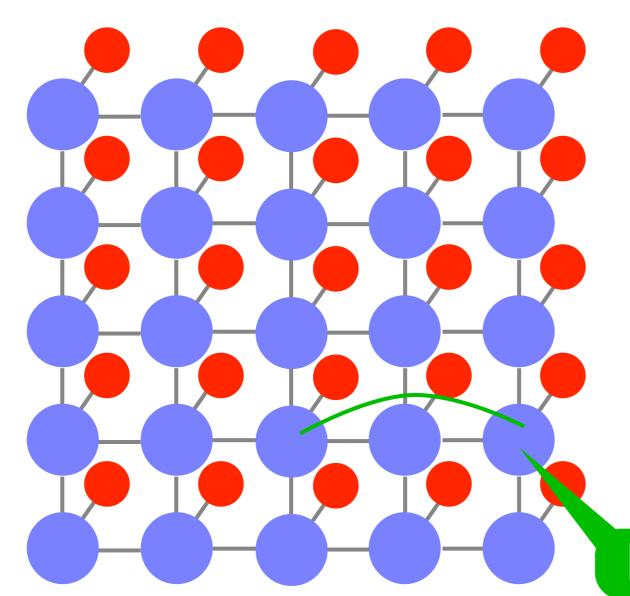


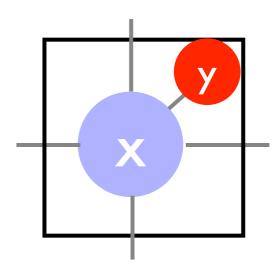


observed pixels real image

$$p(x|y) = \prod_{i,j} \psi^{\text{right}}(x_{ij}, x_{i+1,j}) \psi^{\text{up}}(x_{ij}, x_{i,j+1}) \psi^{xy}(x_{ij}, y_{ij})$$

# Spin Glasses + Images



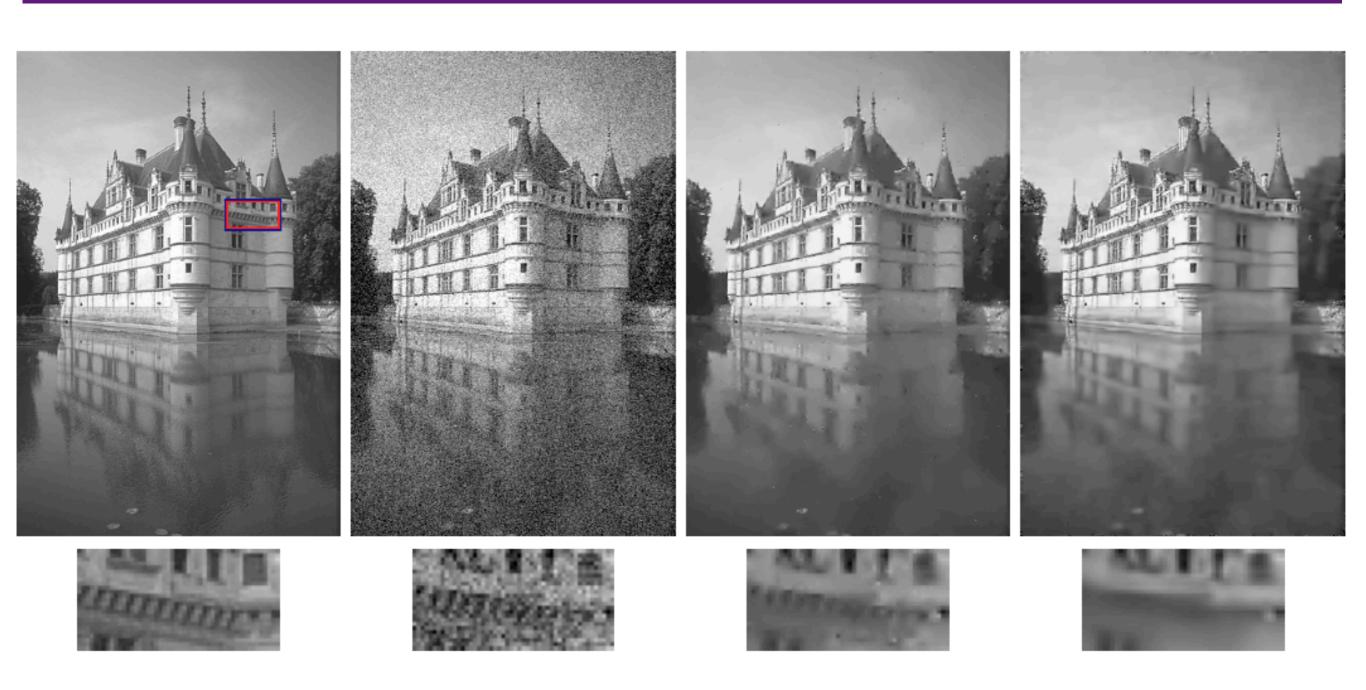


observed pixels real image

long range interactions

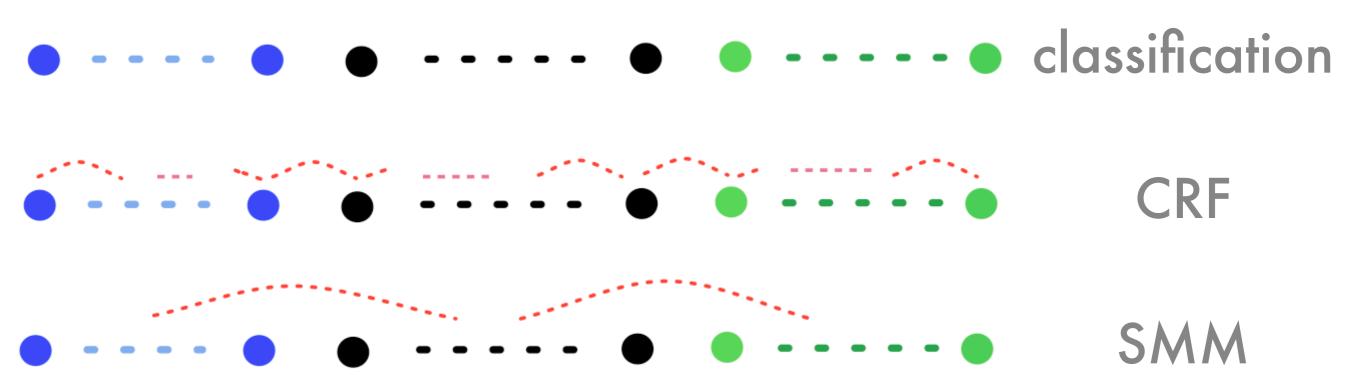
$$p(x|y) = \prod_{i,j} \psi^{\text{right}}(x_{ij}, x_{i+1,j}) \psi^{\text{up}}(x_{ij}, x_{i,j+1}) \psi^{xy}(x_{ij}, y_{ij})$$

# Image Denoising



Li&Huttenlocher, ECCV'08

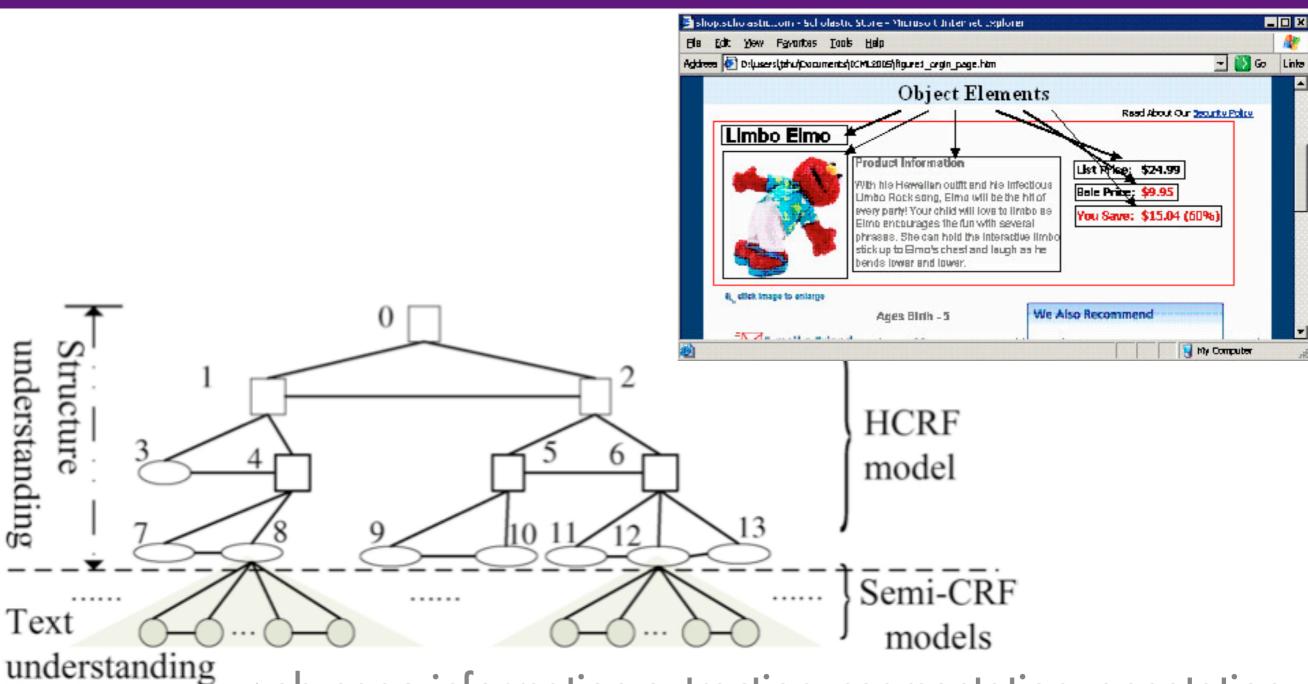
#### Semi-Markov Models



- Flexible length of an episode
- Segmentation between episodes

phrase segmentation, activity recognition, motion data analysis Shi, Smola, Altun, Vishwanathan, Li, 2007-2009

# 2D CRF for Webpages



web page information extraction, segmentation, annotation Bo, Zhu, Nie, Wen, Hon, 2005-2007

# Exponential Families and Graphical Models

# Exponential Family Reunion

Density function

$$p(x; \theta) = \exp\left(\langle \phi(x), \theta \rangle - g(\theta)\right)$$
  
where  $g(\theta) = \log \sum_{x'} \exp\left(\langle \phi(x'), \theta \rangle\right)$ 

Log partition function generates cumulants

$$\partial_{\theta} g(\theta) = \mathbf{E} \left[ \phi(x) \right]$$
  
 $\partial_{\theta}^{2} g(\theta) = \operatorname{Var} \left[ \phi(x) \right]$ 

• g is convex (second derivative is p.s.d.)

## Log Partition Function

$$p(x|\theta) = e^{\langle \phi(x), \theta \rangle - g(\theta)}$$
$$g(\theta) = \log \sum_{x} e^{\langle \phi(x), \theta \rangle}$$

#### Unconditional model

$$\partial_{\theta} g(\theta) = \frac{\sum_{x} \phi(x) e^{\langle \phi(x), \theta \rangle}}{\sum_{x} e^{\langle \phi(x), \theta \rangle}} = \sum_{x} \phi(x) e^{\langle \phi(x), \theta \rangle - g(\theta)}$$

$$p(y|\theta,x) = e^{\langle \phi(x,y),\theta \rangle - g(\theta|x)}$$

Conditional model

$$g(\theta|x) = \log \sum_{y} e^{\langle \phi(x,y), \theta \rangle}$$

$$\partial_{\theta} g(\theta|x) = \frac{\sum_{y} \phi(x,y) e^{\langle \phi(x,y), \theta \rangle}}{\sum_{y} e^{\langle \phi(x,y), \theta \rangle}} = \sum_{y} \phi(x,y) e^{\langle \phi(x,y), \theta \rangle - g(\theta|x)}$$

#### Estimation

#### Conditional log-likelihood

$$\log p(y|x;\theta) = \langle \phi(x,y), \theta \rangle - g(\theta|x)$$

Log-posterior (Gaussian Prior)

$$\log p(\theta|X,Y) = \sum_{i} \log(y_i|x_i;\theta) + \log p(\theta) + \text{const.}$$

$$= \left\langle \sum_{i} \phi(x_i, y_i), \theta \right\rangle - \sum_{i} g(\theta | x_i) - \frac{1}{2\sigma^2} \|\theta\|^2 + \text{const.}$$

• First order optimality conditions

expensive

maxent model

$$\sum_{i} \phi(x_i, y_i) = \sum_{i} \mathbf{E}_{y|x_i} \left[ \phi(x_i, y) \right] + \frac{1}{\sigma^2} \theta$$

prior

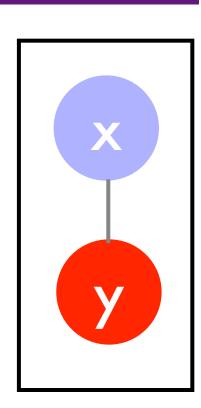
# Logistic Regression

#### Label space

$$\phi(x,y) = y\phi(x)$$
 where  $y \in \{\pm 1\}$ 

Log-partition function

$$g(\theta|x) = \log\left[e^{1\cdot\langle\phi(x),\theta\rangle} + e^{-1\cdot\langle\phi(x),\theta\rangle}\right] = \log 2\cosh\langle\phi(x),\theta\rangle$$



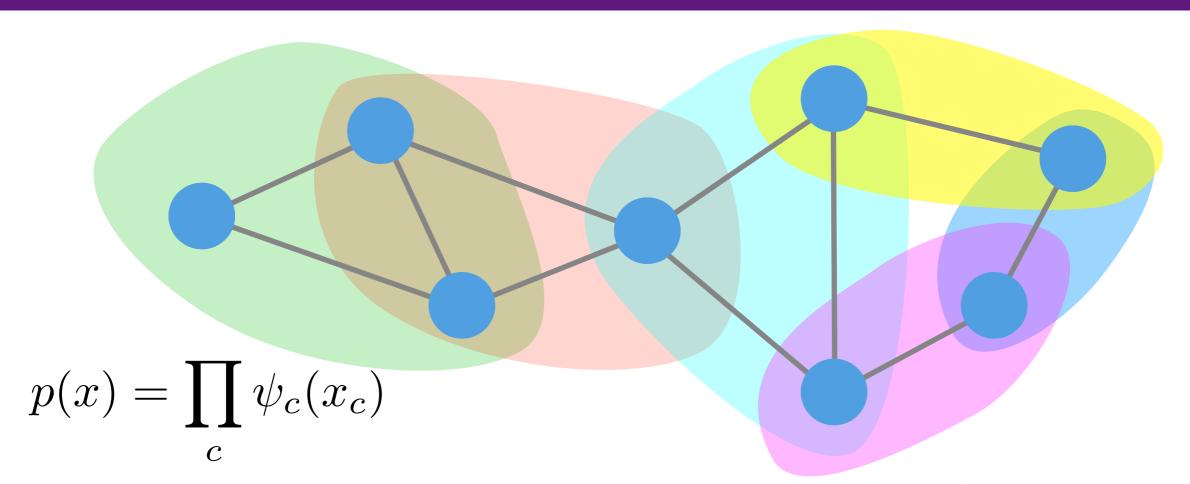
Convex minimization problem

$$\underset{\theta}{\text{minimize}} \frac{1}{2\sigma^2} \|\theta\|^2 + \sum_{i} \log 2 \cosh \langle \phi(x_i), \theta \rangle - y_i \langle \phi(x_i, \theta) \rangle$$

Prediction

$$p(y|x,\theta) = \frac{e^{y\langle\phi(x),\theta\rangle}}{e^{\langle\phi(x),\theta\rangle} + e^{-\langle\phi(x),\theta\rangle}} = \frac{1}{1 + e^{-2y\langle\phi(x),\theta\rangle}}$$

#### Exponential Clique Decomposition



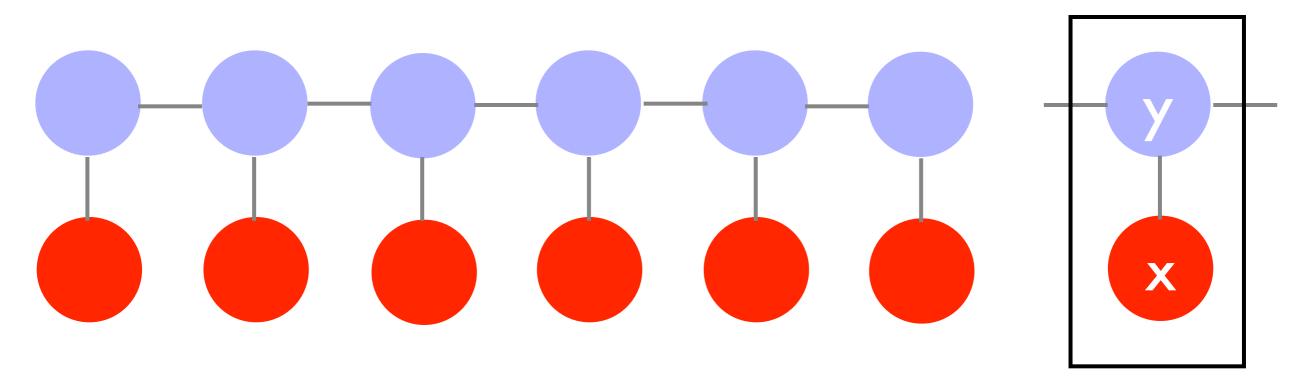
Theorem: Clique decomposition holds in sufficient statistics

$$\phi(x) = (\dots, \phi_c(x_c), \dots) \text{ and } \langle \phi(x), \theta \rangle = \sum \langle \phi_c(x_c), \theta_c \rangle$$

Corollary: we only need expectations on cliques

$$\mathbf{E}_x[\phi(x)] = (\dots, \mathbf{E}_{x_c} \left[\phi_c(x_c)\right], \dots)$$

## Conditional Random Fields



$$\phi(x) = (y_1 \phi_x(x_1), \dots, y_n \phi_x(x_n), \phi_y(y_1, y_2), \dots, \phi_y(y_{n-1}, y_n))$$
$$\langle \phi(x), \theta \rangle = \sum_i \langle \phi_x(x_i, y_i), \theta_x \rangle + \sum_i \langle \phi_y(y_i, y_{i+1}), \theta_y \rangle$$

$$g(\theta|x) = \sum_{y} \prod_{i} f_i(y_i, y_{i+1})$$
 where

$$f_i(y_i, y_{i+1}) = e^{\langle \phi_x(x_i, y_i), \theta_x \rangle + \langle \phi_y(y_i, y_{i+1}), \theta_y \rangle}$$

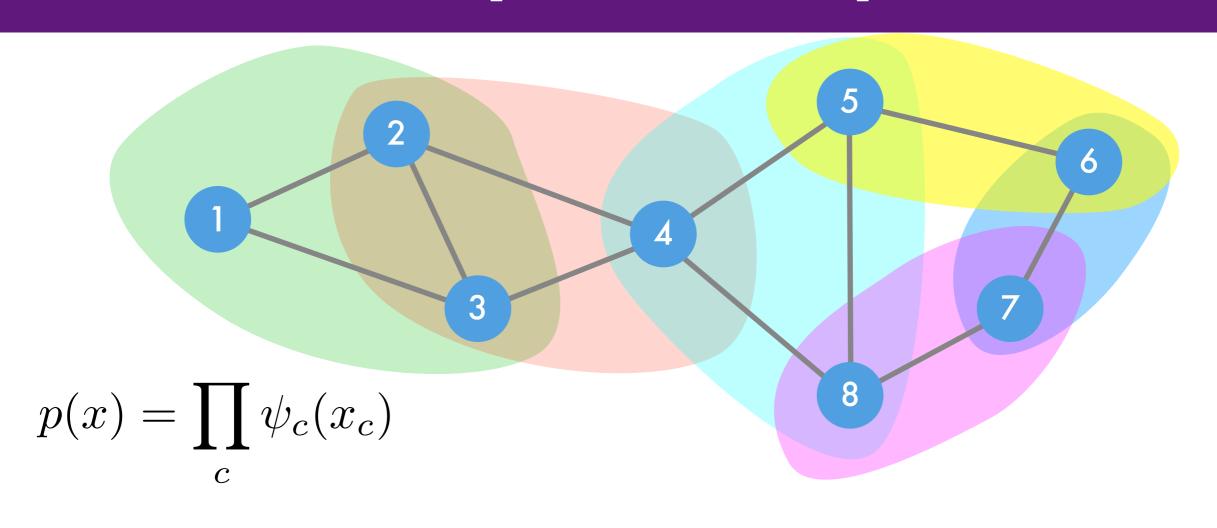
dynamic programming

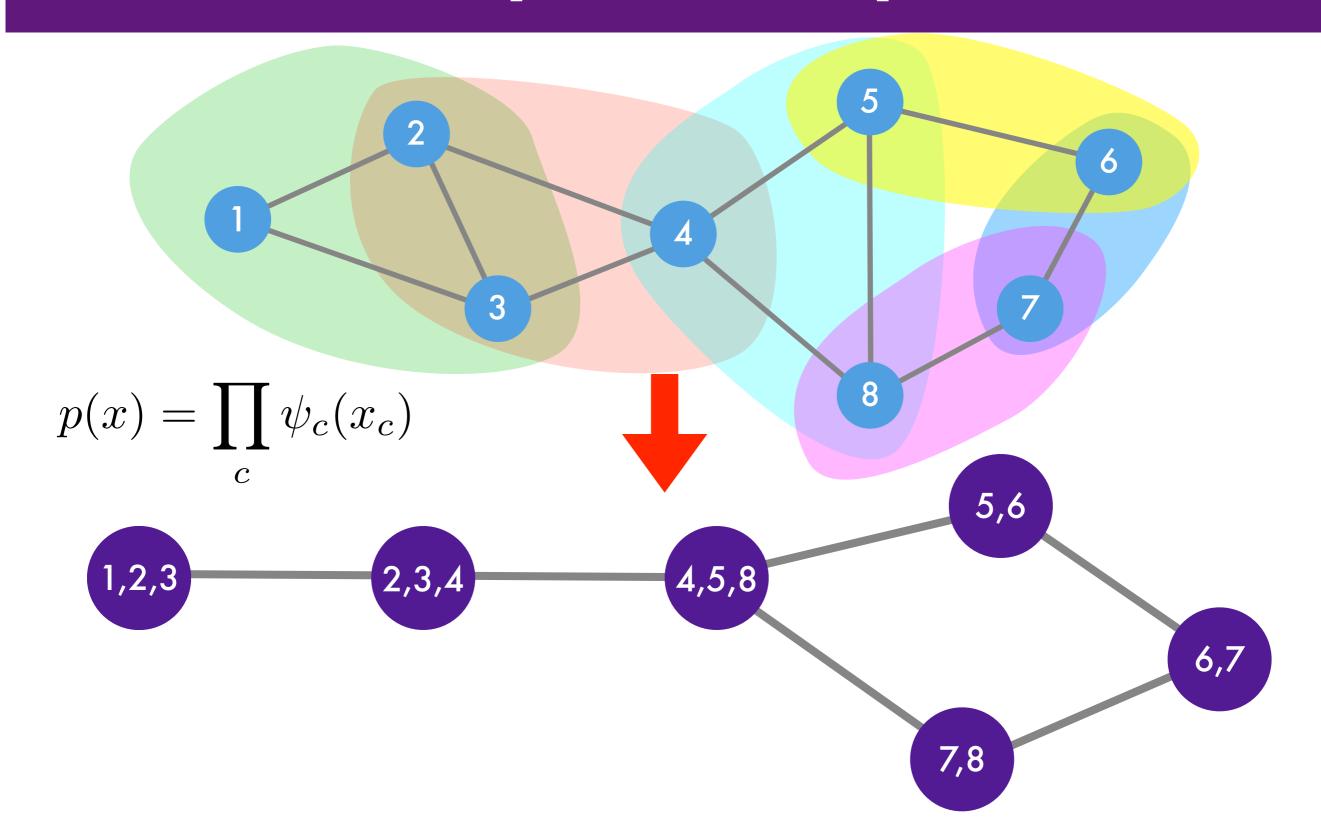
#### Conditional Random Fields

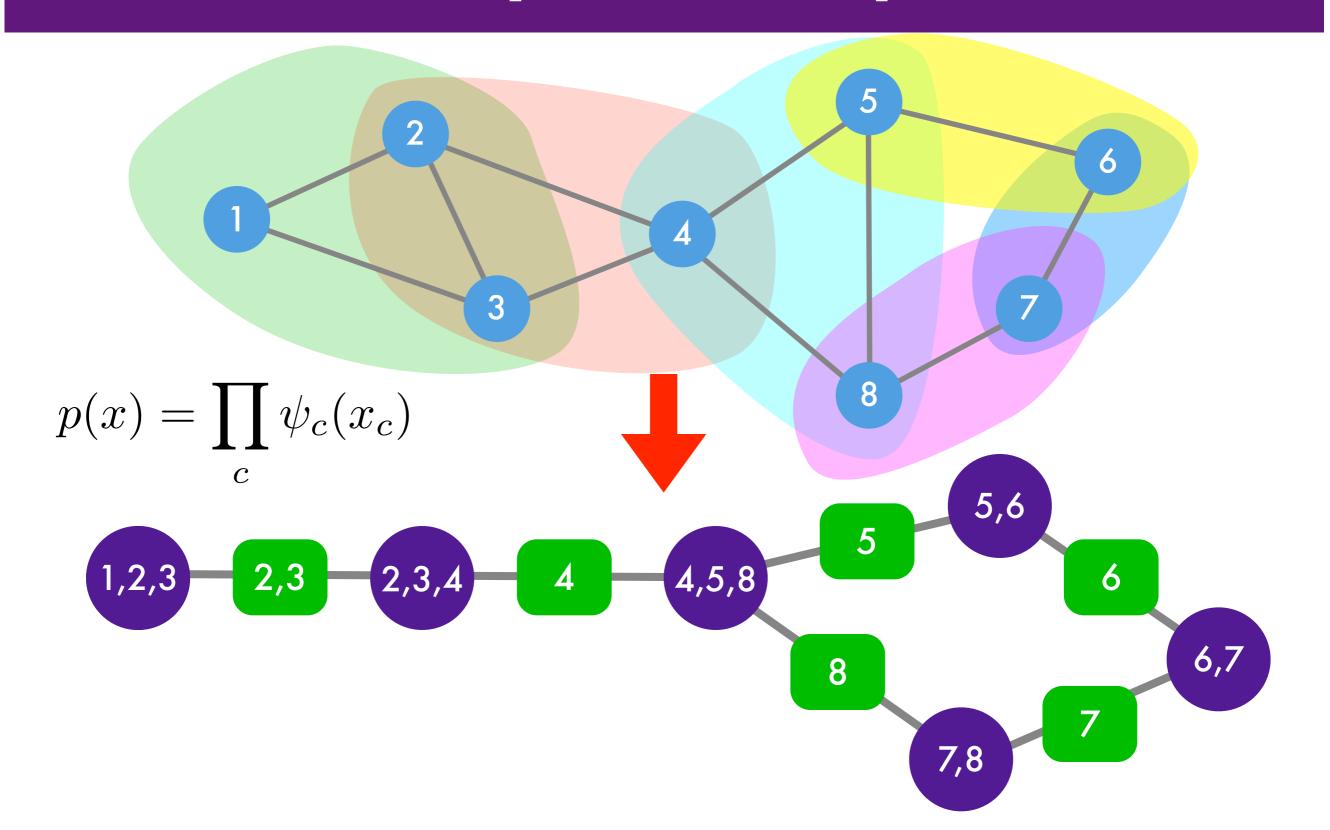
- Compute distribution over marginal and adjacent labels
- Take conditional expectations
- Take update step (batch or online)

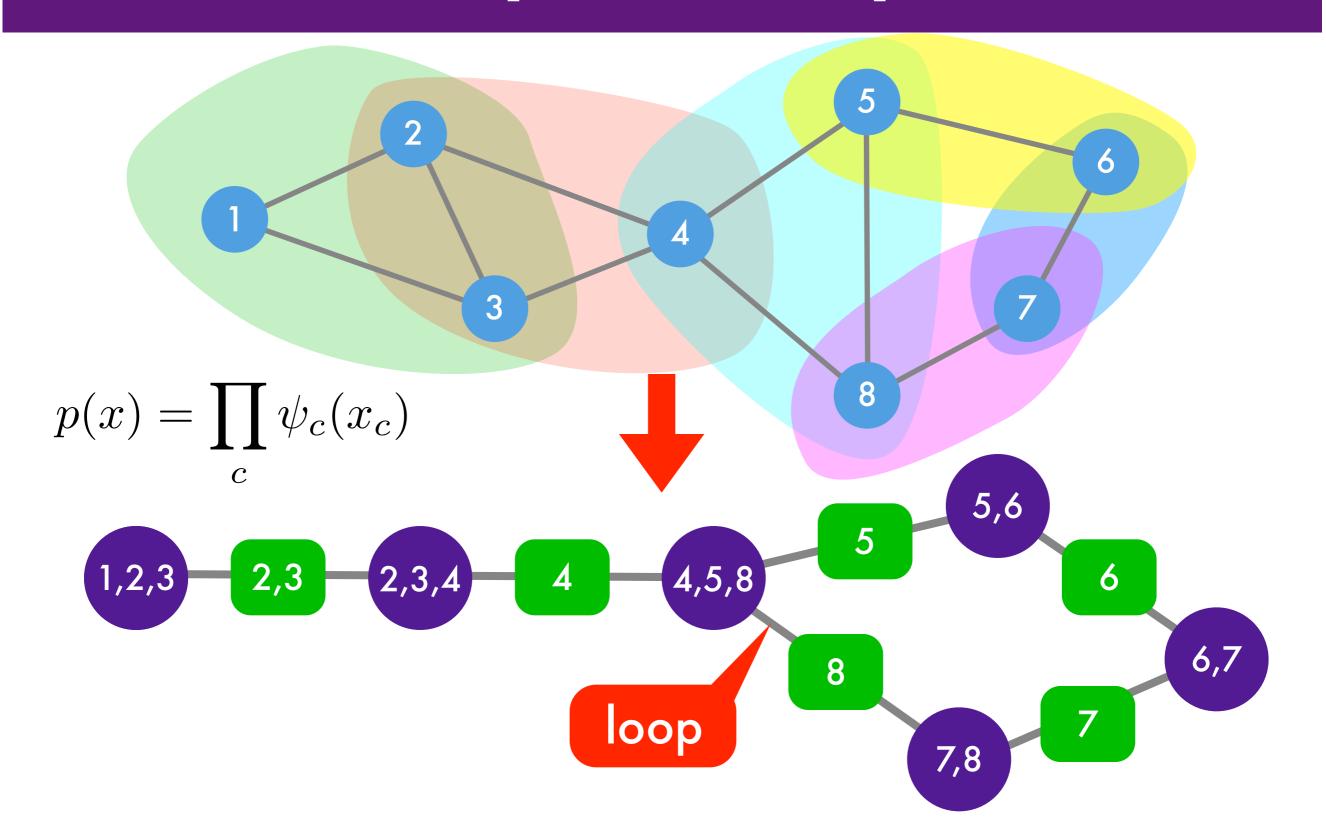
 More general techniques for computing normalization via message passing ...

# Dynamic Programming + Message Passing

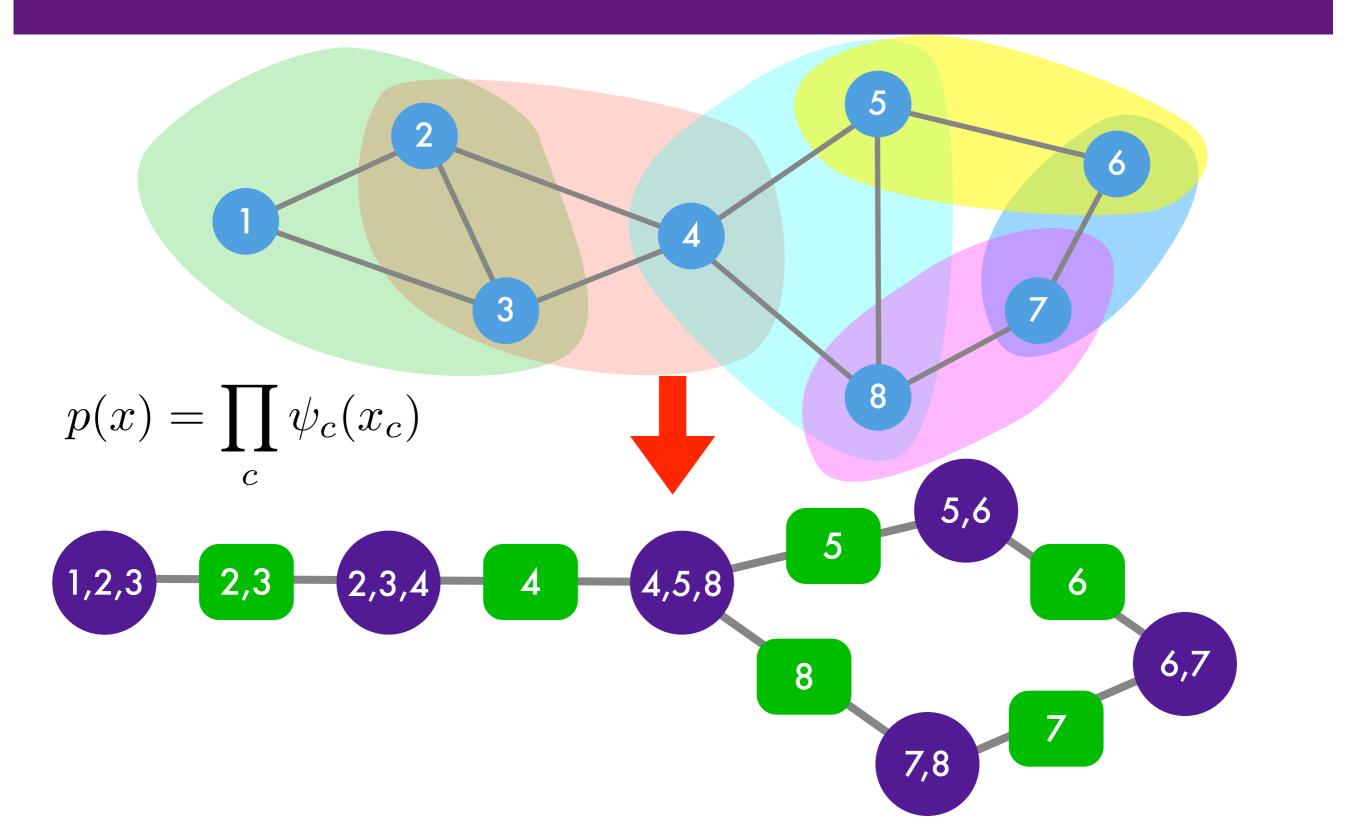




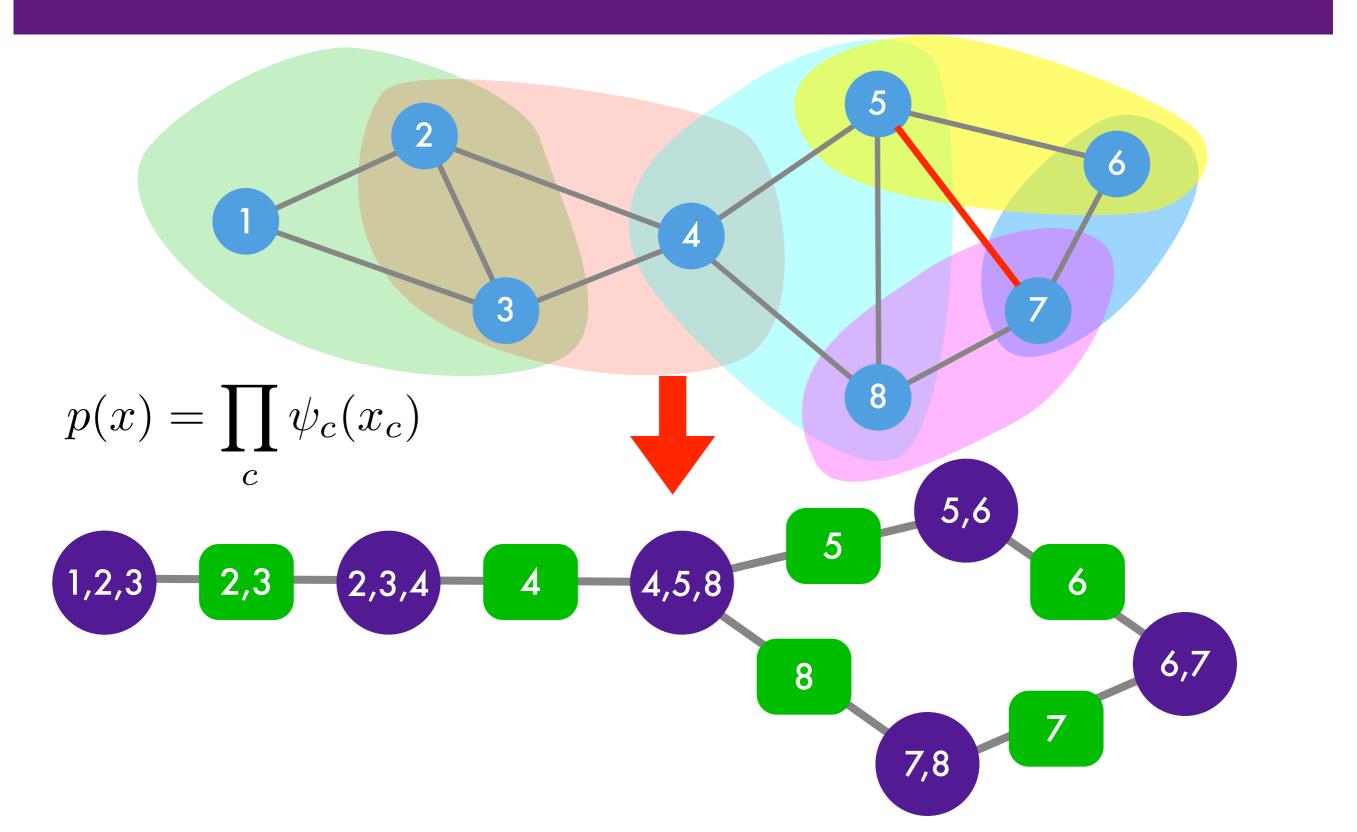




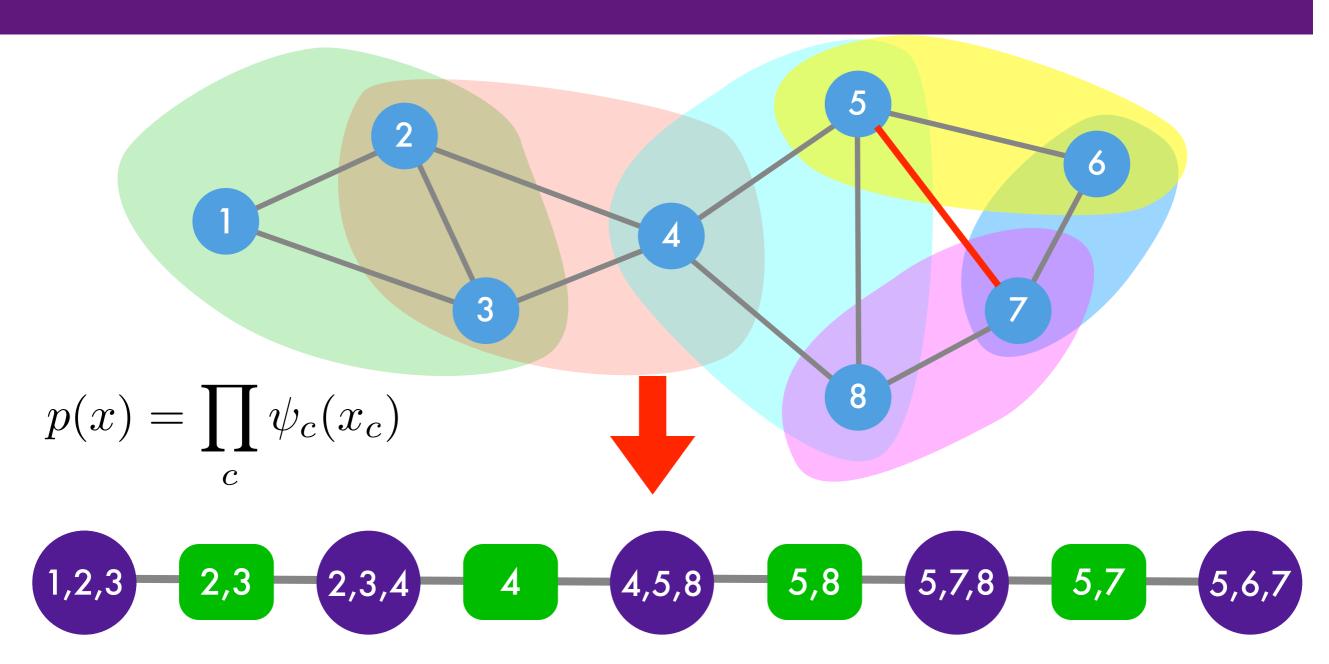
#### Junction Tree / Triangulation



#### Junction Tree / Triangulation

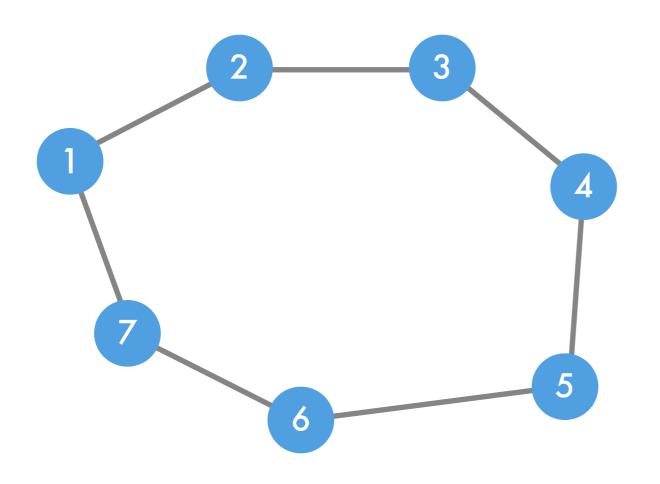


#### Junction Tree / Triangulation



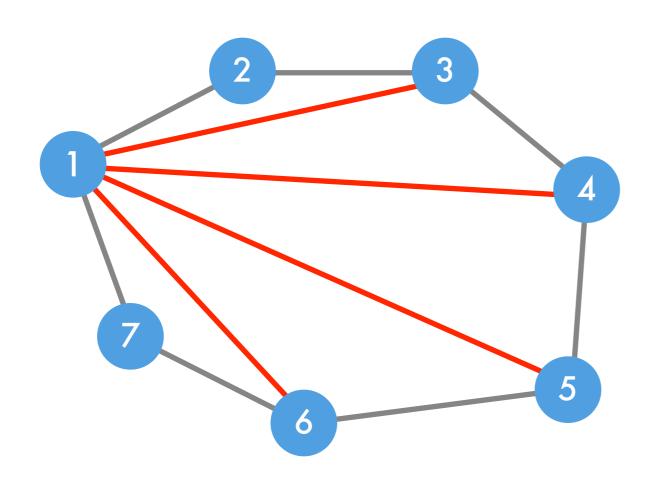
message passing possible

#### Triangulation Examples



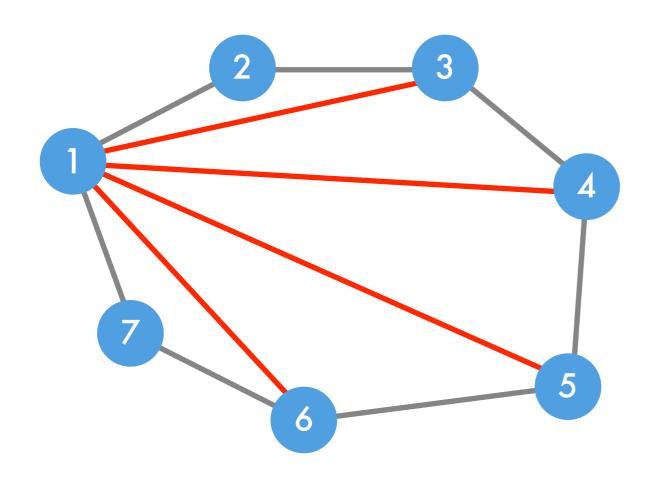
- Clique size increases
- Separator set size increases

### Triangulation Examples



- Clique size increases
- Separator set size increases

### Triangulation Examples



- Clique size increases
- Separator set size increases



## Message Passing



#### Joint Probability

$$p(x) \propto \psi(x_1, x_2, x_3) \psi(x_1, x_3, x_4) \psi(x_1, x_4, x_5) \psi(x_1, x_5, x_6) \psi(x_1, x_6, x_7)$$

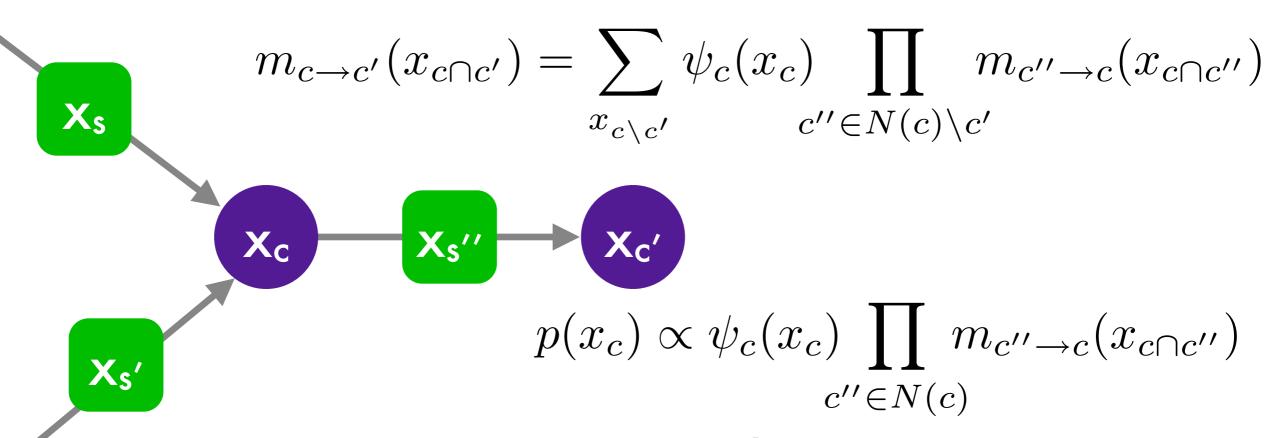
#### Computing the normalization

$$m_{\to}(x_1, x_3) = \sum_{x_2} \psi(x_1, x_2, x_3)$$

$$m_{\to}(x_1, x_4) = \sum_{x_3} m_{\to}(x_1, x_3) \psi(x_1, x_3, x_4)$$

$$m_{\to}(x_1, x_5) = \sum_{x_4} m_{\to}(x_1, x_4) \psi(x_1, x_4, x_5)$$

#### Message Passing



- Initialize messages with 1
- Guaranteed to converge for (junction) trees
- Works well in practice even for loopy graphs
- Only local computations are required

#### Message Passing in Practice

- Incoming messages contain aggregate uncertainty from neighboring random variables
- Message passing combines and transmits this information in both directions

crawler

phase 1 ranker

tiering



#### Message Passing in Practice

- Incoming messages contain aggregate uncertainty from neighboring random variables
- Message passing combines and transmits this information in both directions



#### Related work

- Tools
  - GraphLab (CMU Guestrin, Low, Gonzalez ...)
  - Factorie (UMass McCallum & coworkers)
  - HBC (Hal Daume)
  - Variational Bayes .NET (MSR Cambridge)
- See more on alex.smola.org / blog.smola.org

#### Today's mission

#### Find structure in the data

Human understandable Improved knowledge for estimation

